Perturbative superstring theory and the IKKT Matrix Model

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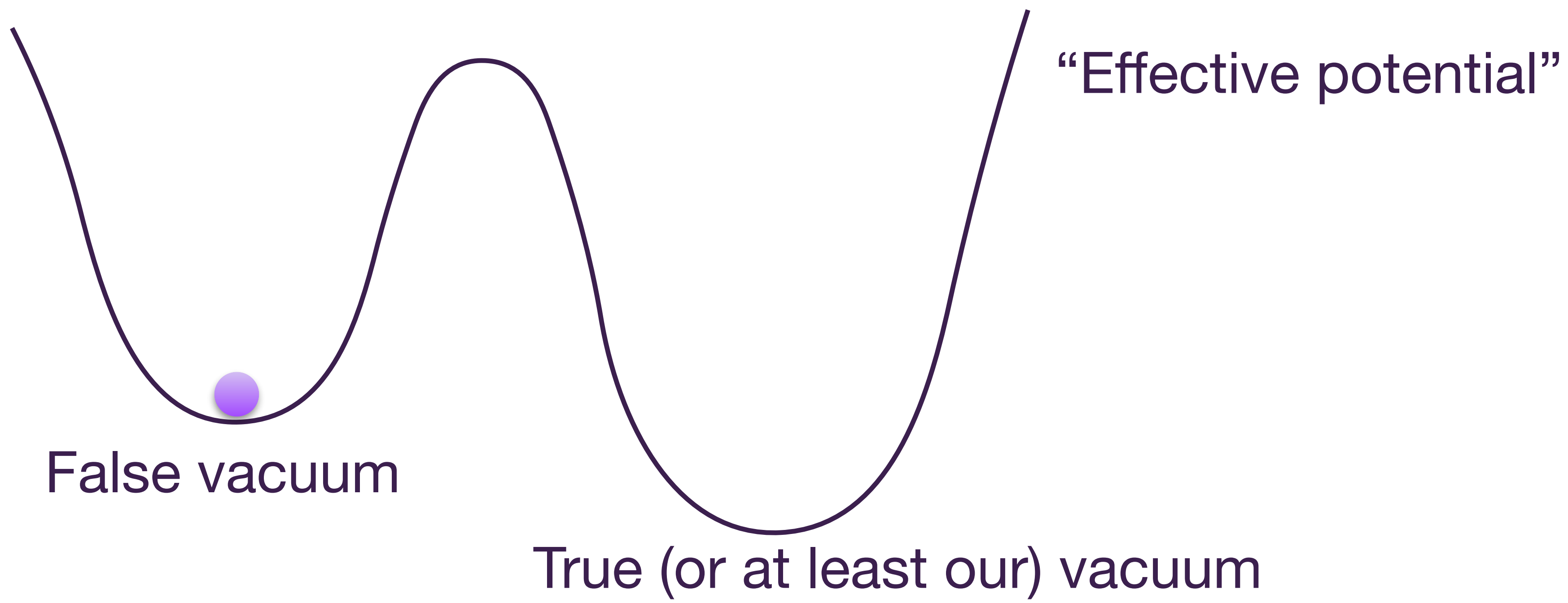
Introduction

What's wrong with superstring theory?

The established string theory is merely based on perturbation theory.

- There're infinitely many candidates of the vacuum, and it's unpredictable.



How is our 4D spacetime realised?

- Non-perturbative physics such as black holes and the early universe is not predictable strictly speaking.

We need non-perturbative formulation
to make it genuinely predictable quantum gravity theory!

