The Causality Road from DT to QG that describes our Universe

Y. Watabiki

Talk @ Discrete Approaches to the Dynamics of Fields and Space-Time held at Univ of Tsukuba on 12/9/2023

Strategy

- We walk on the causality road from DT to QG that describes our Universe.

Conclusion

- The topology of universe is 3D torus.
- Accelerating expansion of universe is caused by Porcupinefish spacetime, not by Dark Energy.

CONTENTS1

離散研究会'17

CONTENTS2

離散研究会 '17

離散研究会 '22

1. Introduction

a. Determinism in Physics (Laplace's demon)

- **How to determine** $\mathcal{X}(x,t) \overset{\text{def}}{=} \{\psi_\alpha(x,t), A_\mu(x,t), g_{\mu\nu}(x,t)\}$.

In classical theory, $\mathcal{X}(^\forall x, ^\forall t)$ is determined by $\mathcal{X}(^\forall x, ^\exists t_0)$.

However, how is $\mathcal{X}(^\forall x, ^\exists t_0)$ chosen?

In quantum theory, the wave function $\Psi(\mathcal{X}(^\forall x, ^\forall t), ^\forall t)$ is determined by $\Psi(\mathcal{X}(^\forall x, ^\exists t_0), ^\exists t_0)$ and $\mathcal{X}(x,t)$ is probabilistically determined by $\Psi(\mathcal{X}(x,t), t)$.

However, how is $\Psi(\mathcal{X}(^\forall x, ^\exists t_0), ^\exists t_0)$ chosen?

- **How to determine** $\mathcal{X}(x,t) \overset{\text{def}}{\equiv} \{\psi_\alpha(x,t), A_\mu(x,t), g_{\mu\nu}(x,t)\}$.

  ⇒ **If the universe starts from a point,**

  all states $\mathcal{X}({}^\forall x, {}^\forall t)$ **are determined by the quantum theory**

  because $\mathcal{X}(x, t=0)$ **is unique.**

  (Note that the point state is prohibited in classical theory

  because of its determinism mechanism.)

  But, new questions arise.

  What was the state of the universe before it was a point?

  Was the universe born from nothing?

b. Overview of Quantum Gravity (QG)

Partition Function

The partition function of QG is defined

by summing up all possible configurations as

[$\mathcal{T}$ represents the topology of spacetime.]

$$Z = \sum_{\mathcal{T}} C_{\mathcal{T}} \int \mathcal{D}\mathcal{X} \exp\Big\{ i \int \mathrm{d}^d x \, \mathrm{d}t \, \mathcal{L}[\mathcal{X}(x,t)] \Big\}.$$

Problems:

$$\mathcal{X}(x,t) \stackrel{\text{def}}{=} \{\psi_\alpha(x,t), A_\mu(x,t), g_{\mu\nu}(x,t)\}$$

1. How to define $\mathcal{L}[\mathcal{X}(x,t)]$.
2. How to define $C_{\mathcal{T}}$ and perform the path-integral $\int \mathcal{D}\mathcal{X}$.

⇒ As far as we know, only 2D QG (= non-critical string theory)

allows us to perform the above-mentioned path integral.

2. Causality Road

a. Creation of our Universe from Emptiness
色 空

- **Our Universe is mathematics with causality**

般若心経 色即是空 空即是色

The Buddhist Heart Sutra is "Form is emptiness, emptiness is form."

In this statement the causality, which is a central teaching of Buddhism, is an inevitable idea. But, what is the emptiness? On the other hand, Mathematics has no substance and exists independently of our universe.

空

⇒ We identify the emptiness as one of the mathematics with causality,

that is, our world is one of the mathematics with causality. (*)

Let us use "time" as a coordinate specifying causality.

"time" becomes "normal time" when Lorentz symmetry exist.

- **Mathematics of QG is simple and extremal**

Mathematics complex enough to describe our universe
is the mathematics that we physicists seek.

String theory is not only a candidate for QG, but also the theory
that we human beings know best in terms of mathematical depth.

⇒ Critical string theory (which has $c = 26$) will be the real QG.

Let us take not only "simplicity" but also "extremity"
素 際
as clues for logical leaps.

⇒ Physics is a simple and extreme theory of mathematics.

b. Proposals to solve the Problems in QG

- **Our universe starts from a point state**

Problem of initial conditions of $\Psi(\mathcal{X}(x, t = 0), t = 0)$.

⇒ This problem is solved because the point state at $t = 0$ is unique.

However, the following new problems arise.

Problem of the singularity [If a conserved quantity exists, the point-state universe becomes a state with a matter memory. If there exists such singularity, those appear everywhere.]

Problem of the topology of the Universe [We need the mechanism to determine $C_{\mathcal{T}}$, i.e. the topology of universe after $t = 0$.]

- **The causal time axis is placed outside spacetime**

Let us abandon the Lorentz symmetry for a while.

Then, the following problem arise.

Problem of the Lorentz symmetry.

