Characterizing Strongly Interacting Matter at Finite Temperature: (2+1)-flavor QCD with Möbius Domain Wall Fermions

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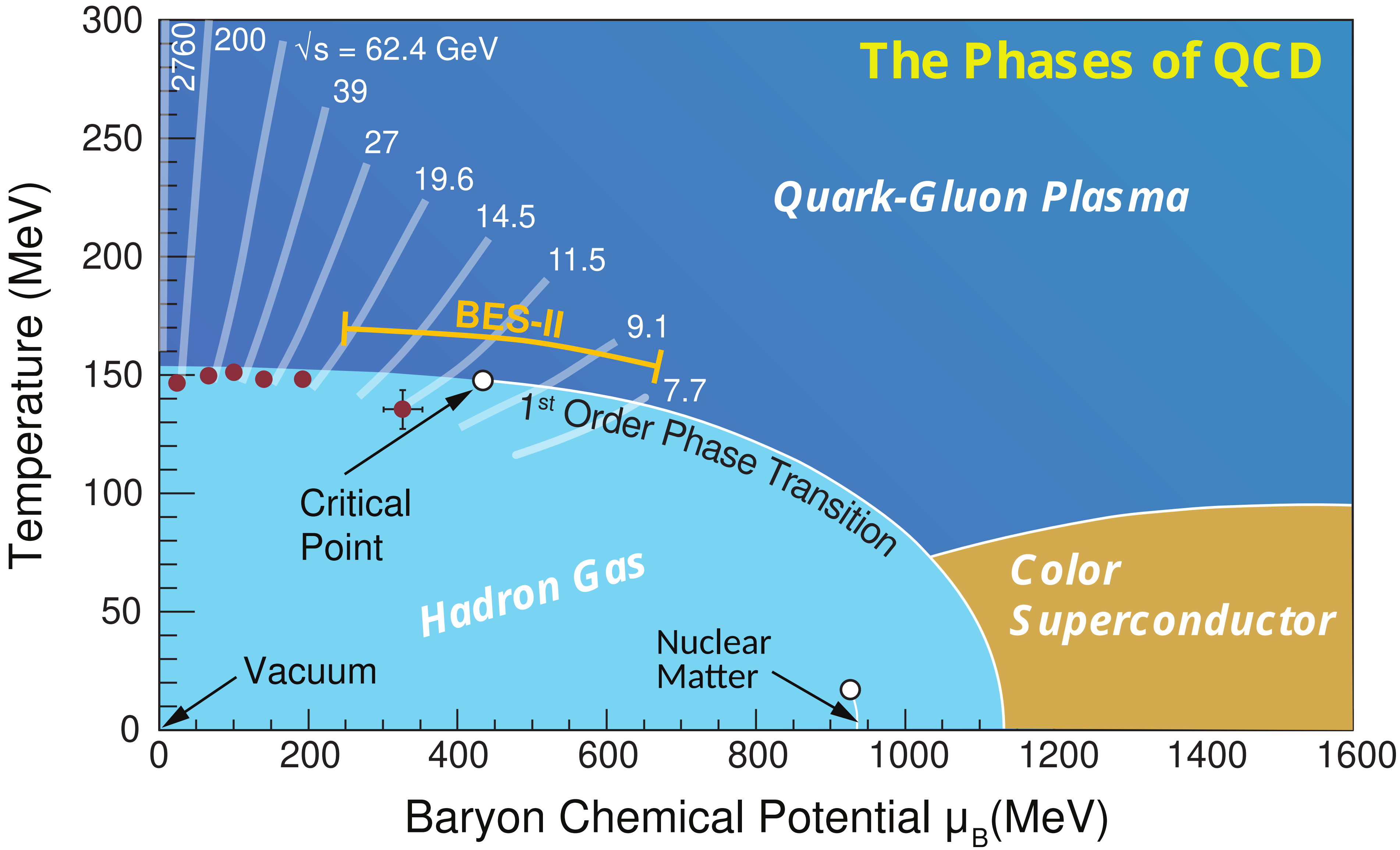
RIKEN R-CCS RIKEN Center for Computational Science

Large-scale lattice QCD simulation and application of machine learning
Center for Computational Sciences, University of Tsukuba

QCD phase diagram : fluctuation of conserved charges

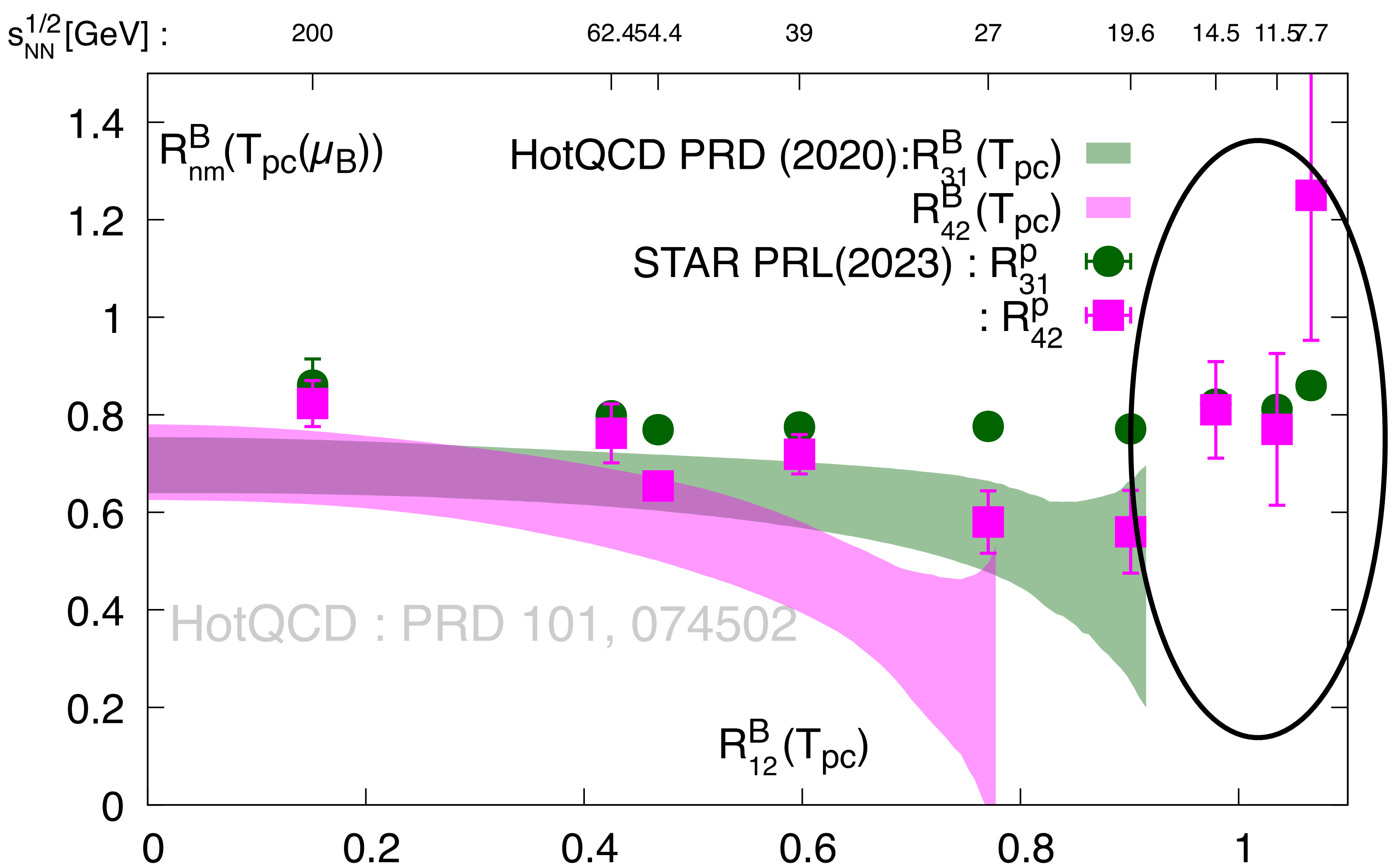
“Mapping the Phases of Quantum Chromodynamics with Beam Energy Scan”, Bzdaket al., Phys. Rept. ‘20