Introduction

1988–1993 Non-pert. formulation of 2D critical string theory and non-critical string theories

… However, this isn't naively applicable to superstring

1994–1995 Progress in superstring theory

- Dualities between superstring theories

IIB string ⟷ IIA string ⟷ M-theory

- D-branes

0-brane 1-brane 2-brane …

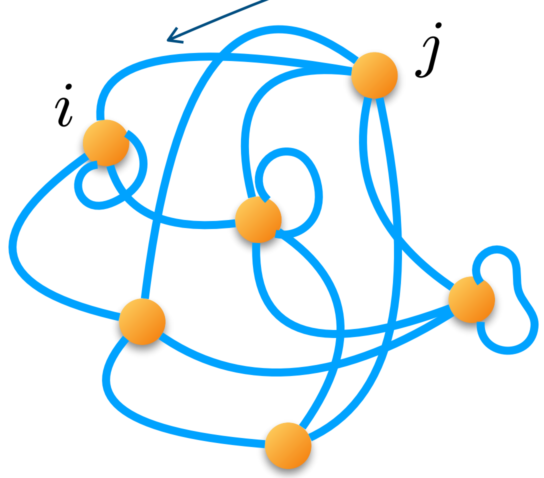
1996 **Non-pert. formulation of superstring theory**

- BFSS matrix model [Banks, Fischler, Shenker, Susskind '96]
- IKKT matrix model [Ishibashi, Kawai, Kitazawa, Tsuchiya '96]

1997
- Gauge/gravity duality [Maldacena '97, …]

Introduction

How matrices appear in superstring theory



branes connected by strings

$$X^\mu = \begin{pmatrix} X^\mu_{11} & X^\mu_{12} & \cdots & \cdots & X^\mu_{1N} \\ X^\mu_{21} & \ddots & & & \vdots \\ \vdots & & \ddots & X^\mu_{ij} & \vdots \\ \vdots & & & \ddots & \vdots \\ X^\mu_{N1} & \cdots & \cdots & \cdots & X^\mu_{NN} \end{pmatrix}$$

The duality between IIA string and M-theory

➡ The non-relativistic D0 dynamics (BFSS matrix model) describes the DLCQ M-theory [Banks, Fischler, Shenker, Susskind '96]

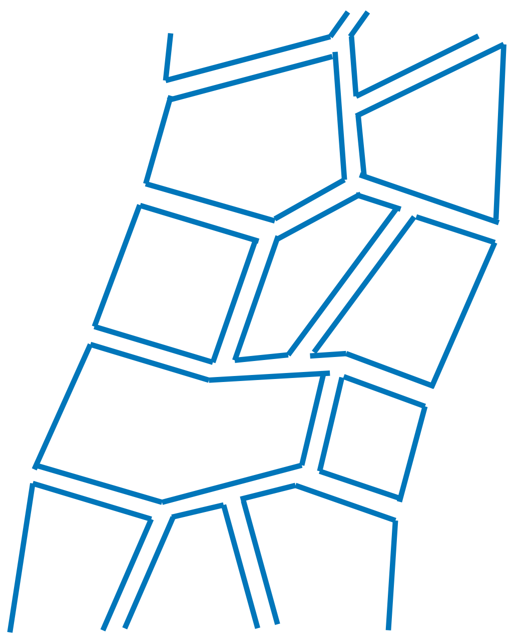
… equivalent to the matrix regularisation of supermembrane [de Wit, Hoppe, Nicolai '88]

➡ The matrix regularisation of superstring (IKKT matrix model) describes the type IIB string theory [Ishibashi, Kawai, Kitazawa, Tsuchiya '96]