Problem of the birth from emptiness [If a conserved quantity exists,
the point-state universe has a matter charge and
we need to produce it when the point-state universe was born.]

On the other hand, the following problem is solved.

Problem of time-closed loop [This picture rules out the existence
of time-closed loop, meaning there are no time machines.]

- **Our Universe started as a one-dimensional space**

Assuming that the universe initially occurs from emptiness as a 1D space and has changed to a high dimensional space during the expansion, the following problems are solved.

Problem of the singularity of spacetime [The conservation law cannot go back to the period when it was 1D space.]

Problem of the topology of the Universe [1D closed space is unique.]

⇒ The knitting mechanism appears and this leads to the toroidal topology.

- **The emergence of critical string theory**

If our theory is equivalent to the critical string theory,

the following problems are solved.

Problem of Lorentz symmetry

[The Lorentz symmetry recovers.]

Problem of the gauge symmetry of Standard Model

[The gauge symmetry appears.]

Problem of background metric independence

[We need the theory which is independent of the background.]

C. Strategy for QG that describes our Universe

We treat the following two points important.

- **2D Euclidean QG, i.e. non-critical string theory is *currently* the only QG theory that has succeeded in calculating the path integral.**

But, there is no concept of time because the metric is Euclidean.

- **critical string theory** [c=26] **has Lorentz symmetry and includes the graviton, so this theory is a candidate of QG that describes our universe.**

But, *so far* the string theory cannot describe the birth of universe.

In both cases the "**time**" looks the key word. $\overset{(*)}{\Rightarrow}$ **"causal time"**

We start from DT to QG which describes our universe as

pure DT ⇒ pure CDT ⇒ CDT with 26 central charge

$[c = 0]$ $[c = 0]$ $[c = 26]$

"pure" means no matter fields.

- **Preparation**

  2D QG ~ Liouville gravity ~ non-critical string theory

  ~ Matrix Model ~ Dynamical Triangulation (DT)

- **Pure DT by non-critical string field theory (SFT)**

  DT is expressed by non-critical SFT

- **Pure DT by W operators**

  non-critical SFT is expressed by W operator.

- **Pure CDT by non-critical SFT**

  The geodesic distance is replaced by the causal time.

- **Pure CDT by W operators**

  ⇒ The phenomena of the birth of universe appear naturally.

- **CDT with matter by W operators**

  ⇒ The Jordan algebra appears naturally.

- **Basic properties of CDT with matter**

  ⇒ The inflation starts after the birth of universe.

  The dimension enhancement by the knitting mechanism

  The vanishing cosmo constant by the Coleman mechanism

- **Phenomenological predictions by CDT with matter**

  ⇒ The accelerating expansion occurs.

  We can predict how our universe will come to an end.

5. Basic of Theory

a. Definition of W and Jordan algebra gravity

- **Transfer Operator**

The partition fun. is derived by the expectation value of $\Theta^{\star}$.

Our model is described by the transfer operator $\Theta^{\star}$

$$\Theta^{\star} \stackrel{\text{def}}{=} e^{W_{-2}^{(3)}} \qquad W_n^{(3)} \stackrel{\text{def}}{=} \frac{1}{3} \sum_{k+l+m=n} \mathrm{Tr} : \alpha_k \alpha_l \alpha_m :$$

$$\alpha_n \stackrel{\text{def}}{=} \sum_{\mu} E_{\mu}\, \alpha_n^{\mu} \qquad [\alpha_m^{\mu}, \alpha_n^{\nu}] = m \delta_{m+n,0} \delta^{\mu,\nu}$$

where E_{μ} is the 3×3 octonian Hermitian matrices.

(m, n are modes [$m, n \in \boldsymbol{Z}$], μ, ν are flavors [$\mu, \nu = 0, 1, \ldots, 26$].)

- **The emergence of time**

We shift α_n and introduce $\phi_n^\dagger$ and ϕ_n as

$$(\alpha_{-n})^\star = 3\lambda_3\delta_{n,3} + \lambda_1\delta_{n,1} + n\sqrt{G}\phi_n \qquad (\alpha_n)^\star = \frac{1}{\sqrt{G}}\phi_n^\dagger$$

$$3\lambda_3 = \frac{\sigma}{2g\sqrt{G}} \quad \lambda_1 = -\frac{\mu}{2g\sqrt{G}} \quad (\alpha_0)^\star = \frac{\omega}{\sqrt{G}}$$

α_0 is commutative with all operators.

Physical vacuum $|\mathrm{vac}\rangle$ is a coherent state,

$$\phi_n|\mathrm{vac}\rangle = 0 \qquad [\phi_m, \phi_n^\dagger] = \delta_{m,n} \qquad [m, n \in \boldsymbol{N}]$$

Under the physical vacuum, the scale symmetry is broken!