Conserve charge in Heavy Ion Collision experiment : Baryon number (B), Electric charge (Q) and Strangeness number (S).

A detailed review : Masayuki Asakawa, Masakiyo Kitazawa, Prog.Part.Nucl.Phys. 90 (2016) 299-342

Current status on the CEP search



A. Bazavov et al, 2001.08530 [hep-lat]

Conserved charges : Baryon number (B), Electric charge (Q) and Strangeness number (S).

Hint for a CEP ??

J. Adam *et al.* (STAR Collaboration) Phys. Rev. Lett. 126, 092301

From QCD EoS calculation

QCD CEP doesn't exist in the BESII range:

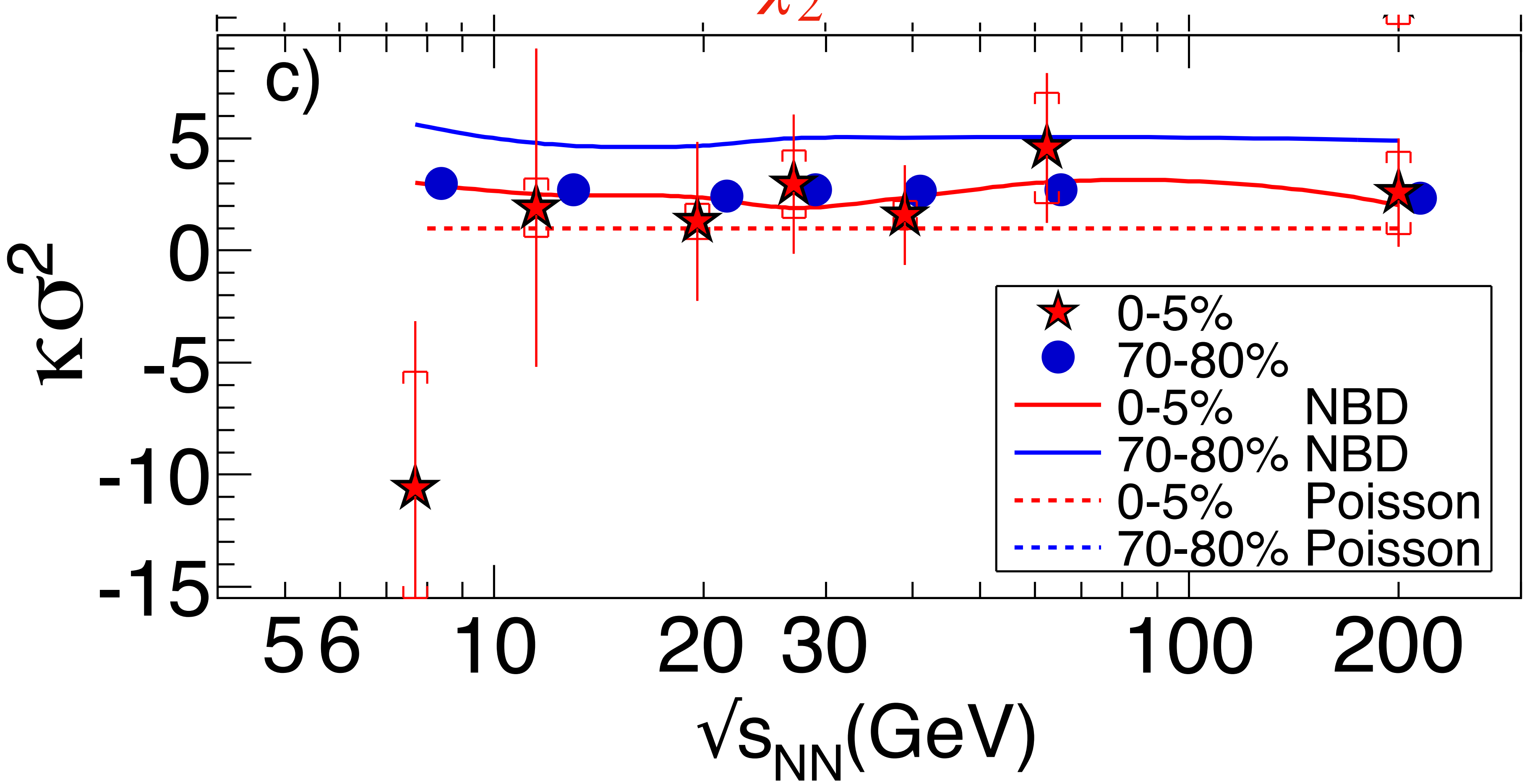
D. Bollweg et. al (HotQCD collaboration), Phys.Rev.D 105 (2022) 7, 074511,

J. Goswami , *PoS* LATTICE2022 (2023) 149

Electric charge fluctuations

Conserved charges : Baryon number (B), Electric charge (Q) and Strangeness number (S).

$$\kappa\sigma^2 = R^Q_{42}(\hat{\mu}_B, T) = \frac{\chi^Q_4}{\chi^Q_2} + O(\hat{\mu}_B^2)$$



Directly accessible in both the theory and experiment!!

L. Adamczyk *et al.* (STAR Collaboration)
Phys. Rev. Lett. 113, 092301, (2014)

A. Adare *et al.* (PHENIX Collaboration)
Phys. Rev. C 93, 011901(R) (2016)

In this talk, we will focus on calculating, χ^Q_2 using Möbius Domain Wall fermions.

Acknowledgments

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  シミュレーションでせまる基礎科学：量子新時代へのアプ

  ローチ課題番号, JPMXP102023041.

- JICFuS.
- JPS KAKENHI(JP20K0396, I. Kanamori).

And to all the JLQCD members for regular meetings and discussions.

