Axial structure of the nucleon in large-volume lattice QCD at the physical point

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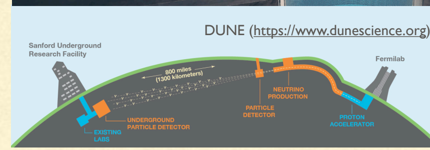
Introduction

Neutrino oscillation and new physics

Seek to answer fundamental questions:

- matter-antimatter asymmetry
- mass ordering
- PMNS unitarity e.t.c.



e.g. Electron neutrino appearance $P_{\nu_\mu \to \nu_e}$

$$P_{\nu_\mu \to \nu_e} \simeq \frac{(\nu_e\text{-flux at the far detector})}{(\nu_\mu\text{-flux at the near detector})}$$

Observable : Interaction rate with atomic nuclei in detectors

$$E_{\rm rec} = E_l + \sum T_i^N + \epsilon_n + \sum_j E_j$$

Neutrino interaction cross section for process i
e.g. quasi-elastic scattering, resonance production
: **Theoretical model/calculation**

$$N_f(E_{\rm rec}, L) \propto \sum_i \int \Phi_f(E_\nu, L)\sigma_i(E_\nu) g_{\sigma_i}(E_\nu, E_{\rm rec}) dE_\nu$$

Interaction rate for flavor f
: **Observable**

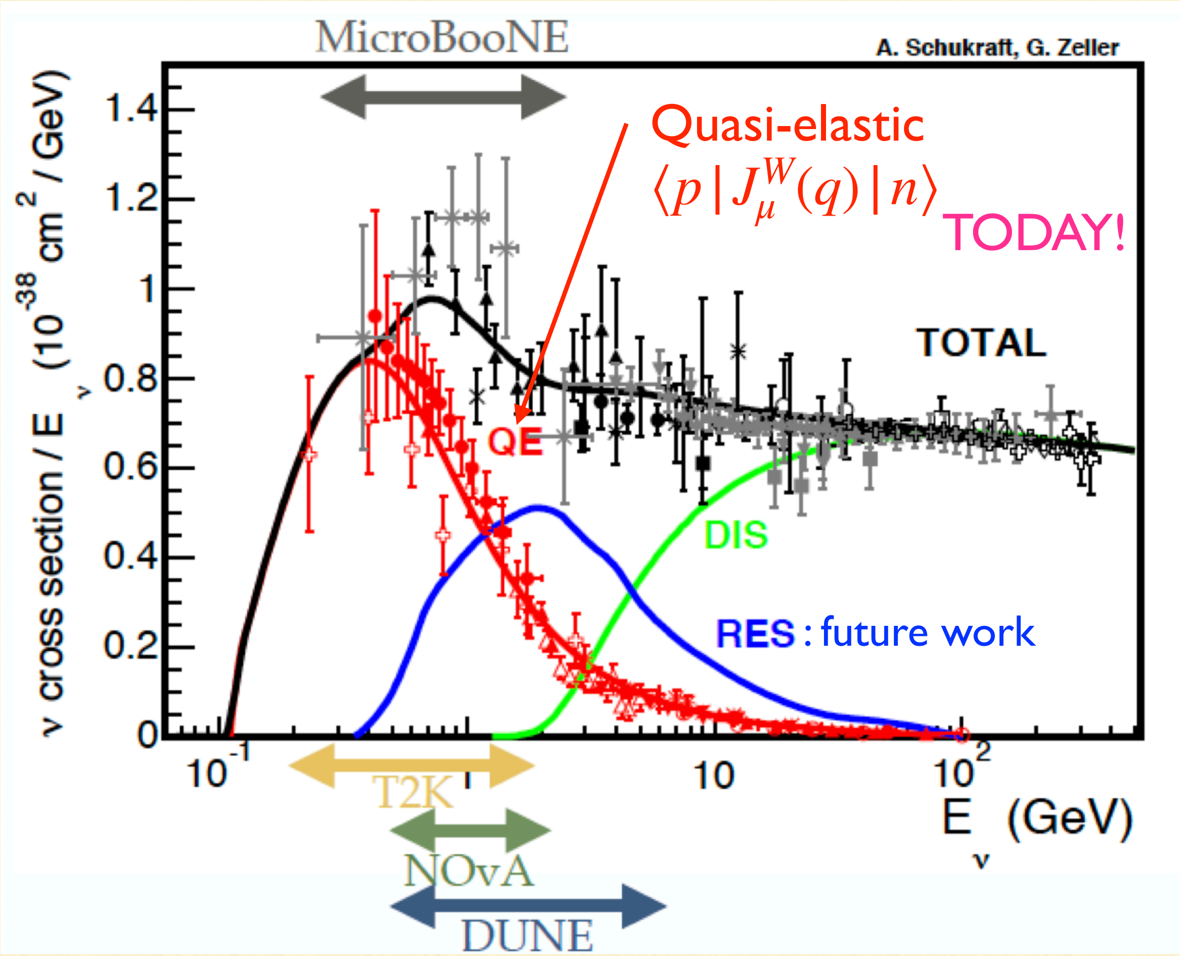
Neutrino flux

Smearing matrix the real E_ν and $E_{\rm rec}$
$E_{\rm rec} \neq E_\nu$ due to experimental and nuclear interaction effect
: **Theoretical model**

M. Khachatryan et al. Nature 599, 565-570 (2021).

Neutrino-nucleon

$$N_f(E_{\text{rec}}, L) \propto \sum_i \int \Phi_f(E_\nu, L)\sigma_i(E_\nu) g_{\sigma_i}(E_\nu, E_{\text{rec}}) dE_\nu$$



Neutrino-nucleon cross section

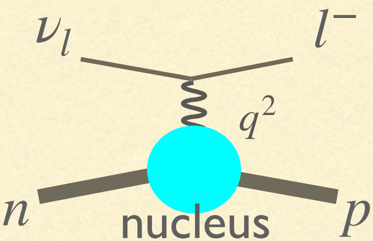
$$\frac{d\sigma}{dq^2}\left(E_\nu, q^2\right) = \frac{G_F^2\left|V_{ud}\right|^2}{2\pi}\frac{M_N^2}{E_\nu^2}\left[\left(\tau + r_\ell^2\right)A\left(\nu, q^2\right) - \frac{\nu}{M_N^2}B\left(\nu, q^2\right) + \frac{\nu^2}{M_N^4}\frac{C\left(\nu, q^2\right)}{1+\tau}\right],$$

$$A = \tau\left(G_M^v\right)^2 - \left(G_E^v\right)^2 + (1+\tau)F_A{}^2 - r_\ell^2\left[\left(G_M^v\right)^2 + F_A{}^2 + 4F_P\left(F_A - \tau F_P\right)\right],$$

$$B = 4\eta\tau G_M^v F_A,$$

$$C = \tau\left(G_M^v\right)^2 + \left(G_E^v\right)^2 + (1+\tau)F_A{}^2,$$

where $\nu = E_\nu/M_N - \tau - r_l^2$, $\tau = q^2/4M_N^2$, $\eta = \pm 1$ and $r_l = m_l/2M_N$ with a lepton mass m_l.