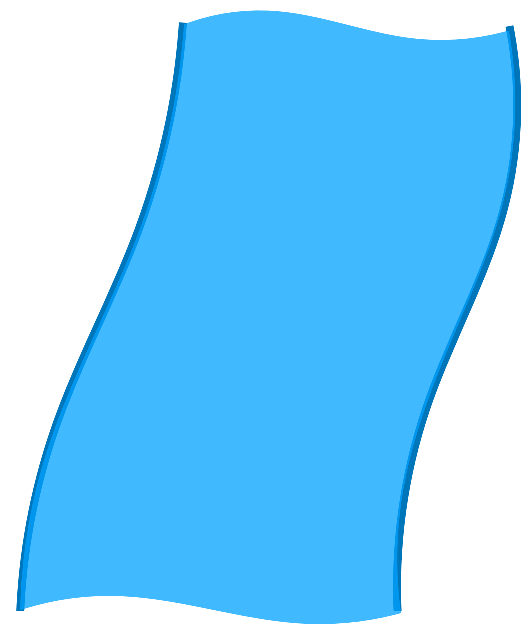
Introduction

Quantum corrections in string theory

Matrix model ⟷ **String theory**



Feynman diagram



string worldsheet

$1/N$ expansion ⟷ genus expansion

$$F = N^2 F_0 + F_1 + \frac{1}{N^2} F_2 + \cdots = \sum_{g=0}^{\infty} N^{2-2g} F_g$$

Introduction

The IKKT matrix model [Ishibashi, Kawai, Kitazawa, Tsuchiya '96]

$$S[X,\Psi] = N\,\mathrm{tr}\left[\frac{1}{4}[X^\mu, X^\nu][X_\mu, X_\nu] + \frac{1}{2}\Psi^T\Gamma^\mu[X_\mu, \Psi]\right]$$

X^μ: bosonic $N \times N$ matrices ($\mu = 0,\cdots,9$) Ψ: Majorana-Weyl fermionic $N \times N$ matrices

This 0-dimensional theory is considered to describe type-IIB superstring theory non-perturbatively. We believe this because it:

- has supersymmetry identical to that of type-IIB string: $\mathcal{N} = (2,0)$ in (9+1)D
- reproduces perturbative results
  (graviton-exchange potential, scattering amplitudes, etc.)
- can reproduce the light-cone string field theory by the Schwinger-Dyson eq.
  [Fukuma, Kawai, Kitazawa, Tsuchiya '97]
- has potential to dynamically realise (3+1)D space-time at large N
  - Dynamics of the diagonal elements of X^μ forms 4D [Aoki, Iso Kawai, Kitazawa, Tada '98]
  - SSB to SO(3) is observed [Anagnostopoulos, Azuma, Ito, Nishimura, Okubo, Papadoudis '20]

Introduction

Matrix identification of gravity

While there is an interpretation of matrices as the coordinates,
there are interpretations in which gravitons are included as matrix elements.

$$X_a \sim i e_a{}^{\mu} \nabla_{\mu} + \cdots$$

- Hanada-Kawai-Kimura interpretation [Hanada, Kawai, Kimura '06;...]

➡ The classical E.o.M. gives the Einstein-Hilbert gravity

- Weitzenböck connection interpretation [Sperling, Steinacker '19; Steinacker '20;...]

➡ The classical E.o.M. gives modified gravity ("pre-gravity") while the one-loop correction gives the Einstein-Hilbert gravity.

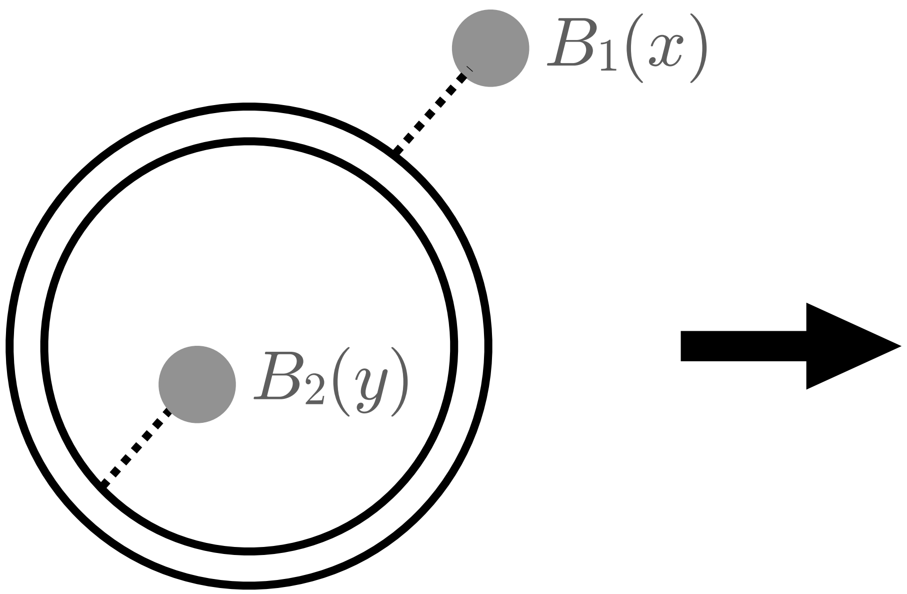
[Fredenhagen, Steinacker '21; Y.A. Steinacker '21; Steinacker '21]