Ongoing work and code bases

Ongoing research on QCD thermodynamics with Möbius Domain Wall fermions:
(JLQCD collaboration)

(1) Finite temperature QCD phase transition with 3 flavors.
(2) Conserved charge fluctuations, (this talk)
(3) Symmetries in $N_f = 2+1$ lattice QCD at high temperatures

Configuration generation: Grid (https://github.com/paboyle/Grid)
Measurements : (i) Hadrons (https://github.com/aportelli/Hadrons)
(ii) Bridge++ (https://bridge.kek.jp/Lattice-code/)
Data Analysis : https://github.com/LatticeQCD/LatticeToolbox

Properties of strongly interacting matter

$T < T_{pc}$: QCD and Hadron resonance gas(HRG)

$T = T_{pc}$: QCD at the transition region; Contact to heavy ion collision data

$T > T_{pc}$: QCD and high T perturbation theory/Ideal fermi gas

Heavy Ion Collision experiment : 3 conserved charges; baryon number (B), electric charge (Q) and Strangeness number (S).

Low T : HRG
B = +/-1
Q= 0, +/-1,+/-2
S=0,+/-1

High T : Ideal fermi gas
B = +/-1/3
Q= +/-1/3,+/-2/3
S=0,+/-1

At, $T = T_{pc}$,

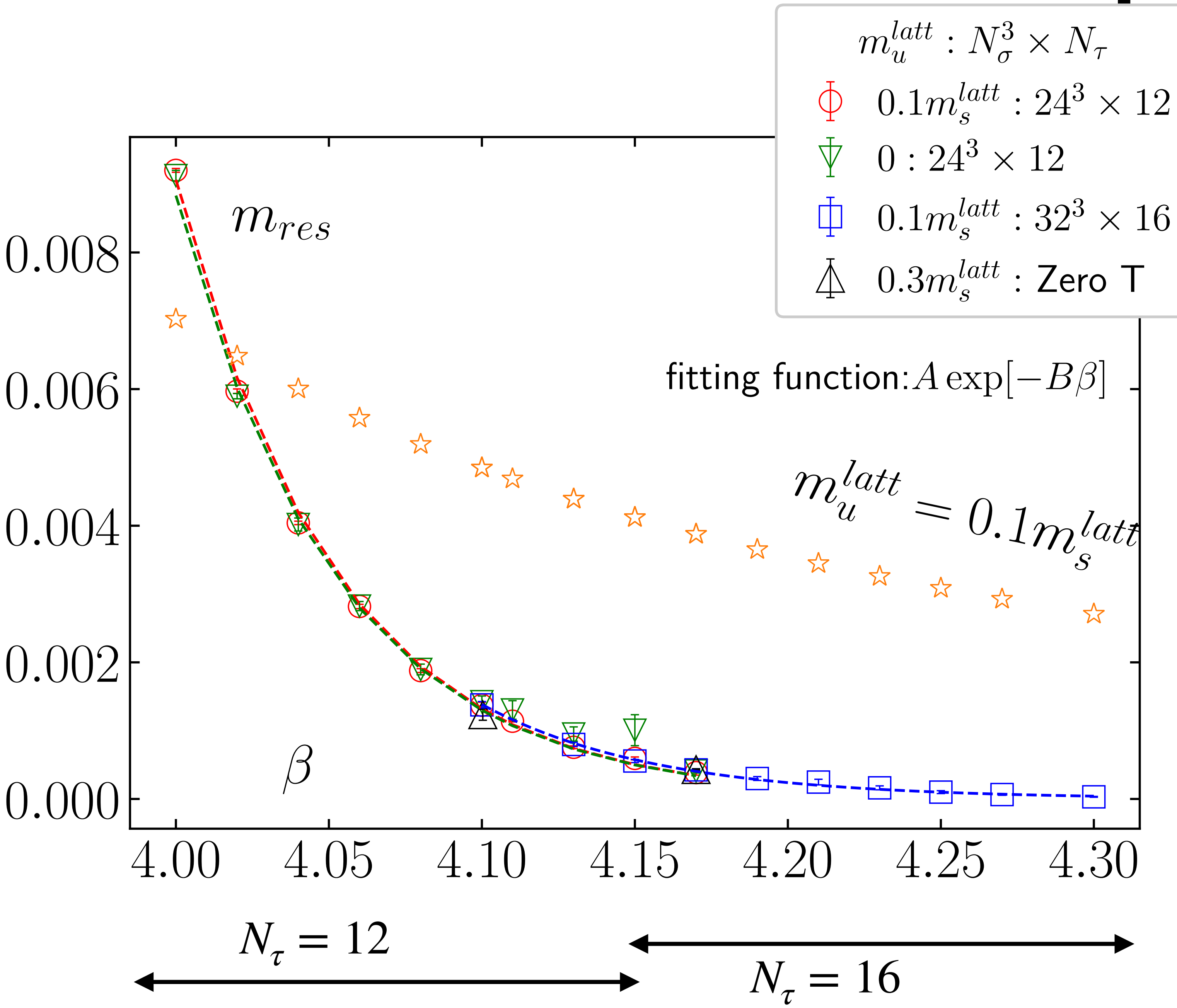
Q -fluctuations are Ideal for making contact with the heavy ion data: Detector can detect all the charge particles.

Extremely challenging in Lattice : χ_2^Q-fluctuations are dominated by the fluctuations of light pions at low T. For ex. Staggered quarks : pion spectrum distorted (i.e. $m_\pi \sim m_\pi^{RMS}$, depends on the lattice spacing).

S-fluctuations also suffer from similar effect, however effects are small as,$m_K > m_\pi$

We propose Möbius Domain Wall Fermions

Tuning of the bare input quark masses on the line of constant physics (LCP)



Tuning of bare input quark masses (m_f^{input}) in the Domain Wall action:

$$m_f^{latt} = m_f^{input} + m_{res} \; , \; f = \{u, d, s\}$$

Y. Aoki et al, *PoS* LATTICE2021 (2022) 609

$$\frac{m_u^{latt}}{m_s^{latt}} = \frac{m_u^{input} + m_{res}}{m_s^{input} + m_{res}} = 0.1$$

For, $N_\tau = 12$, $m_u^{latt} \leq m_{res}$. We tune the m_u^{input} in the LCP.

For, $N_\tau = 16$, $m_u^{latt} > m_{res}$, we perform mass reweighting.