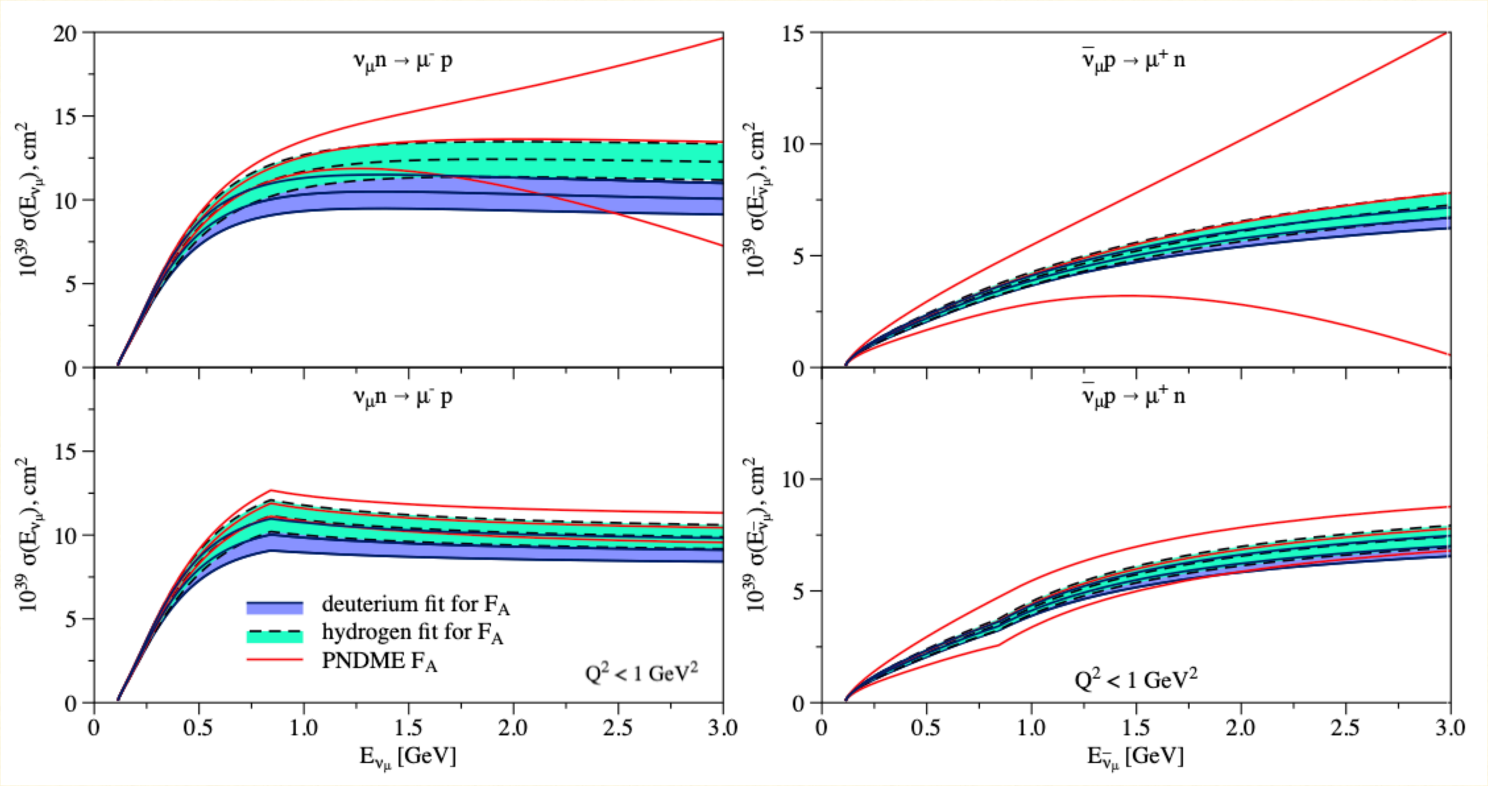
quasi-free nucleon inside nucleus

$G_E^v(q^2)$, $G_M^v(q^2)$: Electric (G_E) and magnetic (G_M) form factors
→Highly constrained by eN experiments

$F_P(q^2)$: Induced pseudoscalar form factor
→subleading in cross section, $\propto F_A(q^2)$

$F_A(q^2)$: Axial form factor
→Leading uncertainty, **Lattice QCD**

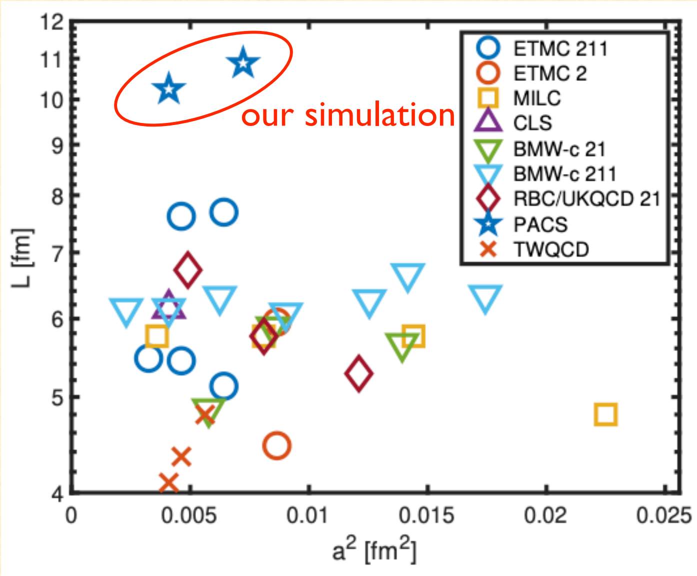
Neutrino-nucleon cross section

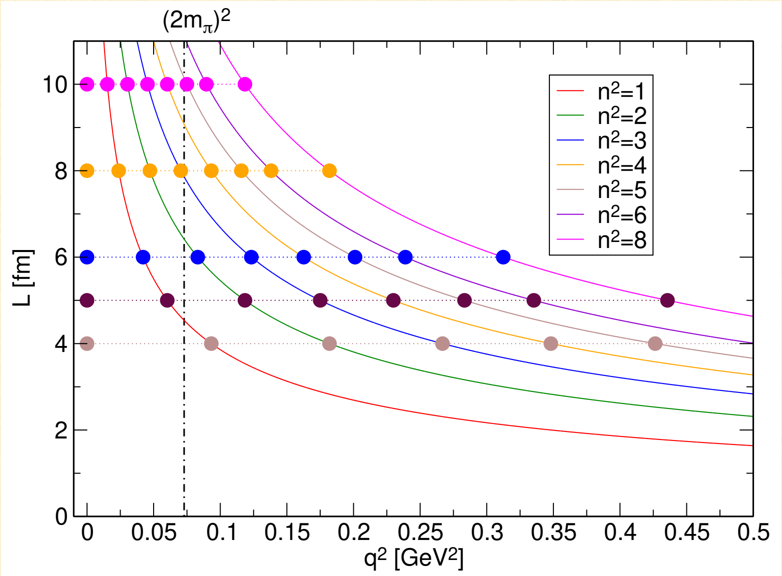


Lattice QCD : good statistical precision data for $q^2 \lesssim 0.2\ \mathrm{GeV}^2$

High precision calculation in low q^2 is demanded!