$\alpha_n \to (gt)^{-n/2}\alpha_n$ **leads to** $W_{-2}^{(3)} \to gt\, W_{-2}^{(3)}$

➡ t **appears in front of** $W_{-2}^{(3)}$
and starts to play the role of time.

b. From the birth of universes to Big Bang

- **Hamiltonian**

Tr is omitted.

$$\mathcal{H}_{\mathrm{W}} \overset{\text{def}}{=} -gW_{-2}^{(3)} = -\frac{g}{3}\sum_{k+l+m=-2} \mathrm{Tr}:\alpha_k\alpha_l\alpha_m:$$

$$= -g\sum_{n=4}^{\infty}\sum_{k=1}^{n-3}\phi_k^\dagger\phi_{n-k-2}^\dagger n\phi_n$$

$$-gG\sum_{n=1}^{\infty}\sum_{k=\max(3-n,1)}^{\infty}\phi_{n+k-2}^\dagger k\phi_k n\phi_n$$

Knitting Mechanism
編み上げ機構

Expansion of Universes

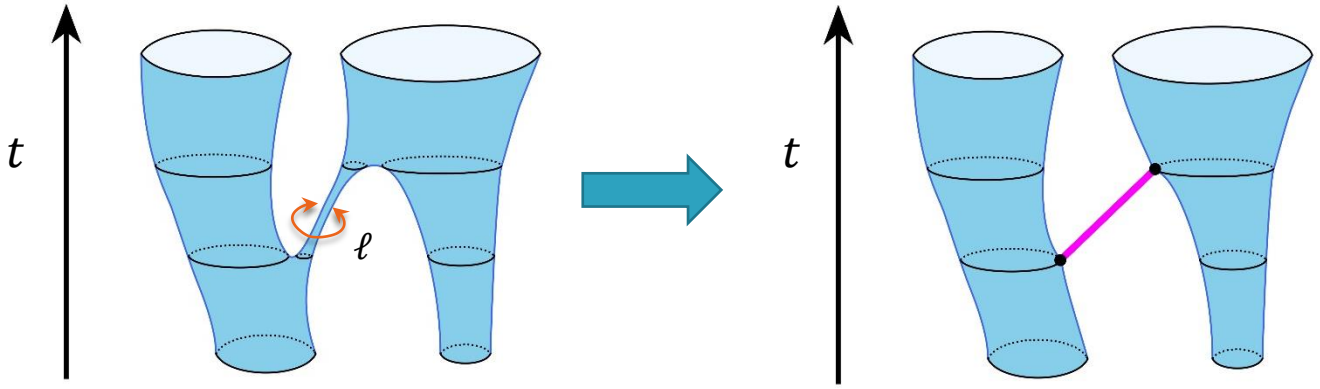
$$-\sigma\sum_{n=1}^{\infty}\phi_{n+1}^\dagger n\phi_n + \mu\sum_{n=2}^{\infty}\phi_{n-1}^\dagger n\phi_n - 2g\omega\sum_{n=3}^{\infty}\phi_{n-2}^\dagger n\phi_n$$

$$+\omega\left(\mu\phi_1 - 2g\omega\phi_2 - gG\phi_1\phi_1\right) + \frac{\sigma}{G}\left(-\frac{\sigma}{4g}\phi_4^\dagger + \frac{\mu}{2g}\phi_2^\dagger - \omega\phi_1^\dagger\right) - \frac{\mu\mu\omega}{4gG}$$

Creation of Universes

- **Knitting mechanism (Dimension Enhancement)**

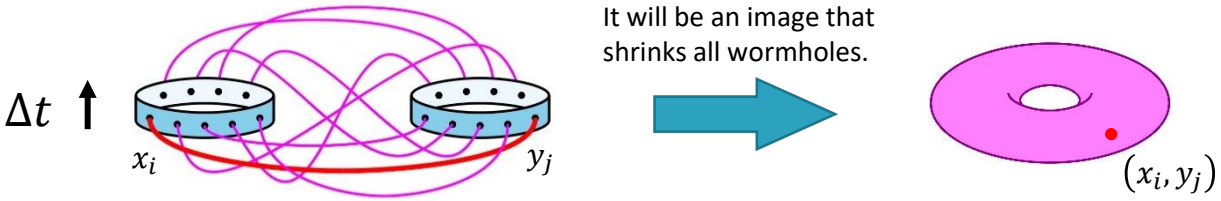
編み上げ機構



(A wormhole with small ℓ is shown by purple line.)

High-dimensional space is formed after the birth of space.