Quark number susceptibility with Domain wall fermions

$$Z = \int DU \prod_{f=u,d,s} detM(m_f)\exp[-S_g], \qquad \det M(m_f, \hat{\mu}_f) = \left[\frac{\det D(m_f, \hat{\mu}_f)^{DWF}}{\det D(m_{PV}, \hat{\mu}_f)^{DWF}}\right]$$

$$U_4(x) \to \exp(\hat{\mu}_f)U_4(x),\ U_4^\dagger(x) \to \exp(-\hat{\mu}_f)U_4^\dagger(x),$$

J. Bloch and T. Wettig, Phys. Rev. Lett. 97, 012003 (2006)

$\hat{\mu}_f = \mu_f/T$, **where , μ_f is the quark chemical potential for flavor f.**

The diagonal and off-diagonal quark number susceptibilities can be written as,

$$\chi_2^f = \frac{N_\tau}{N_\sigma^3}\frac{\partial^2 \ln Z}{\partial \hat{\mu}_f^2}\bigg|_{\hat{\mu}_f=0} = \frac{N_\tau}{N_\sigma^3}\left[\left\langle \frac{\partial^2}{\partial \hat{\mu}_f^2}\ln\det M\right\rangle + \left\langle\left(\frac{\partial}{\partial \hat{\mu}_f}\ln\det M\right)^2\right\rangle\right]$$

$$= \frac{N_\tau}{N_\sigma^3}\langle D_2^f\rangle + \langle (D_1^f)^2\rangle,\ f = \{u,d,s\}$$

M. Cheng et al, Phys.Rev.D81:054510,2010 ; P. Hegde et al, PoS LATTICE2008:187,2008

$$\chi_{11}^{fg} = \frac{N_\tau}{N_\sigma^3}\frac{\partial^2 \ln Z}{\partial \hat{\mu}_f \partial \hat{\mu}_g}\bigg|_{\hat{\mu}_f=0} = \frac{N_\tau}{N_\sigma^3}\langle D_1^f D_1^g\rangle,\ f \neq g,\ f,g = \{u,d,s\}$$

$(D_1^f)^2$ and $D_1^f D_1^g$ are the most noisy part in our calculation

Stochastic trace estimation

Matrix size : $12V_5 \times 12V_5$

$V_5 = N_\sigma^3 \times N_\tau \times L_s$

Error reduction in stochastic trace estimators ??

Each trace needs proper subtraction from the unphysical degrees of freedom

$$D_1^f = \mathrm{Tr}\left[D(m_f)^{-1}\frac{dD(m_f)}{d\hat{\mu}_f} - D(m_{pv})^{-1}\frac{dD(m_{pv})}{d\hat{\mu}_f}\right]$$

$$D_1^f = \frac{1}{N_n}\sum_j^{N_\mathrm{n}}\left[\eta_j^\dagger D(m_f)^{-1}\frac{dD(m_f)}{d\hat{\mu}_f}\eta_j - \eta_j^\dagger D(m_{pv})^{-1}\frac{dD(m_{pv})}{d\hat{\mu}_f}\eta_j\right]$$

η_j **is the gaussian random noise.**

Stochastic error reduction using dilution vectors :

$$D_1^f = \frac{1}{N_n}\sum_j^{N_\mathrm{n}}\left[\sum_{a=1}^{N_p}\eta_{aj}^\dagger D(m_f)^{-1}\frac{dD(m_f)}{d\mu_f}\eta_{aj} - \sum_{a=1}^{N_p}\eta_{aj}^\dagger D(m_{pv})^{-1}\frac{dD(m_{pv})}{d\mu_f}\eta_{aj}\right]$$

η_{aj} **is the diluted gaussian random noise.**

$(D_1^f)^2$ we need to employ the unbiased estimator method.

We present calculations of qns and charge fluctuations for the lattice size,

$$N_\sigma^3 \times N_\tau \times L_s = 24^3 \times 12 \times 12,\ 32^3 \times 16 \times 12$$