Large-volume lattice QCD



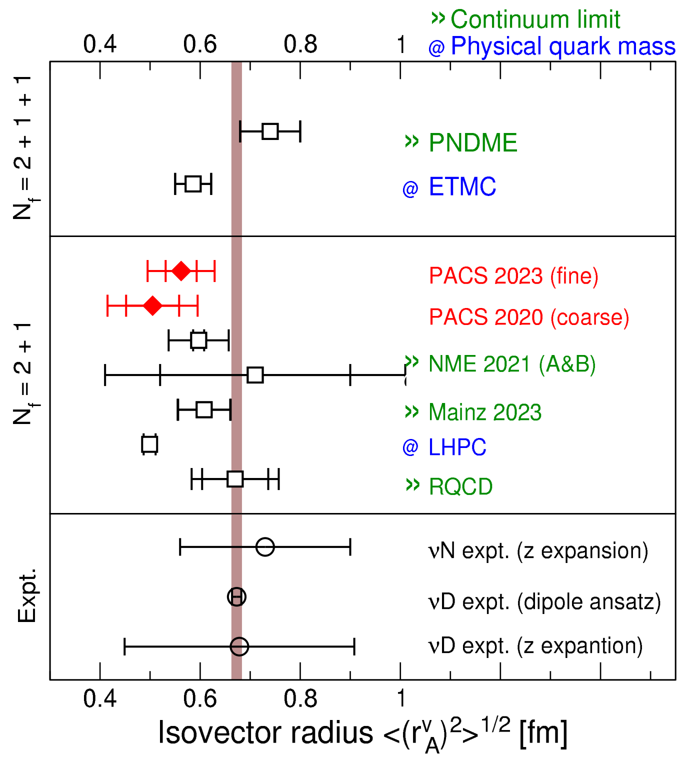


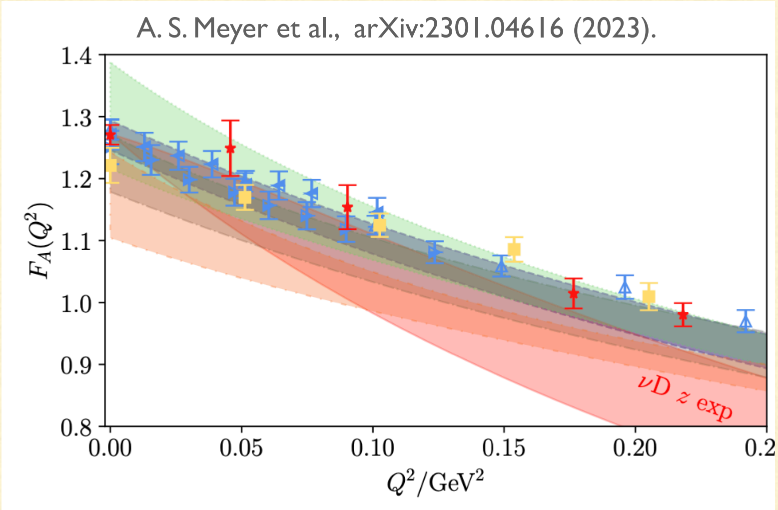
The momentum is discretized as $q^2 = \left(\frac{2\pi}{L}\right)^2 \times |\boldsymbol{n}|^2$

Low q^2 data are accessible by a large-volume lattice QCD

i.e. our simulation is the BEST to seek the low q^2 region!

Precise measurement at low-Q^2





Useful probe at low q^2 :

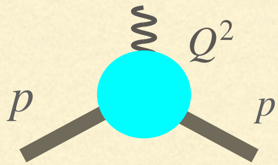
$$\langle r_A^2 \rangle = -\frac{6}{F_A(0)} \frac{dF_A}{dq^2}\bigg|_{q^2=0}$$ axial radius

PACS achieves *fully-dynamical* simulation at the physical points,

High-precision(PACS'18) + Continuum limit = second PACS(this work)

Calculation strategy for form factors

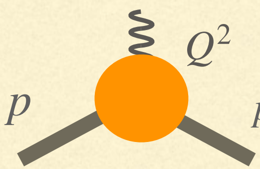
Targets : RMS radius of $G_l(q^2)$: $\sqrt{\langle r_{l^2} \rangle} = -\frac{6}{G_l(0)} \frac{dG_l(q^2)}{dq^2}\Big|_{q^2 \to 0}$



$$= \bar{u}(p')\left[\frac{(p'+p)^\mu}{2M}\frac{G_E(q^2) - \frac{q^2}{4M^2}G_M(q^2)}{1-\frac{q^2}{4M^2}} + i\sigma^{\mu\nu}\frac{q_\nu}{2M}G_M(q^2)\right]u(p)$$

$\langle N(p')|\bar{q}\gamma_\mu q|N(p)\rangle$

$\rightarrow \langle r_E^2\rangle,\ \mu,\ \langle r_M^2\rangle$



$$= \bar{u}(p')\left[\gamma_\mu\gamma_5 F_A(q^2) + iq^\mu\gamma_5 F_P(q^2)\right]u(p)$$

$\langle N(p')|\bar{q}\gamma_\mu\gamma_5 q|N(p)\rangle$ $\rightarrow \langle r_A^2\rangle$, g_A

*Local current

Determination of the RMS radius $\langle r_l^2\rangle$: z-expansion today

$$G_l(z) = \sum_{k=0}^{\infty} c_k z^k,\ z = (\sqrt{t_{\rm cut}+q^2} - \sqrt{t_{\rm cut}})/(\sqrt{t_{\rm cut}+q^2} + \sqrt{t_{\rm cut}}) \text{ with } t_{\rm cut} = \begin{cases} 4m_\pi^2 \ (l=E,M) \\ 9m_\pi^2 \ (l=A) \end{cases}$$

Numerical results

arXiv:hep-lat/2311.10345

Simulation details -PACS10 configuration[1][2]

Eliminate major uncertainties

Finite size effect
Chiral extrapolation

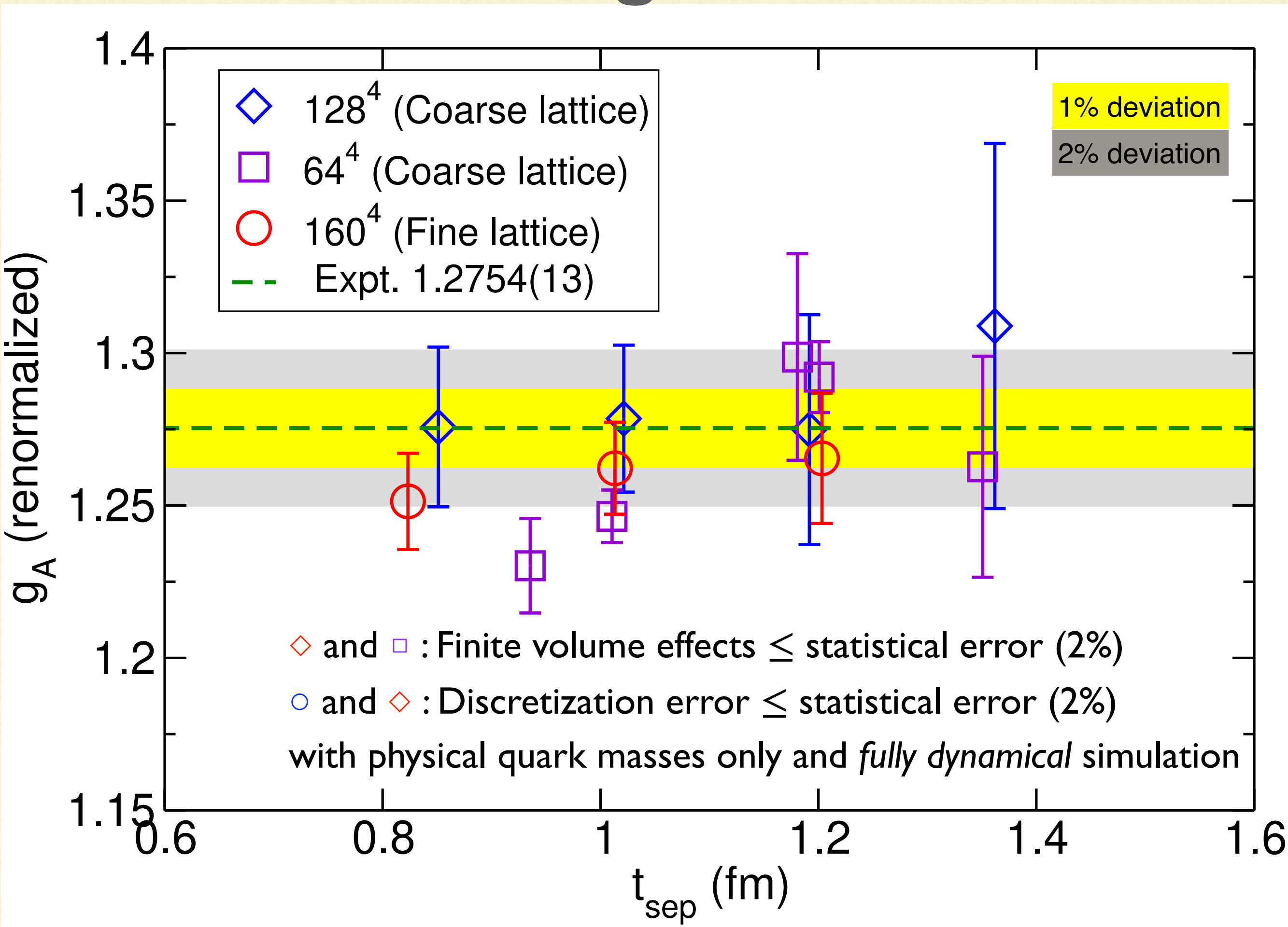
$\otimes$ Low q^2 data are accessible $q^2 = (2\pi/L)^2 \times |\boldsymbol{n}|^2$ = PACS10