Introduction

Matrix identification of gravity

Also, the interpretations seem to give a solution to the naturalness problem. The quantum corrections in HKK produce a multi-local effective action, which implies that the coupling constants are dynamically fine-tuned.

[Coleman '88; Kawai, Okada '11, '13; Y.A., Kawai, Tsuchiya '12; Hamada, Kawai, Oda '18;…]



$$\int d^d x \, \underset{\substack{\Cup \\ B_1(x)}}{\mathcal{O}_1(x)} \int d^d y \, \underset{\substack{\Cup \\ B_2(y)}}{\mathcal{O}_2(y)} = S_{\text{int},1} \, S_{\text{int},2}$$

$$w(S_{\text{int}}) = e^{i(c_i S_{\text{int},i} + c_{ij} S_{\text{int},i} S_{\text{int},j} + \cdots)} = \int [d\lambda] \, \tilde{w}(\lambda) e^{i\lambda_i S_{\text{int},i}}$$

Very attractive! But how can one be certain about the interpretations of X?

$\cdots$ My motivation to revisit its relationship to the perturbative string

Introduction

Problem: How is the 0D theory defined?

The IKKT action: [Ishibashi, Kawai, Kitazawa, Tsuchiya '96]

$$S[X,\psi;G_{\mu\nu}] = N\,\mathrm{tr}\left[\frac{1}{4}G_{\mu\rho}G_{\nu\sigma}[X^\mu,X^\nu][X^\rho,X^\sigma] + \frac{1}{2}\psi^T G_{\mu\nu}\Gamma^\mu[X^\nu,\psi]\right]$$

X^μ: bosonic $N\times N$ matrices ($\mu = 0,\cdots,9$) ψ: Majorana-Weyl fermionic $N\times N$ matrices

However, we don't really know how the IKKT action enters in the partition fn.

$$Z = \int [dX][d\psi]\, e^{iS[X,\psi;\eta_{\mu\nu}]} \text{ ?} \qquad \left(\eta_{\mu\nu} = \mathrm{diag}(-1,1,\cdots,1)_{\mu\nu}\right)$$

		metric in the action	
		Euclidean	Lorentzian
weight	Euclidean	$e^{-S[X,\psi;\delta_{\mu\nu}]}$	$e^{-S[X,\psi;\eta_{\mu\nu}]}$
	Lorentzian	$e^{iS[X,\psi;\delta_{\mu\nu}]}$	$e^{iS[X,\psi;\eta_{\mu\nu}]}$

"Euclidean IKKT model" (Euclidean weight, Euclidean metric)

"Lorentzian IKKT model" (Lorentzian weight, Lorentzian metric)

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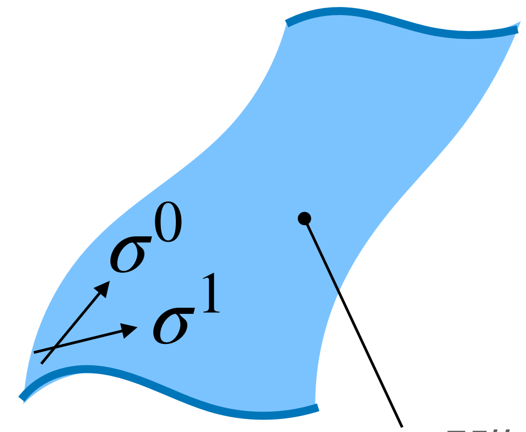
Green-Schwarz formalism