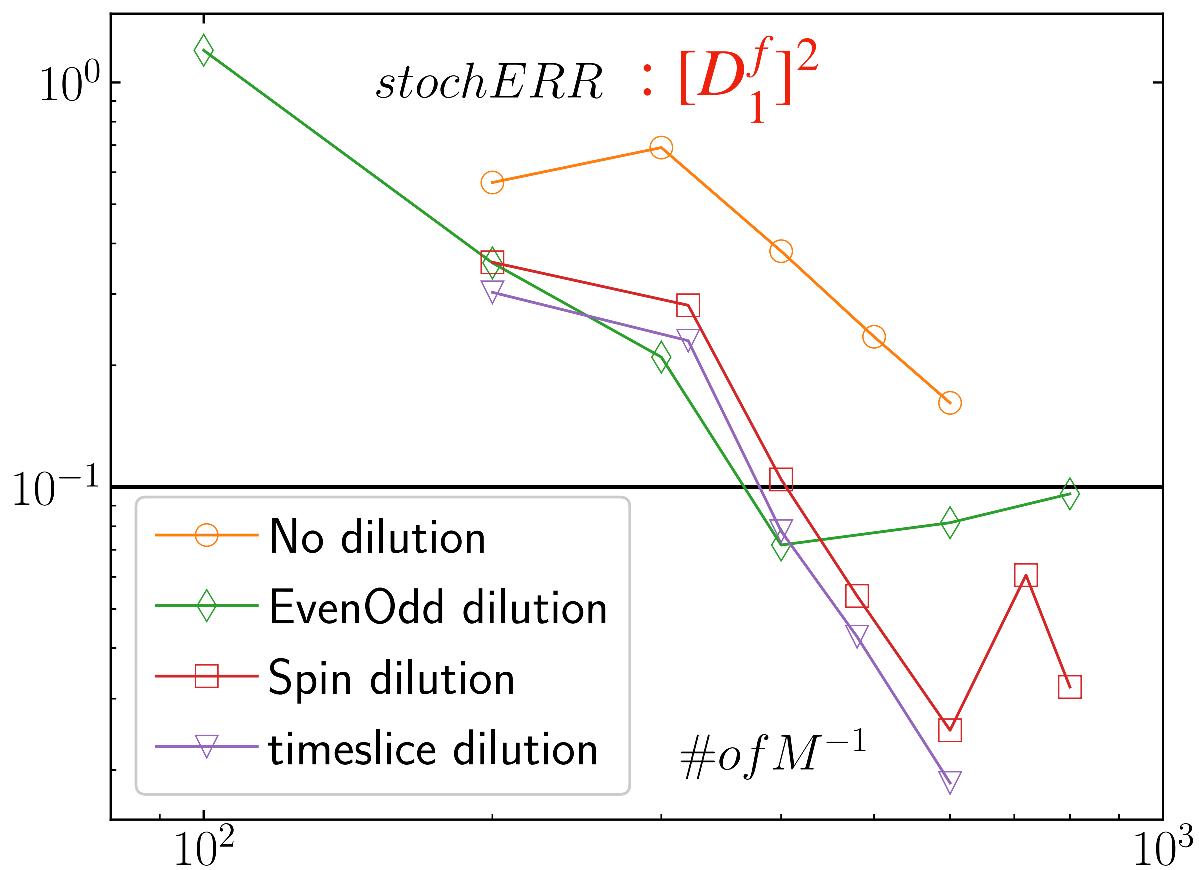
Stochastic error reduction

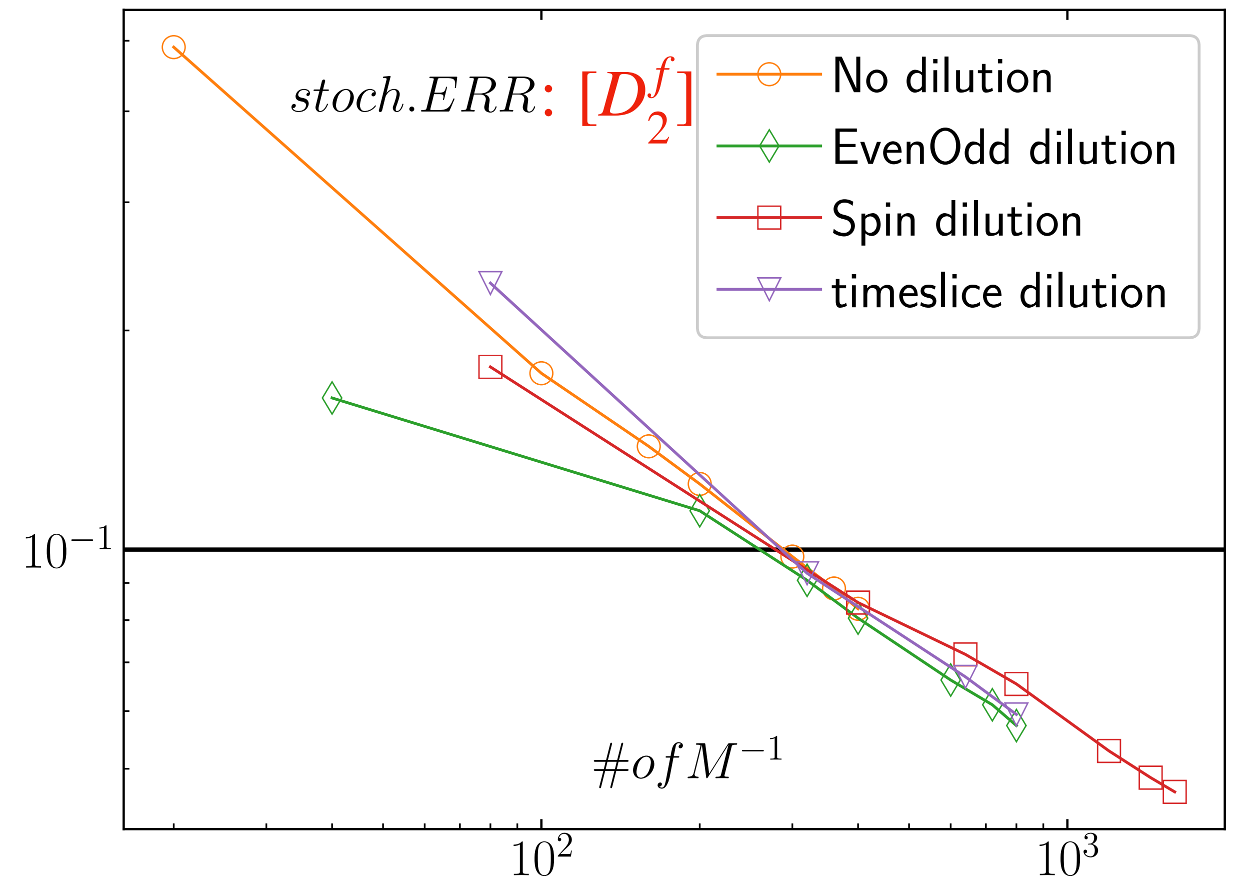
We examine three dilution methods:

(i) Even Odd dilution : splitting the η_j into two parts, in even and Odd lattice sites.

(ii) Spin dilution : splitting the η_j into four spinor components.

(iii) Timeslice dilution : splitting the η_j into four parts, using $(N_\tau \mod 4)$.

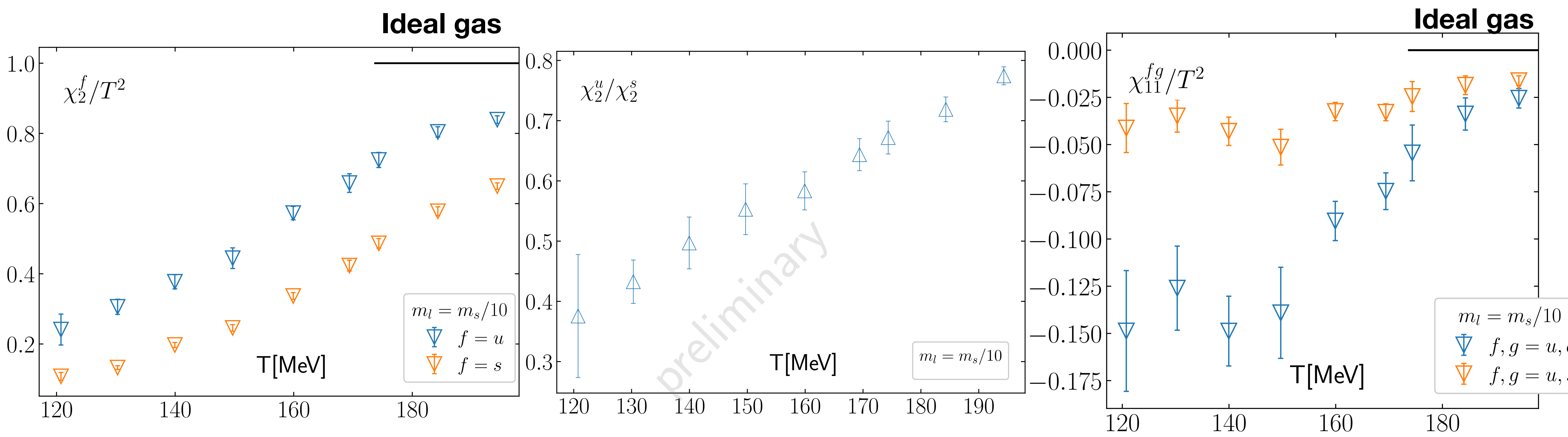




Spin and timeslice dilution : Efficient for $(D_1^f)^2$, $2-3$ times error reduction.

EvenOdd dilution : Efficient for (D_2^f).

Quark number susceptibility: $24^3 \times 12 \times 12$



Spin dilution method and 150 gaussian random noises for $(D_1^f)^2$.

Even-Odd dilution and 100 gaussian random noises for D_2^f .

χ_2^f's rise rapidly in the vicinity of the T_{pc}.

At high T: χ_2^f's are smaller than the Ideal gas limit.

χ_{11}^{fg} reaches closer to Ideal gas limit.