Lattice size	128^4 [1]	160^4 [2]
Spacial vol. $\gg$ nucleon	$\sim (10.9\ \mathrm{fm})^3$	$\sim (10.1\ \mathrm{fm})^3$
Pion mass $\sim m_\pi^{\mathrm{exp.}}$	135 MeV	138 MeV
Nucleon mass	~ 0.935 GeV	~ 0.947 GeV
$\|t_{sink}-t_{src}\|/a$	10, 12, 14, 16	13,16,19
Lattice spacing	coarse ~ 0.085 fm	fine ~ 0.063 fm

[1] E. Shintani et al., Phys. Rev. D **99,** 014510(2019), (Erratum; Phys. Rev. D 102 (2020) 019902.)
[2] E. Shintani and Y.Kuramashi, Phys.Rev. D **100,** 034517(2019)

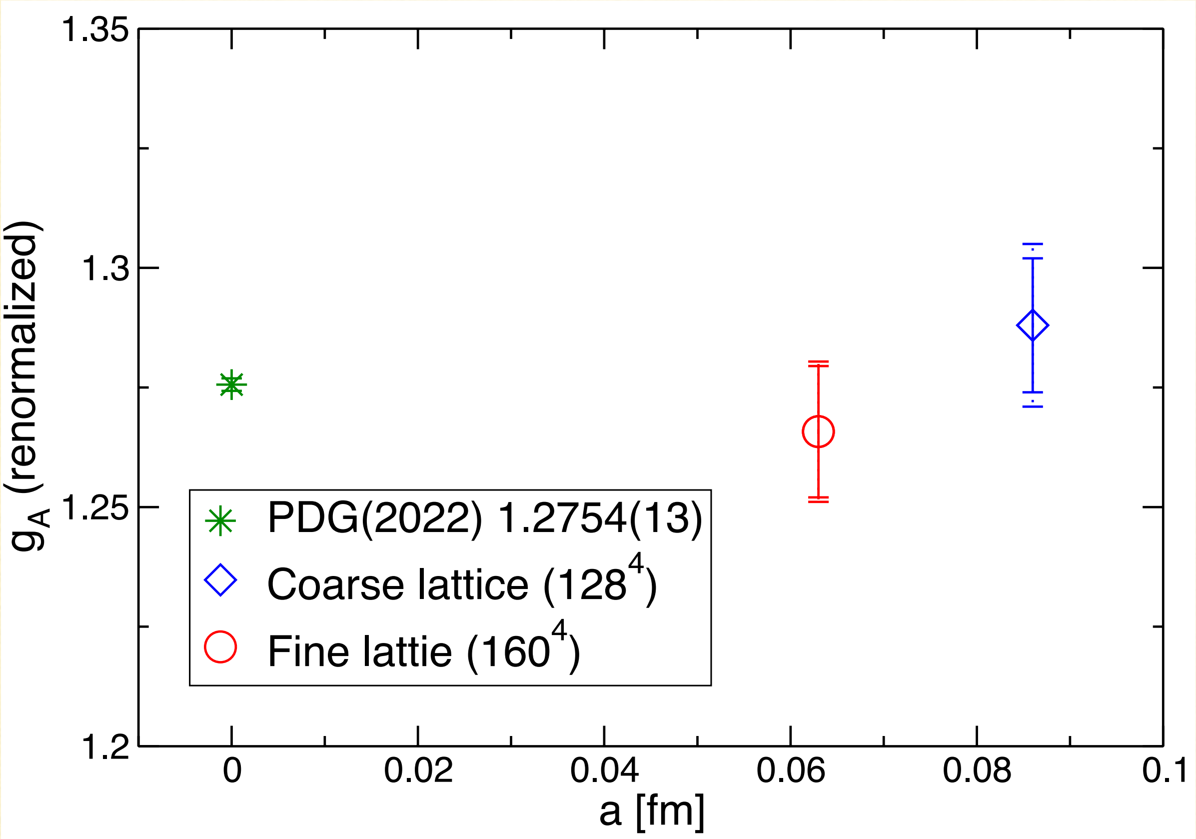
The stout-smeared $O(a)$ improved Wilson fermions and Iwasaki gauge action.

Nucleon axial charge



Nucleon axial charge

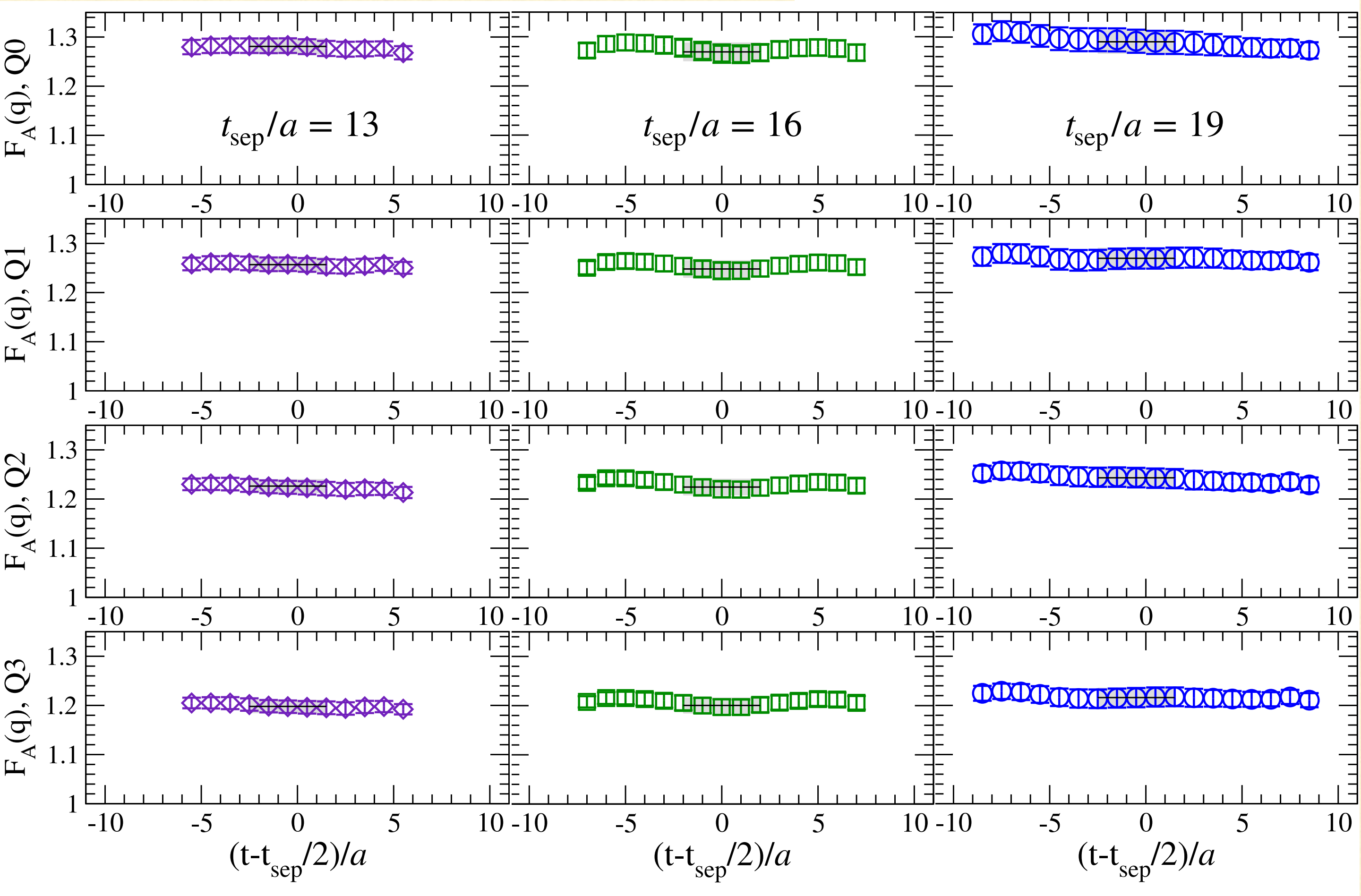
$$\text{Disc. err.} \equiv \frac{|(\text{coarse}) - (\text{fine})|}{(\text{coarse})}$$