Nambu-Goto-type action

The following respects target-space supersymmetry.

$$S_{\text{GS}} = -\frac{1}{2\pi}\int d^2\sigma \Big\{ \underbrace{\sqrt{-\det(\eta_{\mu\nu}\Pi^\mu_a \Pi^\nu_b)}}_{\text{area of the worldsheet}} - i\varepsilon^{ab}\partial_a X^\mu (\theta^{1T}\Gamma_\mu \partial_b \theta^1 - \theta^{2T}\Gamma_\mu \partial_b \theta^2) + \varepsilon^{ab}\theta^{1T}\Gamma^\mu \partial_a \theta^1\, \theta^{2T}\Gamma_\mu \partial_b \theta^2 \Big\}$$

X^μ: bosons (position of a string) θ^A: Majorana-Weyl fermions ($A = 1,2$)

worldsheet index: $a = 0,1$ target space index: $\mu = 0,\cdots,9$

$$\Pi^\mu_a = \partial_a X^\mu - i(\theta^{1T}\Gamma^\mu \partial_a \theta^1 + \theta^{2T}\Gamma^\mu \partial_a \theta^2)$$

SUSY: $\delta^s \theta^A = \epsilon^A$ $\delta^s X^\mu = i(\epsilon^{1T}\Gamma^\mu \theta^1 + \epsilon^{2T}\Gamma^\mu \theta^2)$

(10D type II SUSY)

[Green, Schwarz '84]

Green-Schwarz formalism

$$S_{\rm GS} = -\frac{1}{2\pi}\int d^2\sigma \Big\{ \sqrt{-\det(\underbrace{\underbrace{\eta_{\mu\nu}\Pi^\mu_a\Pi^\nu_b}_{h_{ab}})}_{h}} - i\varepsilon^{ab}\partial_a X^\mu(\theta^{1T}\Gamma_\mu\partial_b\theta^1 - \theta^{2T}\Gamma_\mu\partial_b\theta^2)$$
$$+\varepsilon^{ab}\theta^{1T}\Gamma^\mu\partial_a\theta^1\,\theta^{2T}\Gamma_\mu\partial_b\theta^2 \Big\}$$

X^μ: 10 bosons

→ 8 { left-moving, right-moving }

θ^A: 32 fermions

→ 8 + 8

They match!

Gauge symmetries

Reparametrisation symmetry:

$$\delta^{\rm b}\theta^A = -\delta\sigma^a\partial_a\theta^A \qquad \delta^{\rm b}X^\mu = -\delta\sigma^a\partial_a X^\mu$$

2 bosons are redundant

"κ symmetry" (local fermionic symmetry):

$$\delta^{\rm f}\theta^1 = (\mathbf{1}+\tilde\Gamma)\kappa^1 \qquad \delta^{\rm f}\theta^2 = (\mathbf{1}-\tilde\Gamma)\kappa^2 \qquad \delta^{\rm f}X^\mu = -i(\delta^{\rm f}\theta^{1T}\Gamma^\mu\theta^1 + \delta^{\rm f}\theta^{2T}\Gamma^\mu\theta^2)$$

$$\left(\tilde\Gamma = \frac{\varepsilon^{ab}}{2\sqrt{-h}}\Gamma_{\mu\nu}\Pi^\mu_a\Pi^\nu_b, \quad \tilde\Gamma^2 = \mathbf{1}\right)$$

16 fermions are redundant

Green-Schwarz formalism

Algebra of kappa symmetry

There have been obstacles of quantisation in the G-S formalism.