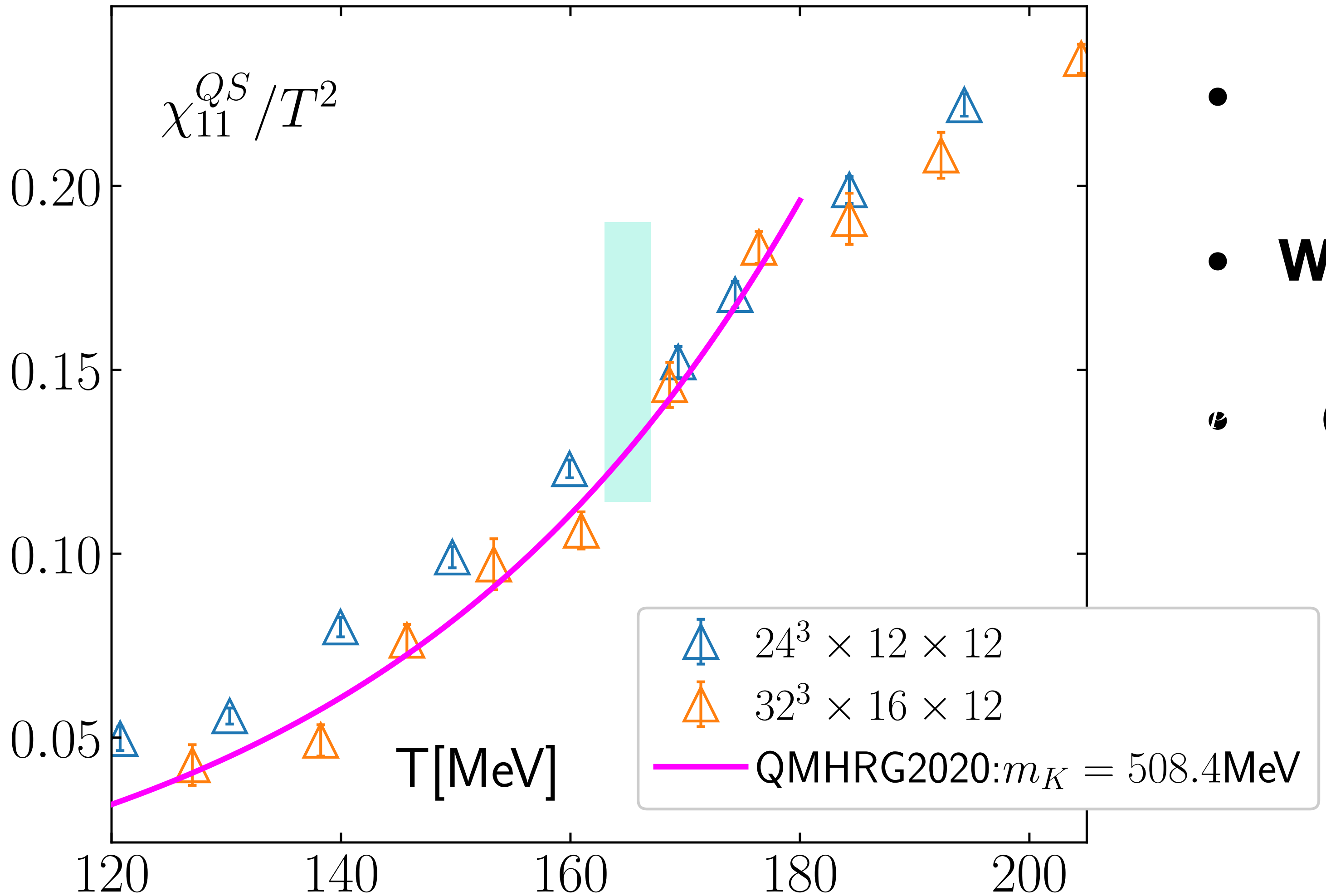
In high T PT : $\chi_2^f \sim \chi_2^{f,ideal} + O(g^2)$, $\chi_{11}^{fg} \sim O(g^6 lng)$ A. Vuorinen, PRD68, 054017 (2003)

χ_2^u/χ_2^s **: Reduction in the mass difference between u, s quarks at high temperatures.**

Results are qualitatively consistent with,
S. Borsanyi et al, JHEP 1201 (2012) 138

Conserved charge fluctuations : electric charge-strangeness correlations

$$\chi_{11}^{QS} = \frac{1}{3}(\chi_2^s - \chi_{11}^{us})$$



- At $T < T_{pc}$, χ_{11}^{QS} is dominated by the ground state kaons and $K^*(892)$.
- We use LO Chpt to estimate the ground state for $m_l = m_s/10$.
- Good agreement between QCD data and the HRG curve for $T \leq T_{pc}$.

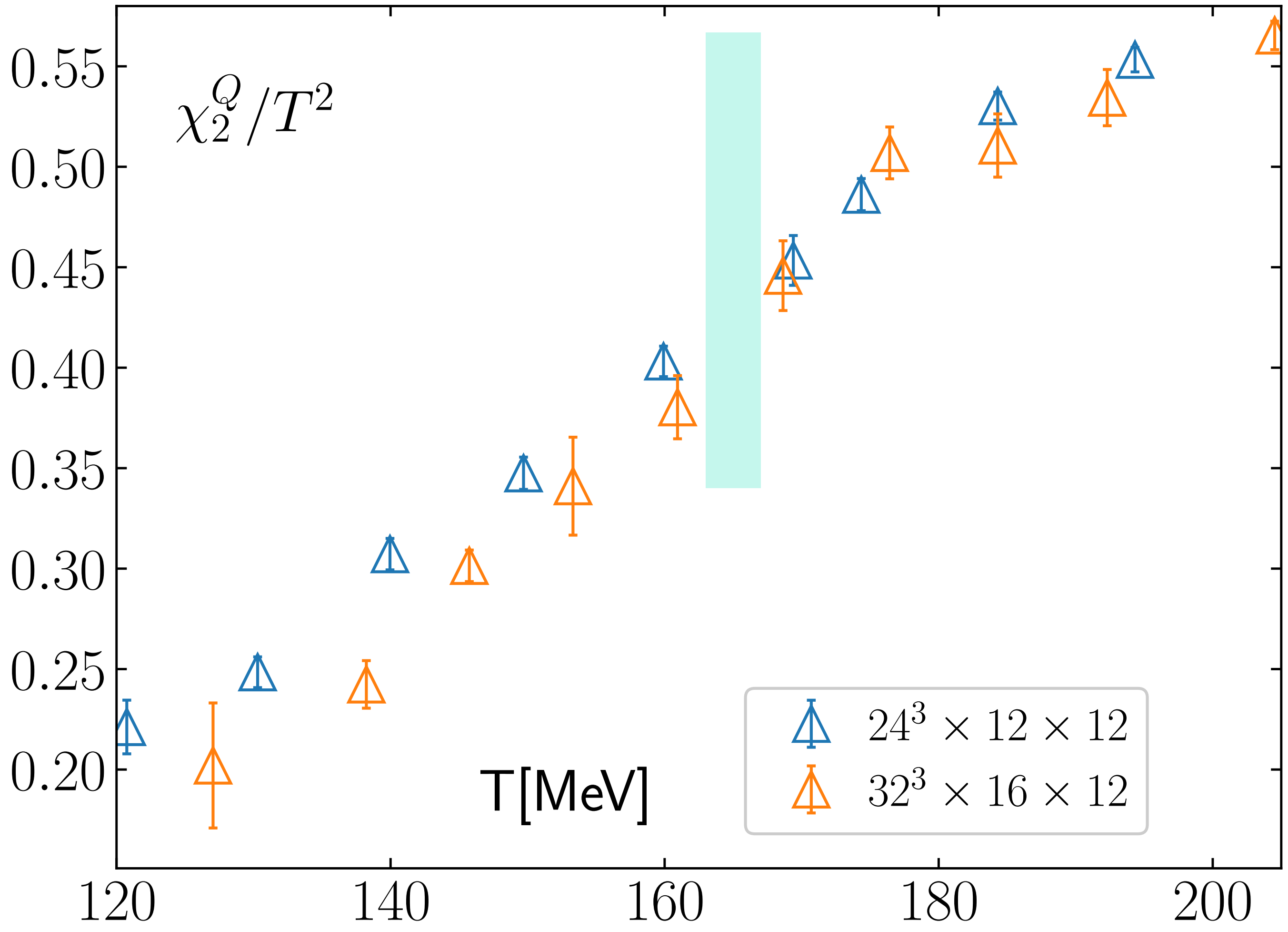
$N_\tau = 16$, data needs more statistics for understanding the cut-off effects.