Contributions by tiny wormholes are dominant. $G(\ell,\ell;t) \sim \frac{1}{\sqrt{4\pi\ell t}}$ $[t \sim 0]$

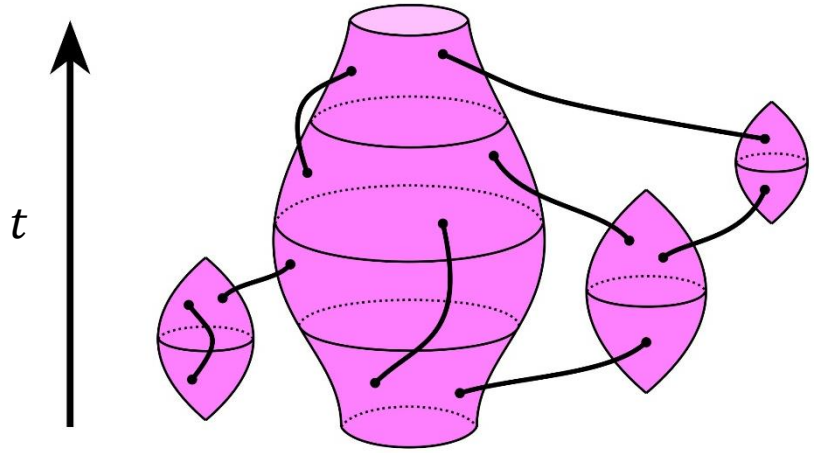


(The set of tiny wormholes gives a torus topology.)

- **Coleman mechanism (Vanishing cosmo const.)**

  大爆発機構

  Connection by wormholes with finite t will give vanishing the cosmological constant μ.



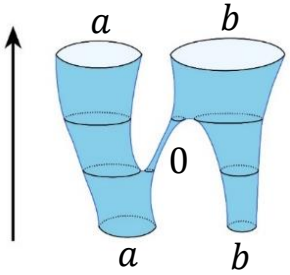
➡ Vanishing the cosmological constant μ will give the Big Bang energy and will deny the existence of dark energy.

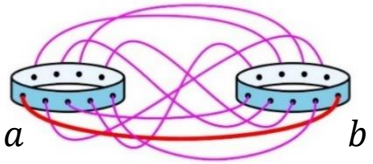
- **The conditions for the knitting mechanism** 編み上げ機構

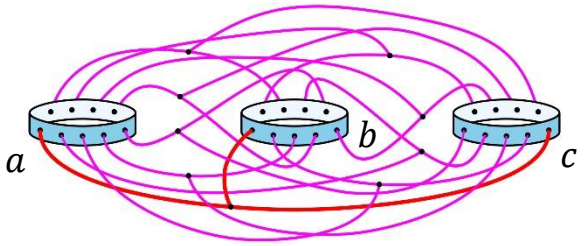
$G(\ell, \ell; T) \longrightarrow +\infty \quad \text{for} \quad T \longrightarrow 0$

$c_3 = 0 \quad \text{or} \quad g \sim 0$

$$g\, d_{\mu\nu\rho} = g\left(c_2 \sum_a \left(\delta_{\mu 0}\delta_{\nu a}\delta_{\rho a} + \text{cyclic terms}\right) + c_0\, \delta_{\mu 0}\, \delta_{\nu 0}\, \delta_{\rho 0} + c_3 \sum_{a,b,c} \delta_{\mu a}\, \delta_{\nu b}\, \delta_{\rho c}\right) \qquad [\, g \neq 0\,]$$

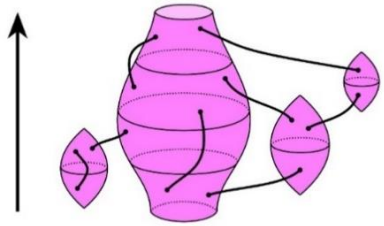






- **The conditions for the Coleman mechanism** 大爆発機構

$G(\ell, \ell; T) \longrightarrow \text{finite} \quad \text{for} \quad T \longrightarrow +\infty$



6. Modified Friedmann Equation

a. Expansion of our Universe

- **Derivation of Modified Friedman equation**

$$\mathcal{H}_{\rm kin} = -\sum \phi_{n+1}^{\dagger} n \phi_n + \mu \sum \phi_{n-1}^{\dagger} n \phi_n - 2g \sum \phi_{n-2}^{\dagger} n \phi_n$$

leads to the classical Hamiltonian $\quad \{L, \Pi\} = 1$

$$\mathcal{H}_{\rm c} = L\left(\pm(\Pi^2 - \mu) + \frac{2g}{\Pi} \right)$$

then, we have equation of motion

$$H^2 = \frac{\kappa \rho_{\rm m}}{3} + \frac{B}{H} \frac{1 + 3F(x)}{(F(x))^2} \qquad B \stackrel{\rm def}{=} -8g$$

Cosmo const $\boldsymbol{\mu}$ is replaced by the matter energy $\boldsymbol{\rho_{\rm m}}$ by Coleman mechanism.

$$4\mu \to \frac{\kappa \rho_{\rm m}}{3}$$

This eq. is invariant under

$$(4B)^{1/3} H \leftrightarrow \frac{\kappa \rho_{\rm m}}{3}$$

$$H \stackrel{\rm def}{=} \frac{\dot{L}}{L} \qquad (F(x))^2 - (F(x))^3 = x \qquad x \stackrel{\rm def}{=} \frac{B}{H^3}$$