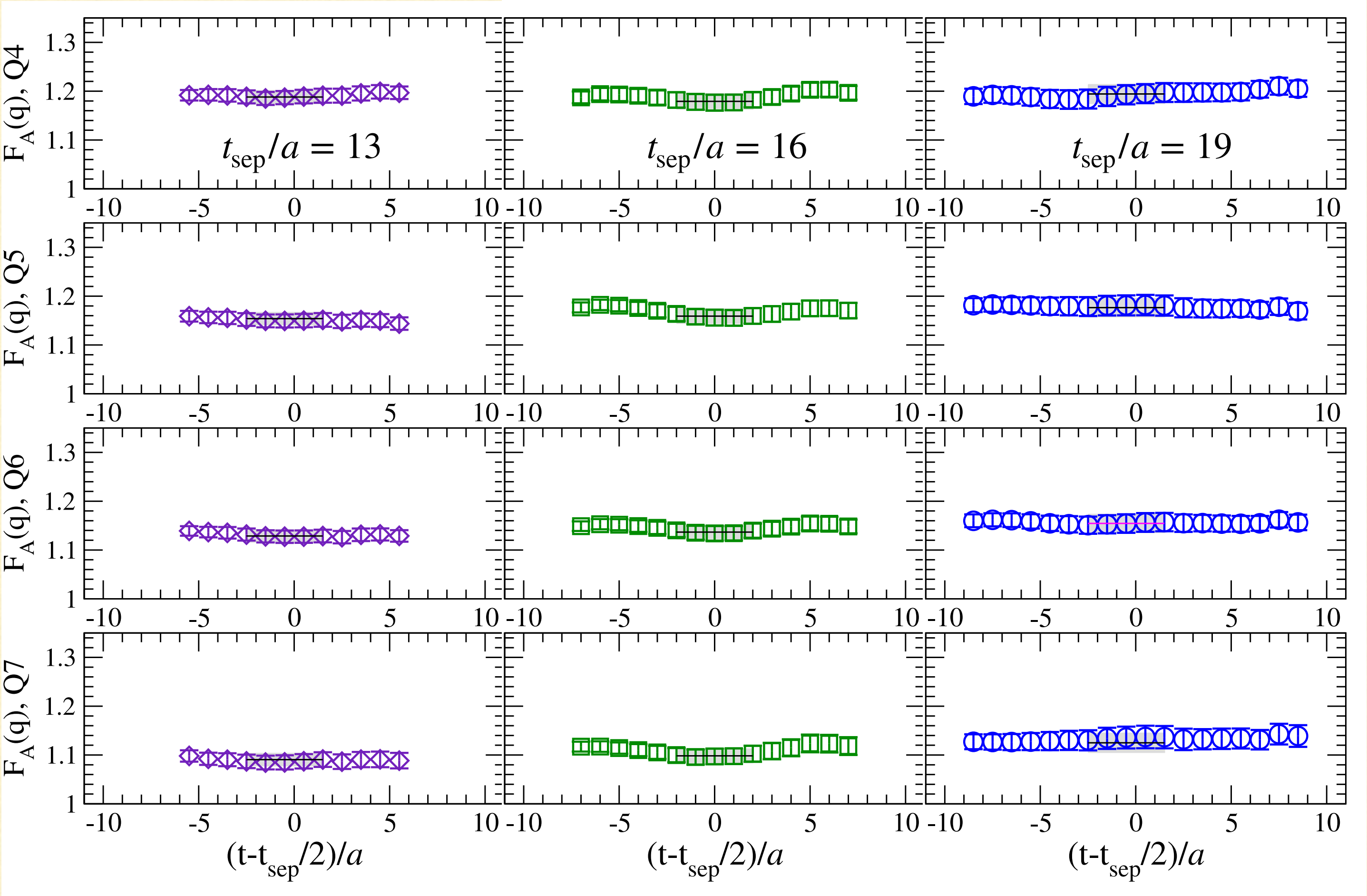
Both coarse & fine reproduce PDG within statistical errors

Discretization error (1.6%) $\lesssim$ Statistical error (1.9%)