QMHRG2020 : D. Bollweg el al, *Phys.Rev.D* 104 (2021) 7, 074512

R. Bellwied et al, Phys. Rev. D 92, 114505 (2015)
D. Bollweg el al, *Phys.Rev.D* 104 (2021) 7, 074512

Conserved charge fluctuations : electric charge cumulant

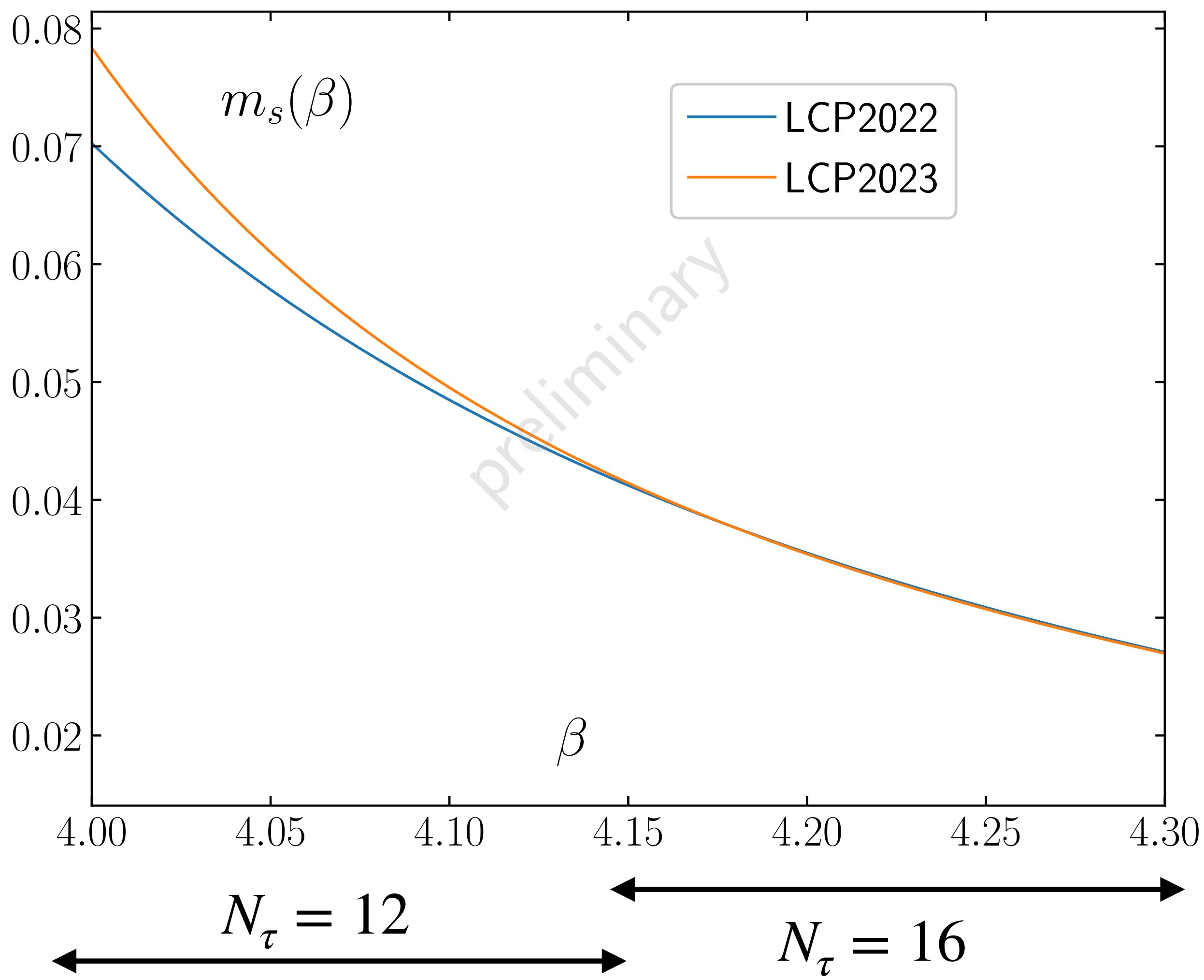
$$\chi_2^Q = \frac{1}{9}(5\chi_2^u + \chi_2^s - 4\chi_{11}^{ud} - 2\chi_{11}^{us})$$



- **Non interacting HRG framework, At low T, χ_2^Q is dominated by charged pions and Kaons.**
- **Using LO chpt we obtained pion mass for slightly heavy light quarks ($m_l = 0.1 m_s$) is 223 MeV.**
- **We will use HRG to understand the cut-off effects in this observables.**

Line of constant physics

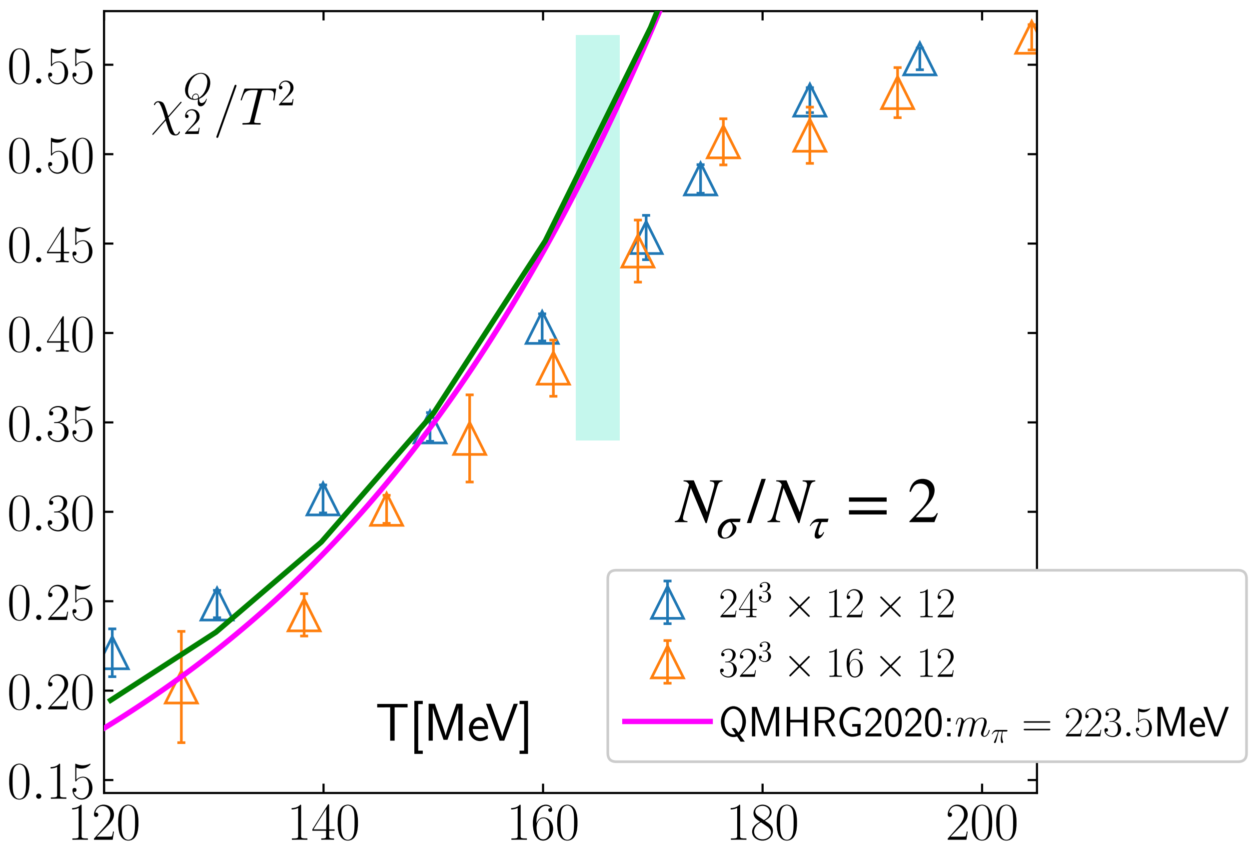
$N_\tau = 12$, the LCP deviates at the lower temperatures. Which will lead to smaller pion and Kaon masses.