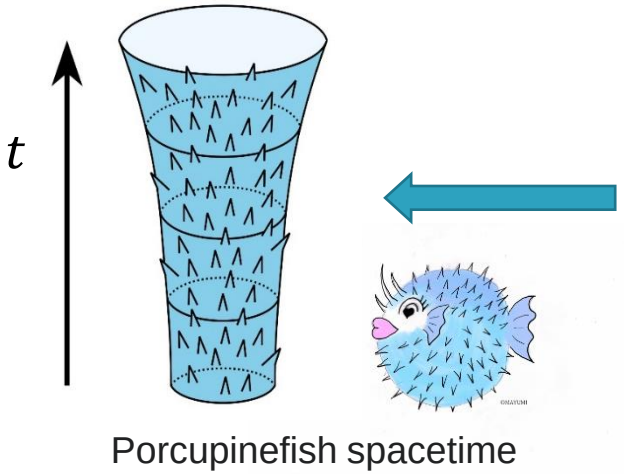
- **The geometrical meaning of** $-2g\phi_0^\dagger \sum \phi_{n-2}^\dagger n\phi_n$

This term comes from the leading term of disk amplitude $F(L)$ $(\phi_0^\dagger = 1)$

$$F(L) = \delta(V) + \cdots \quad \Leftarrow \quad \tilde{F}(\xi) = \langle \tilde{\Psi}^\dagger \rangle = \xi^{-1} + \cdots = \frac{1}{\xi + \sqrt{\mu}}$$

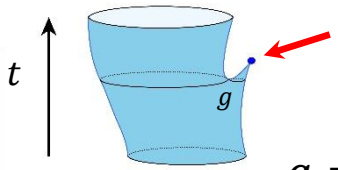
$F^{(\text{baby})}$

$$\left(\tilde{\Psi}^\dagger = \sum_{n=0}^{\infty} \xi^{-1-n} \phi_n^\dagger \right)$$



Porcupinefish spacetime

The creation of one baby universe



$F^{(\text{baby})} = \delta(V)$

$$g = -\frac{B}{8} < 0$$

Negative g gives the accelerating expansion of Universe.

This term is the quantum effect of quantum gravity!

b. The accelerating expansion of Universe

- H_0 tension (problem) and S_8 tension

$$H_0 \overset{\text{def}}{=} H(t_0), \quad S_8 \overset{\text{def}}{=} \sigma_8(t_0)\sqrt{\Omega_{\mathrm{m}}(t_0)/0.3}, \quad \Omega_{\mathrm{m}} \overset{\text{def}}{=} \frac{\kappa\rho_{\mathrm{m}}}{3H^2}$$

Data from Planck satellite (Early Universe w/ ΛCDM model)

$$H_0^{(\mathrm{CMB})} = 67.3 \pm 0.6 \text{ [km/sec/Mpc]}$$

$$S_8^{(\mathrm{CMB})} = 0.835 \pm 0.014$$

Data from Standard candles (Late Universe)

$$H_0^{(\mathrm{SC})} = 73.0 \pm 1.0 \text{ [km/sec/Mpc]}$$

(ArXiv:2112.04510)

$$S_8^{(\mathrm{SC})} = 0.769 \pm 0.005$$

5.0σ

3.4σ

- **Boundary Condition 1** (CDM is assumed)

Data from Planck satellite

$t_0^{(\mathrm{CMB})} = 13.8 \times 10^9$ [year]

$H_0^{(\mathrm{CMB})} = 67.3 \pm 0.6$ [km/sec/Mpc]

$z_{\mathrm{LS}}^{(\mathrm{CMB})} = 1089.95$

$$\frac{a_{\Lambda^{(\mathrm{CMB})}}\left(t_0^{(\mathrm{CMB})}\right)}{a_{\Lambda^{(\mathrm{CMB})}}\left(t_{\mathrm{LS}}^{(\mathrm{CMB})}\right)} = 1 + z_{\mathrm{LS}}^{(\mathrm{CMB})} \qquad H_{\Lambda^{(\mathrm{CMB})}}\left(t_0^{(\mathrm{CMB})}\right) = H_0^{(\mathrm{CMB})}$$

$t_{\mathrm{LS}}^{(\mathrm{CMB})}$ and $\Lambda^{(\mathrm{CMB})}$ are determined.

(These are hidden information in CMB observed by Planck satellite.)

- **Boundary Condition 2** (CDM is assumed)

Data from Standard candles

$$H_0^{(\mathrm{SC})} = 73.0 \pm 1.0 \ [\mathrm{km/sec/Mpc}]$$

We also use $t_{\mathrm{LS}}^{(\mathrm{CMB})}$ and $z_{\mathrm{LS}}^{(\mathrm{CMB})}$. ← No difference between ΛCDM model and our model before $t_{\mathrm{LS}}^{(\mathrm{CMB})}$

$$\frac{a_{\Lambda(\mathrm{SC})}\left(t_0^{(\mathrm{SC})}\right)}{a_{\Lambda(\mathrm{SC})}\left(t_{\mathrm{LS}}^{(\mathrm{CMB})}\right)} = 1 + z_{\mathrm{LS}}^{(\mathrm{CMB})} \qquad H_{\Lambda(\mathrm{SC})}\left(t_0^{(\mathrm{SC})}\right) = H_0^{(\mathrm{SC})}$$

$$\frac{a_B\left(t_0^{(\mathrm{B})}\right)}{a_B\left(t_{\mathrm{LS}}^{(\mathrm{CMB})}\right)} = 1 + z_{\mathrm{LS}}^{(\mathrm{CMB})} \qquad H_B\left(t_0^{(\mathrm{B})}\right) = H_0^{(\mathrm{SC})}$$