Plateaus of F_A -$q^2 = 0.0 \sim 0.044$ GeV^2

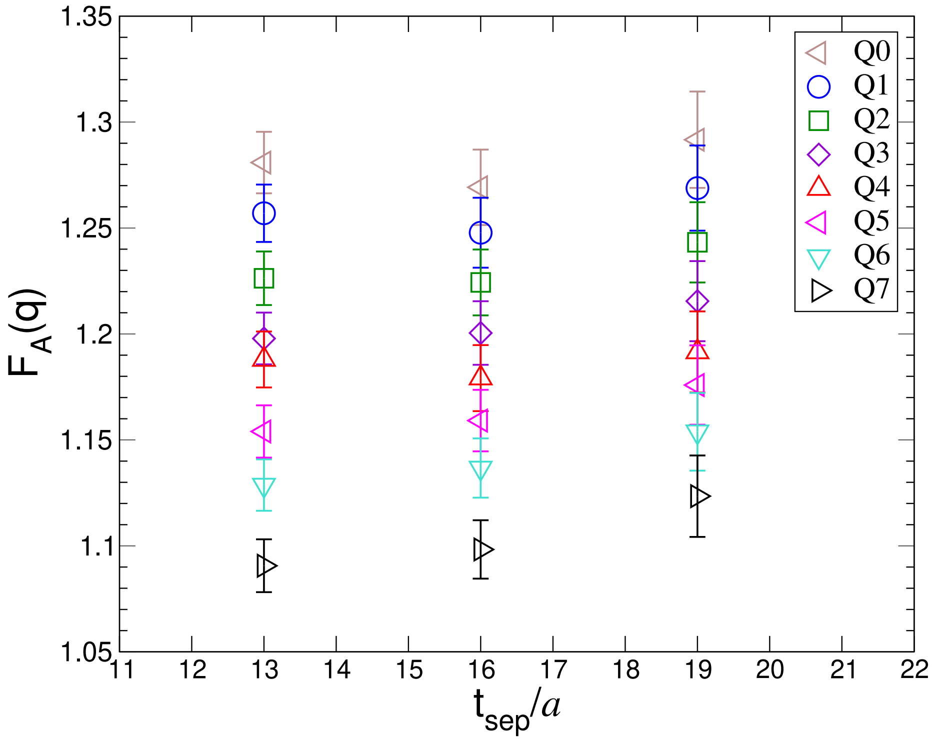


Plateaus of F_A -$q^2 = 0.059 \sim 0.116$ GeV^2



t_{sep}-dependences F_A

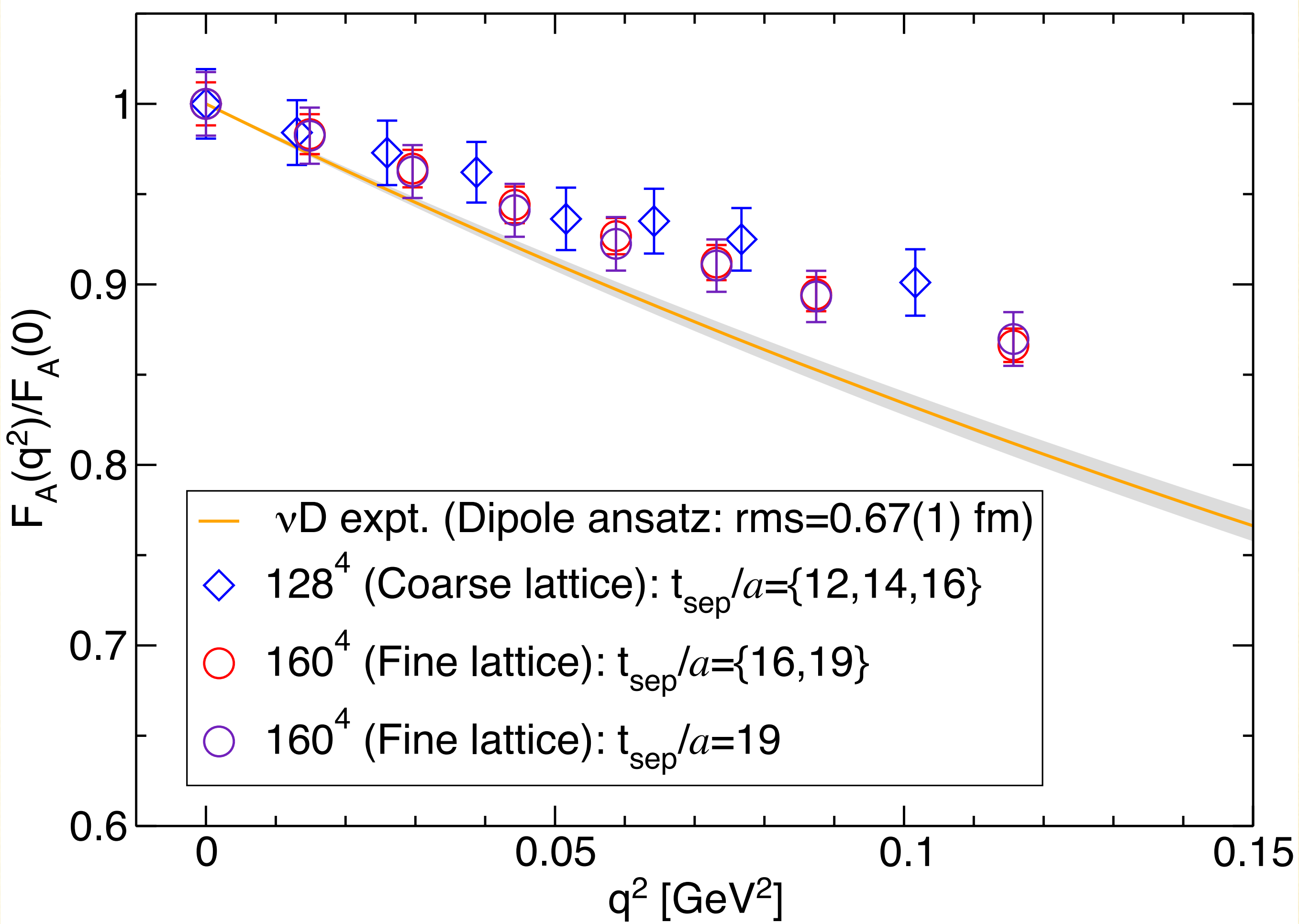
$$\langle N(p')|A_\mu(q)|N(p)\rangle = \bar{u}(p')\left[\gamma_\mu F_A(q^2) + iq^\mu F_P(q^2)\right] u(p)$$



t_{sep}-dependences < statistical err → ground-state saturation

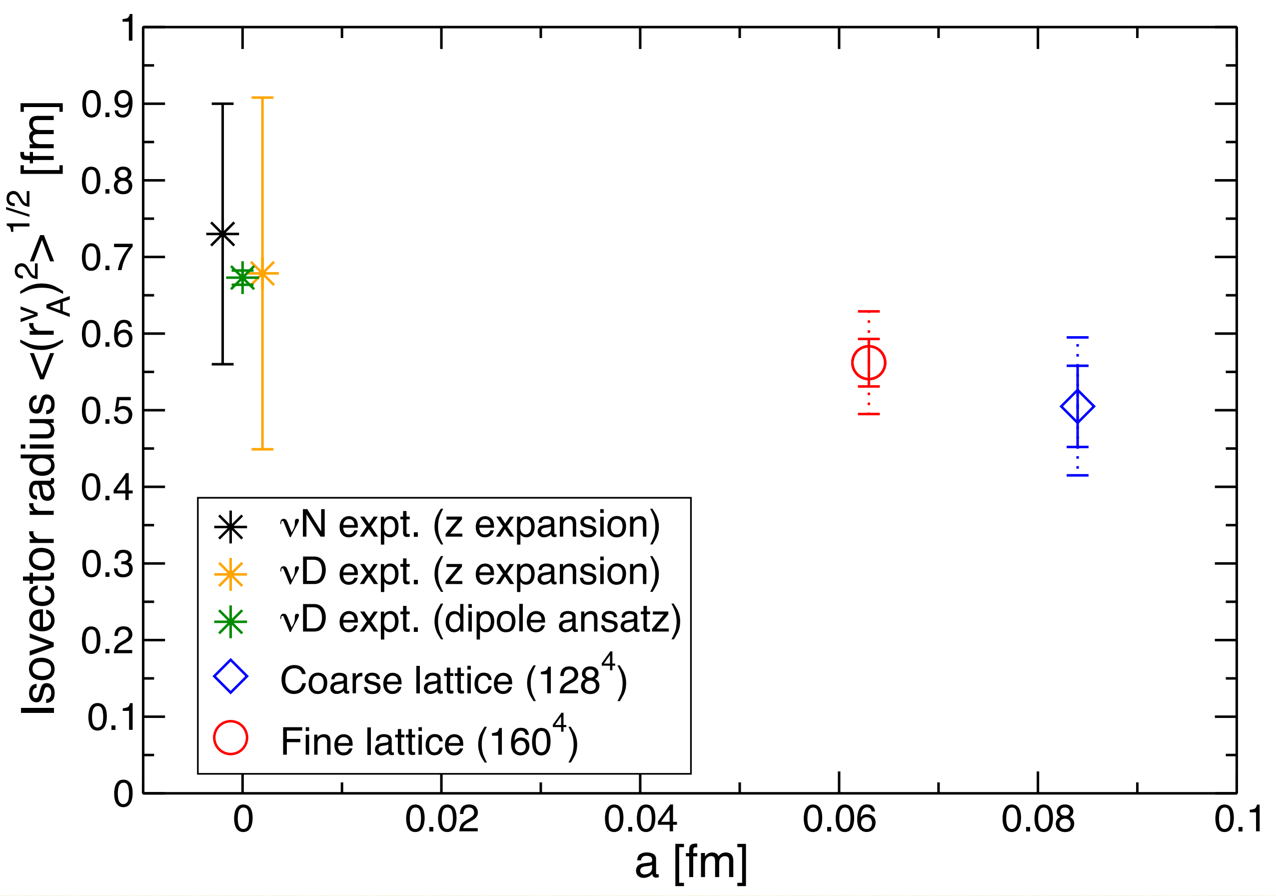
Excited-states could be unproblematic in our precision at low Q^2.