- **Non interacting HRG framework, At low T, χ_2^Q is dominated by charged pions and Kaons.**
- **Using LO chpt we obtained pion mass for slightly heavy light quarks ($m_l = 0.1 m_s$) is 223 MeV.**
- **We also use the LO chpt to estimate the ground state pion and Kaons.**

Conserved charge fluctuations : electric charge cumulant

$$\chi_2^Q = \frac{1}{9}(5\chi_2^u + \chi_2^s - 4\chi_{11}^{ud} - 2\chi_{11}^{us})$$



- At low temperatures, χ_2^Q is dominated by pions.
- Using LO chpt we obtained pion mass for slightly heavy light quarks ($m_l = 0.1 m_s$) is $223\ \mathrm{MeV}$.

- Unlike calculations with staggered fermions, we see a good agreement with hadron resonance gas (HRG) with χ_2^Q at $T < T_{pc}$at $N_\tau = 12$.
- However, close to the T_{pc} the deviations seems to be robust.

D. Bollweg el al, *Phys.Rev.D* 104 (2021) 7, 074512

$N_\tau = 16$, data needs more statistics for better understanding the cut-off effects.

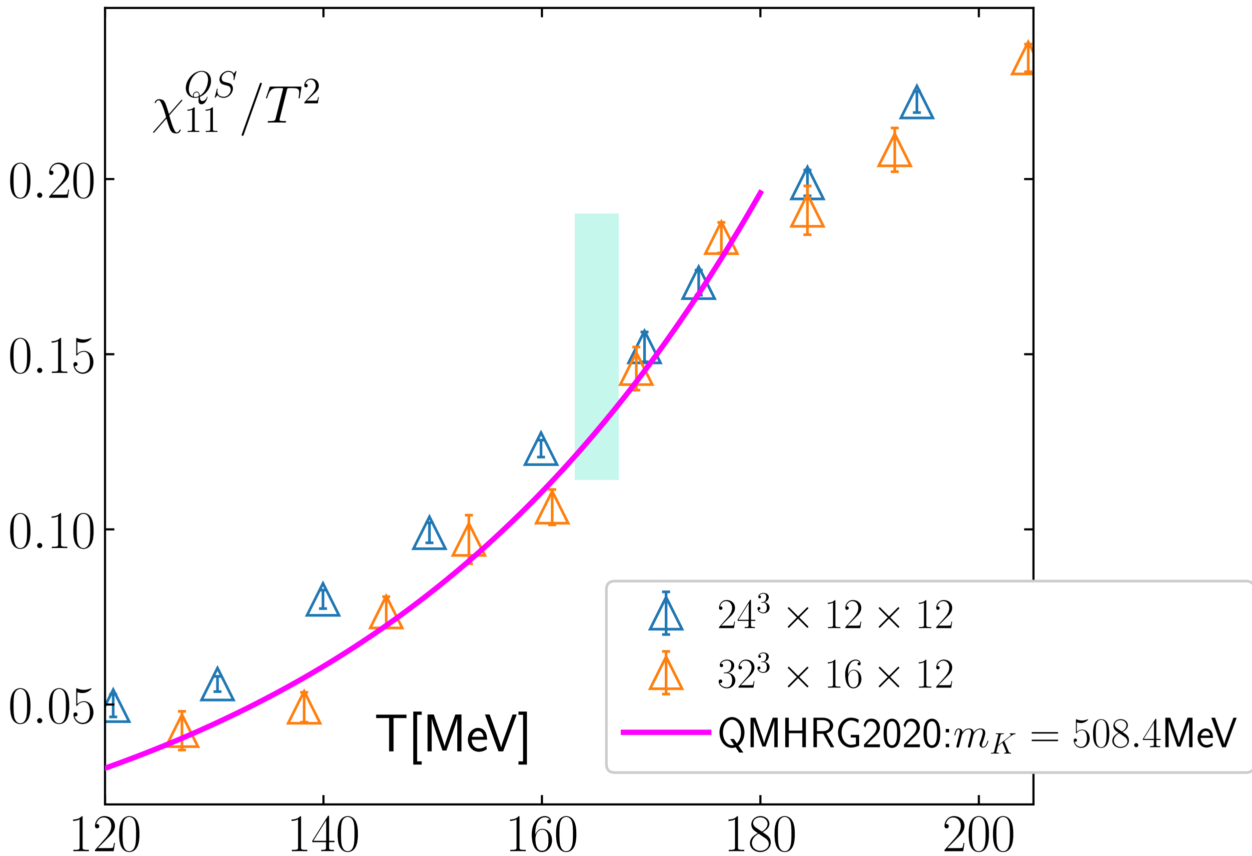
Summary and future work

- We present preliminary results of conserved charge fluctuations using (2+1)-flavor QCD with a chiral fermion formalism, specifically Möbius Domain Wall Fermions.
- For benchmarking and understanding all the systematics of Möbius Domain Wall Fermions, we consider slightly heavier light quark mass on the line of constant physics.
- We also compare our results with HRG (Hadron Resonance Gas) model.
- Calculations with physical quark masses are currently ongoing.

Thank you for your attention !!

Conserved charge fluctuations : electric charge cumulant

$$\chi_2^Q = \frac{1}{9}(5\chi_2^u + \chi_2^s - 4\chi_{11}^{ud} - 2\chi_{11}^{us})$$



$N_\tau = 16$**, data needs more statistics for understanding the cut-off effects.**