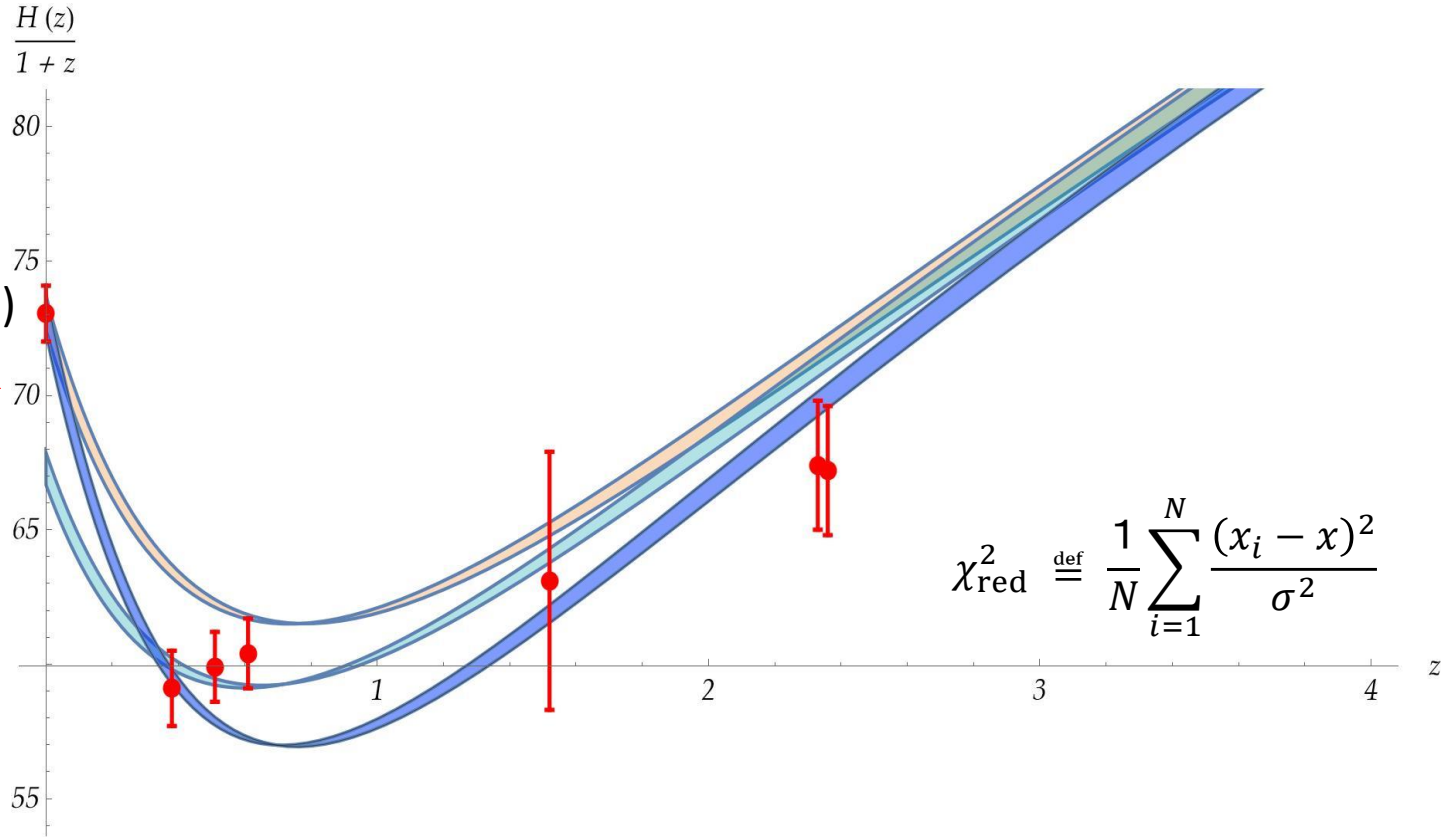
$t_0^{(\mathrm{SC})}$, $\Lambda^{(\mathrm{SC})}$, $t_0^{(\mathrm{B})}$, B are determined.

13.3[Gyr] $\quad \frac{2.2}{t_0^{(\mathrm{SC})2}} \quad$ 13.9[Gyr] $\quad \frac{0.15}{t_0^{(\mathrm{B})3}}$

Blue is our model using Standard Candle data.
Orange is ΛCDM model using Standard Candle data.
Green is ΛCDM model by Planck satellite data only.