Axial form factor



Axial radius

$$\text{Disc. err.} \equiv \frac{|(\text{coarse}) - (\text{fine})|}{(\text{coarse})}$$



Statistical error (10.5%) $\lesssim$ Discretization error (11.3%)

Discretization error

Error budget	g_A	$\sqrt{\langle (r^v_E)^2 \rangle}$	$\sqrt{\langle (r^v_M)^2 \rangle}$	$\sqrt{\langle (r^v_A)^2 \rangle}$
Statistical:	1.9%	5.6%	8.3%	10.5%
Discretization:	1.6%	8.3%	9.0%	11.3%

Statistical error $\lesssim$ Discretization error

Check

1. Dispersion relation of nucleon

2. $O(a)$ improved current $A^{\rm imp}_\alpha = A_\alpha + c_A a \partial_\alpha P$ → PCAC relation

$$m^{\rm pion}_{\rm PCAC} \equiv \frac{m^2_\pi f_\pi}{2\langle 0|P^+(0)|\pi\rangle} \longleftrightarrow m^{\rm nucl}_{\rm PCAC} \equiv \frac{\langle N_{\rm snk}\partial_\mu A_\mu(x)\overline{N}_{\rm src}\rangle}{2\langle N_{\rm snk}P(x)\overline{N}_{\rm src}\rangle}$$

- Pion 2-pt function
- Zero momentum
- Improvement is helpless

$m^{\rm pion}_{\rm PCAC} = (m^{\rm pion}_{\rm PCAC})^{\rm imp}$

$\bar{c}_A$ s.t. $m^{\rm pion}_{\rm PCAC} \sim (m_{\rm PCAC})^{\rm imp}$

$\rightarrow \bar{c}_A \propto m^{\rm PCAC}_{\rm AWTI} - (m^{\rm PCAC}_{\rm AWTI})^{\rm imp}$

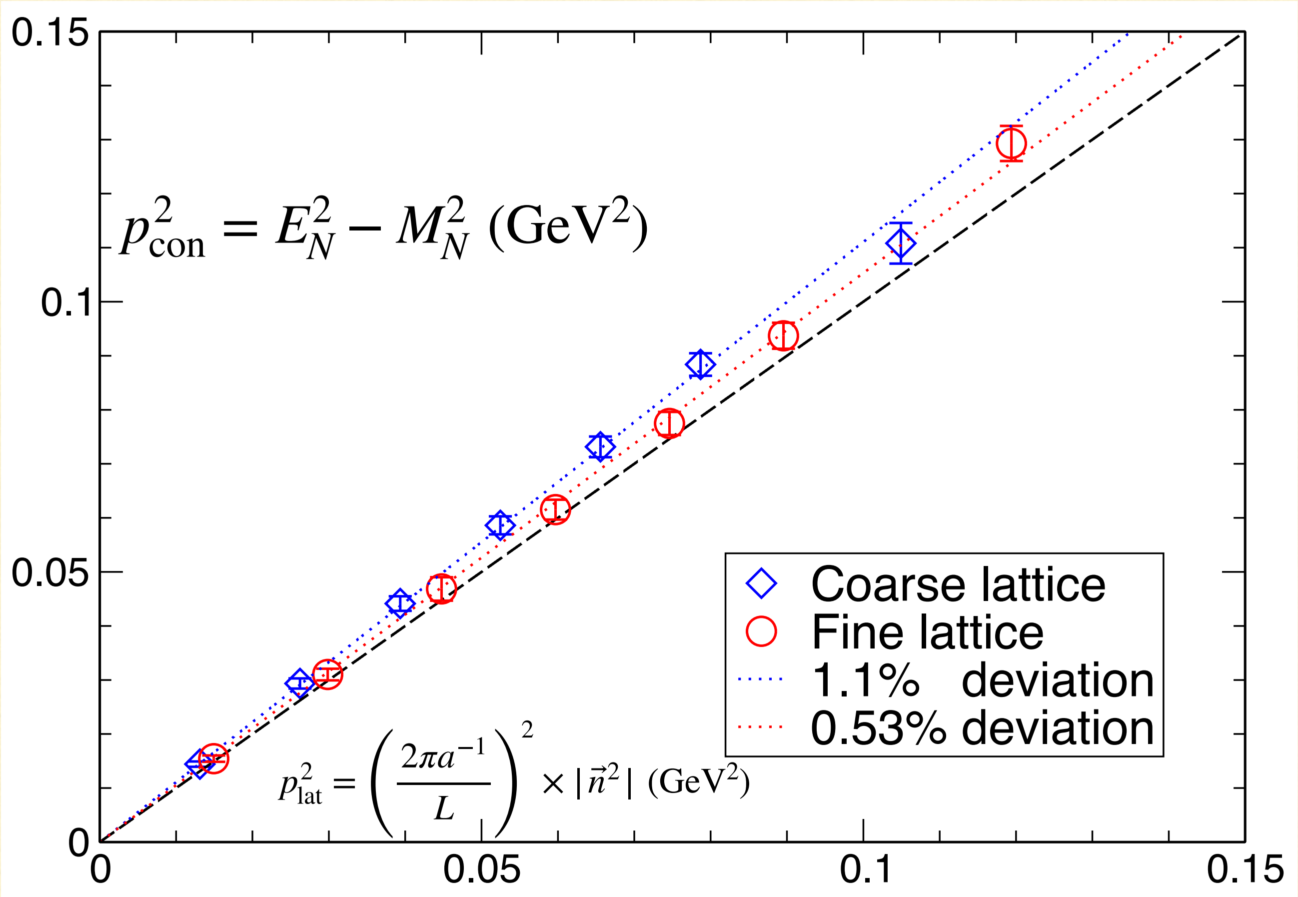
$\sim m^{\rm PCAC}_{\rm AWTI} - m_{\rm PCAC}$

- Nucleon 3-pt function
- Nonzero momentum
- Improvement works

$(m^{\rm nucl}_{\rm PCAC})^{\rm imp} = m^{\rm nucl}_{\rm PCAC} - a c_A q^2/2$

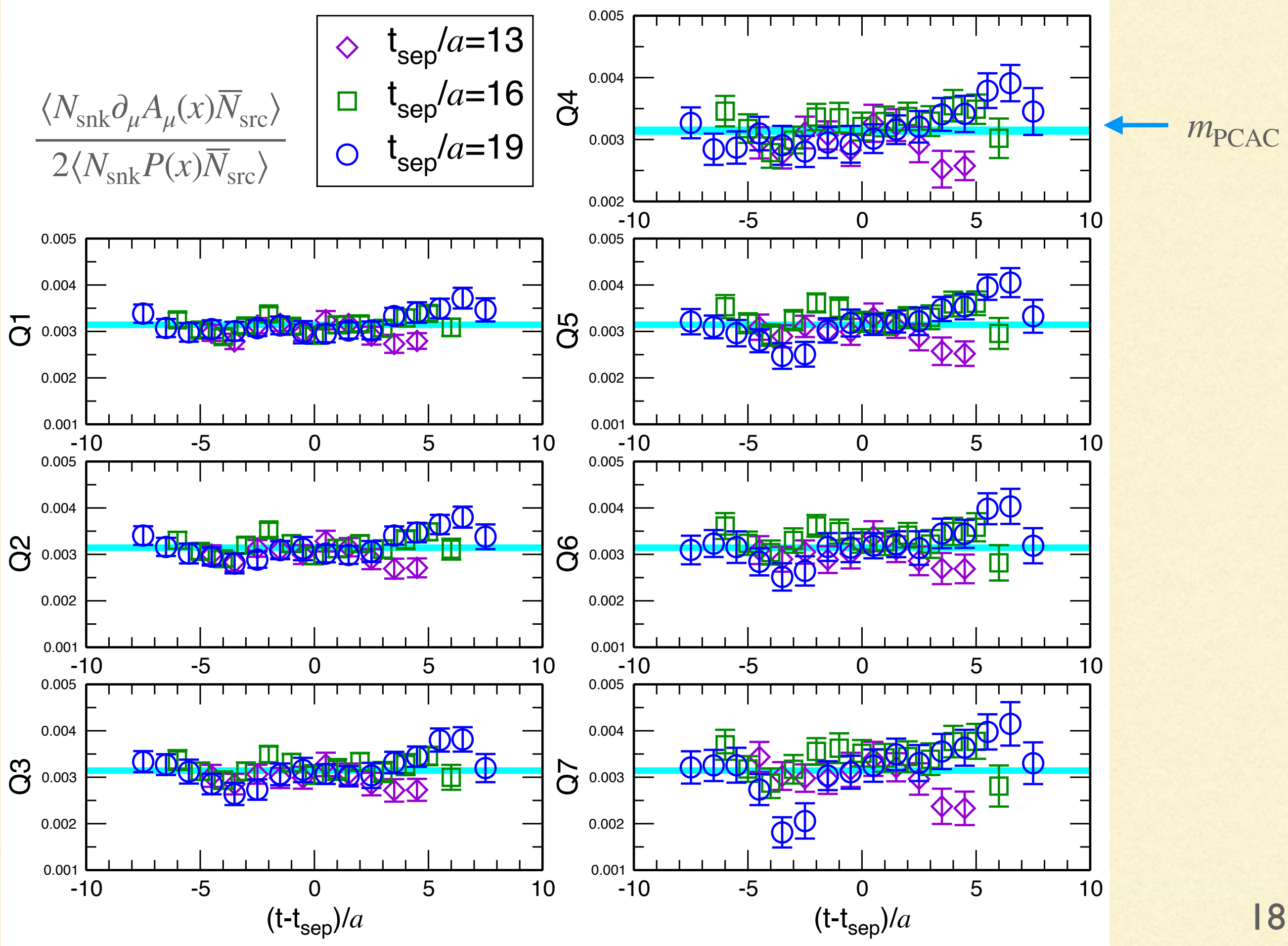
Dispersion relation

Discretization effect is small

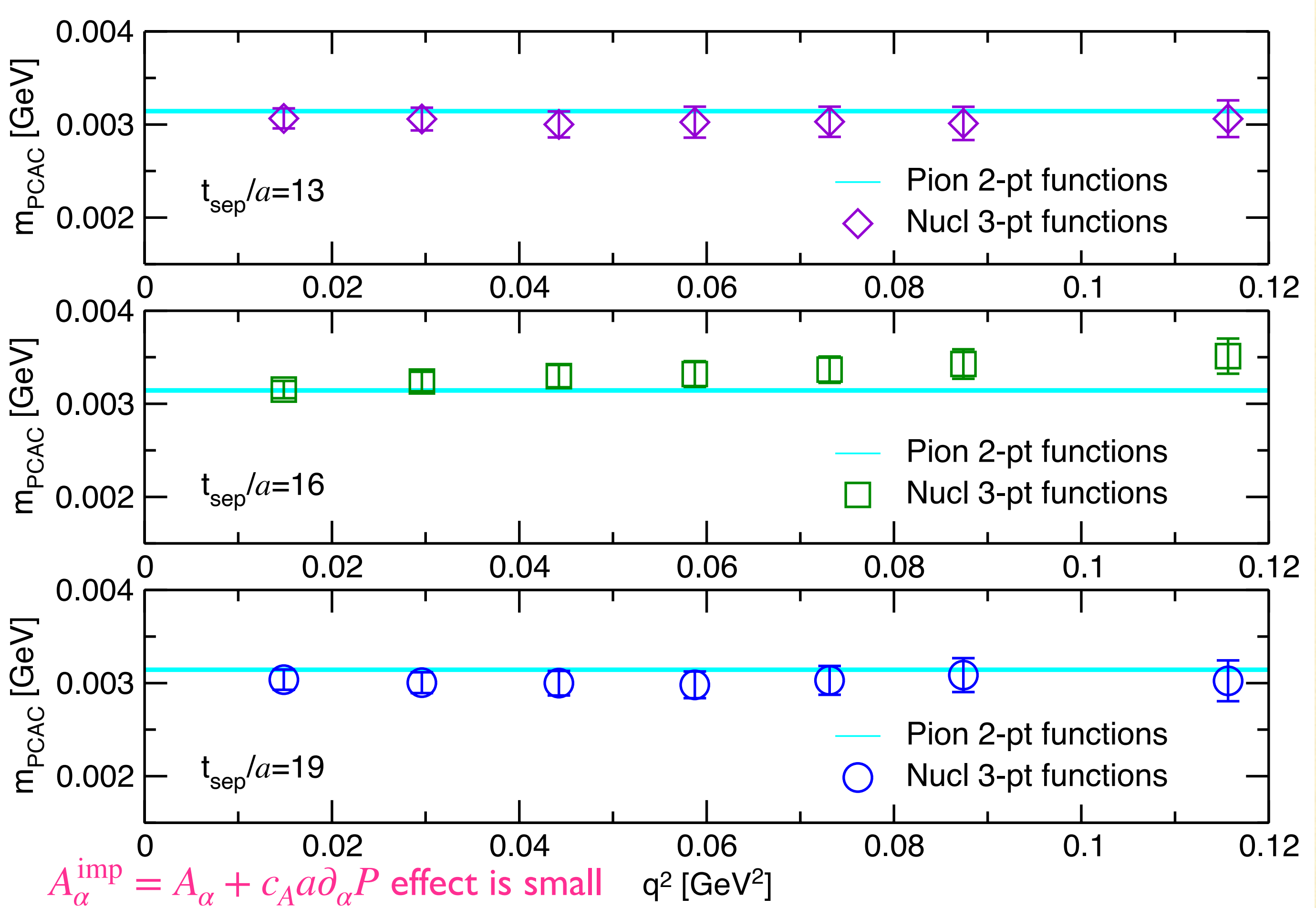


[1] E. Shintani et al., Phys. Rev. D **99,** 014510(2019), (Erratum; Phys. Rev. D 102 (2020) 019902.)

PCAC satisfying correlation functions



PCAC satisfying correlation functions



$A_\alpha^{\rm imp} = A_\alpha + c_A a \partial_\alpha P$ effect is small

Summary

Summary of form factor studies

PACS Collaboration for Nucleon projects:

- AMA technique → High statistical precision
- Physical point → No chiral extrapolations
- Large physical volume ($\sim 10^4\ \mathrm{fm}^4$) → Low Q^2 information
- *Fully dynamical lattice QCD simulations* towards **continuum limit**

Our results:

- For g_A, both coarse & fine reproduce PDG within stat. err. (2%).
- Large discretization error appears on the radii compared to the error on the dispersion relation, and it would not be resolved by the $O(a)$ improved current.

Error budget	g_A	$\sqrt{\langle (r_E^v)^2 \rangle}$	$\sqrt{\langle (r_M^v)^2 \rangle}$	$\sqrt{\langle (r_A^v)^2 \rangle}$
Statistical:	1.9%	5.6%	8.3%	10.5%
Discretization:	1.6%	8.3%	9.0%	11.3%