- $\frac{H(z)}{1+z}$

H_0 tension
(5σ difference)



$$\chi^2_{\text{red}} \overset{\text{def}}{=} \frac{1}{N}\sum_{i=1}^{N}\frac{(x_i - x)^2}{\sigma^2}$$

$$\chi^{(\text{B})2}_{\text{red}} = 1.3^2 \qquad \chi^{(\text{SC})2}_{\text{red}} = 1.9^2 \qquad \chi^{(\text{CMB})2}_{\text{red}} = 2.3^2$$

(⇒ Standard Candle data represents the accel. expansion well.)

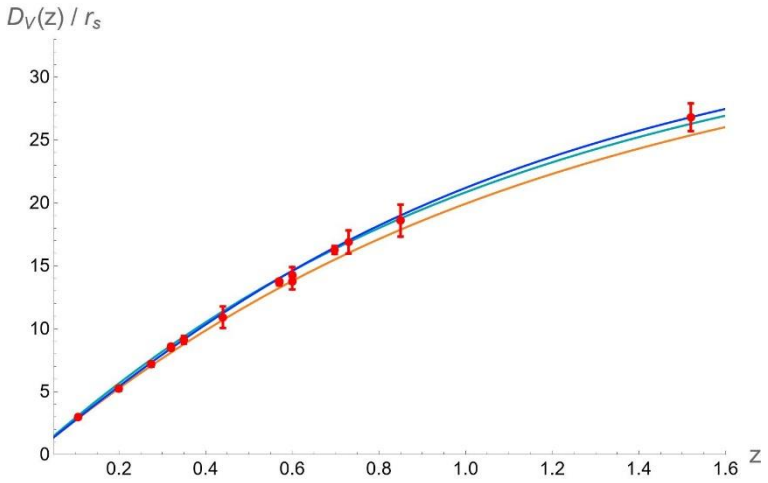
Blue is our model using Standard Candle data.
Orange is ΛCDM model using Standard Candle data.
Green is ΛCDM model by Planck satellite data only.

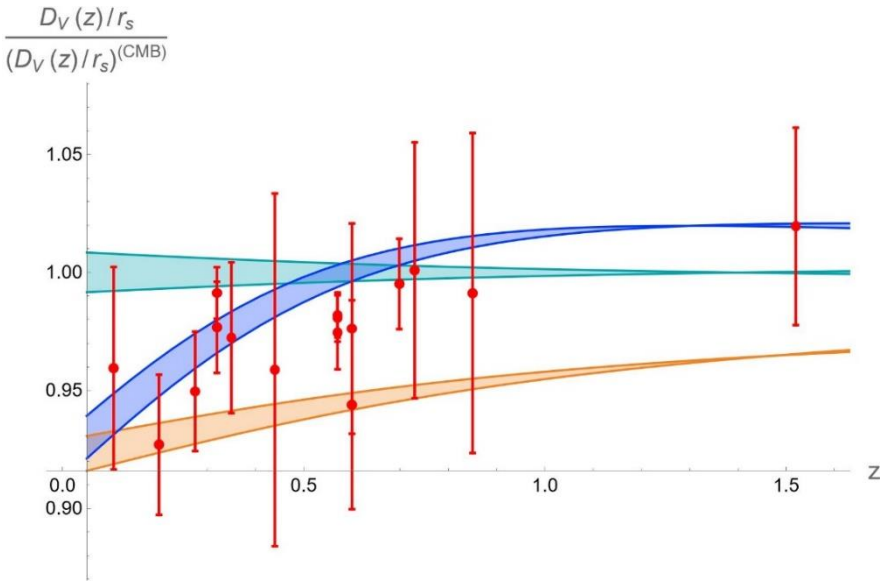
- $\dfrac{D_V(z)}{r_s}$ **(BAO)**

$$r_s^{(\mathrm{B})} \backsim r_s^{(\mathrm{SC})} \sim r_s^{(\mathrm{CMB})} = 147.05 \pm 0.30\ [\mathrm{Mpc}]$$

Data from Planck satellite

r_s is the sound horizon at $z = z_{\mathrm{drag}}$





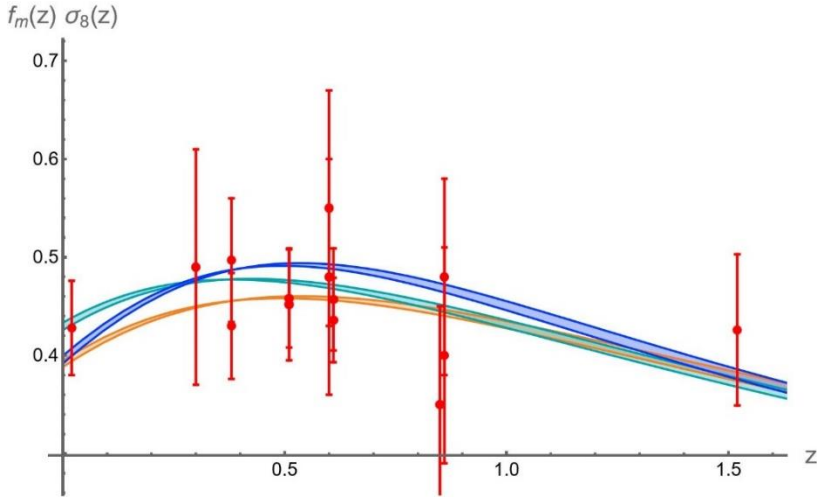
$$\chi^{(\mathrm{B})2}_{\mathrm{red}} = 1.0^2 \qquad \chi^{(\mathrm{SC})2}_{\mathrm{red}} = 2.1^2 \qquad \chi^{(\mathrm{CMB})2}_{\mathrm{red}} = 1.3^2$$

(⇒ BAO is related with the early stage of Universe.)

- $f_{\mathrm{m}}(z)\,\sigma_8(z)$

Blue is our model using Standard Candle data.
Orange is ΛCDM model using Standard Candle data.
Green is ΛCDM model by Planck satellite data only.

$$\sigma_8^{(\mathrm{B})}(0) \sim \sigma_8^{(\mathrm{SC})}(0) \sim \sigma_8^{(\mathrm{CMB})}(0) = 0.8120 \pm 0.0073$$



Data from Planck satellite

(⇒ Error bars are large.)

$$\chi_{\mathrm{red}}^{(\mathrm{B})2} = 0.70^2 \quad \chi_{\mathrm{red}}^{(\mathrm{SC})2} = 0.51^2 \quad \chi_{\mathrm{red}}^{(\mathrm{CMB})2} = 0.54^2$$

- $S_8 \stackrel{\mathrm{def}}{=} \sigma_8(0)\sqrt{\Omega_{\mathrm{m}}(0)/0.3}$

S_8 tension

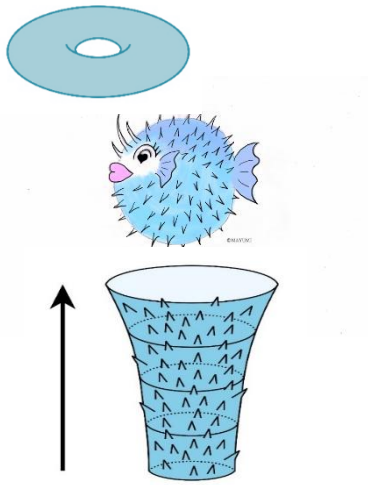
$$\chi_{\mathrm{red}}^{(\mathrm{B})2} = 0.75^2 \quad \chi_{\mathrm{red}}^{(\mathrm{SC})2} = 0.75^2 \quad \chi_{\mathrm{red}}^{(\mathrm{CMB})2} = 3.35^2$$

(⇒ S_8 looks related with the late stage of Universe.)

7. Conclusions

a. Emergence of space

- High-dimensional space is formed by the direct product of several 1D loop spaces S^1.
- The topology of our universe is 3D torus. Therefore, the spacetime is flat. $(K = 0)$



b. Identity of Dark energy

- Accelerating expansion of Universe is caused by Porcupinefish spacetime.
- No tensions appear in $(H_0$, BAO, $f_{\mathrm{m}}\sigma_8$, $S_8)$.
- Dark energy does not exist. (because of Coleman mechanism)

c. Destiny of our Universe

- There exist two scales:

  $t_\mu \stackrel{\text{def}}{=} |\mu|^{-1/2}, \quad t_g \stackrel{\text{def}}{=} |g|^{-1/3}$

- We need to neglect the interaction of 3-universes.
  The condition is $t_0 \lesssim t_g \sim 3.8\, t_0 \sim 52\ [\mathrm{Gyr}]$

d. The anthropic principle 人間原理

- If we assume the anthropic principle,
  we human beings encounters the era $t_0 \sim t_g$ and
  we human beings cannot survive beyond t_g.

8. Overview

a. Change of Vacuum and Birth of Time

- **We need the change of vacuum in order to birth the time.**

$$|0\rangle \rightarrow |\mathrm{vac}\rangle$$

This transition looks a sudden change.

Is it possible to incorporate the concept of SSB?

b. Cosmic age division

- **Pre- and Post-world** $[t \lesssim 0]$
- **Cosmic dawn age** $[0 \lesssim t \lesssim t_{\mu}]$
  - **Space-birth period**
  - **Wormhole period**
- **Cosmic growth age** $[t_{\mu} \lesssim t \lesssim t_{g}]$
  - **Big-bang period**
  - **Transition period**
- **Cosmic dusk age** $[t_{g} \lesssim t \lesssim t_{\mathrm{c}}]$
  - **Chaos period**
  - **Doomsday period**

C. Further problems

- Why $\frac{t_g}{t_{\text{planck}}} \sim 3 \times 10^{61}$ is so large?
- Our model is equivalent to the string theory?

  Only the conformal dimensions coincide.
- What happens beyond t_c?