- κ symmetry has an infinite series of gauge symmetry

$$\delta^{\rm f}\theta^1 = (\mathbf{1}+\tilde{\Gamma})\kappa^1 \qquad \kappa^1 \sim \kappa^1 + (\mathbf{1}-\tilde{\Gamma})\kappa'^1 \qquad \kappa'^1 \sim \kappa'^1 + (\mathbf{1}+\tilde{\Gamma})\kappa''^1 \qquad \cdots$$

- κ symmetry is not closed off-shell

$$[\delta^{\rm f}_{\kappa_1}, \delta^{\rm f}_{\kappa_2}] = \delta^{\rm b}_{v_3} + \delta^{\rm f}_{\kappa_3} + \delta^{\lambda}_{\lambda_3} + \text{(E.o.M.)}$$

➡ Batalin-Vilkoviski quantisation w/ an infinite tower of ghosts

[Kallosh '89;…]

Schild-type action

The Nambu-Goto-type action is equivalent to the following Schild-type action:

$$S_{\text{Schild}} = -\frac{1}{2\pi}\int d^2\sigma\left\{ -\frac{1}{2}\left(\frac{h}{e_g} - e_g\right) - i\varepsilon^{ab}\partial_a X^\mu(\theta^{1T}\Gamma_\mu\partial_b\theta^1 - \theta^{2T}\Gamma_\mu\partial_b\theta^2) + \varepsilon^{ab}\theta^{1T}\Gamma^\mu\partial_a\theta^1\,\theta^{2T}\Gamma_\mu\partial_b\theta^2 \right\}$$

e_g: a Lagrange multiplier or "gauge field"

E.o.M. for e_g: $e_g^2 = -h$

※ Integrating out e_g brings this back to the Nambu-Goto action.

Remarkably, the fermionic gauge symmetry is formally enhanced:

$$\delta^{\text{f}} X^\mu = -i(\delta^{\text{f}}\theta^{1T}\Gamma^\mu\theta^1 + \delta^{\text{f}}\theta^{2T}\Gamma^\mu\theta^2)$$

$$\delta^{\text{f}} e_g = \frac{4ie_g^2}{e_g^2 + h}\sum_{A=1}^{2}\left(\frac{-h}{e_g}h^{ab} + (-1)^{A+1}\varepsilon^{ab}\right)\delta^{\text{f}}\theta^{AT}\Gamma_\mu\Pi_a^\mu\partial_b\theta^A$$

$\delta^{\text{f}}\theta^A$ is not projected by $\frac{1}{2}(\mathbf{1} \pm \tilde{\Gamma})$.

[Y.A. to appear]

Enhanced kappa symmetry

Algebra of the enhanced kappa symmetry

Each sector of the enhanced κ symmetry

$$\delta^{\mathrm{f}} = \delta^{\mathrm{f}\varphi} + \delta^{\mathrm{f}\psi} \qquad \text{with} \qquad \varphi = \frac{1}{2}(\theta_1 + i\theta_2) \qquad \psi = \frac{1}{2}(\theta_1 - i\theta_2)$$

is closed off-shell, with a “trivial” gauge symmetry $\delta^{\mathrm{g}} e_g = \frac{e_g^2}{e_g^2 + h}\partial_a(e_g \mu^a)$

$$[\delta^{\mathrm{f}\varphi}_{\kappa_1}, \delta^{\mathrm{f}\varphi}_{\kappa_2}] = \delta^{\mathrm{f}\varphi}_{\kappa_3} + \delta^{\mathrm{g}}_{\mu_3}$$

$$[\delta^{\mathrm{b}}_{\nu_1}, \delta^{\mathrm{b}}_{\nu_2}] = \delta^{\mathrm{b}}_{\nu_3}, \qquad [\delta^{\mathrm{b}}_{\nu}, \delta^{\mathrm{f}\varphi}_{\kappa}] = \delta^{\mathrm{f}\varphi}_{\kappa'} - \delta^{\mathrm{b}}_{\delta_\kappa \nu},$$

$$[\delta^{\mathrm{b}}_{\nu}, \delta^{\mathrm{g}}_{\mu}] = \delta^{\mathrm{g}}_{\mu'} - \delta^{\mathrm{b}}_{\delta_\mu \nu}, \qquad [\delta^{\mathrm{f}\varphi}_{\kappa}, \delta^{\mathrm{g}}_{\mu}] = \delta^{\mathrm{f}\varphi}_{\kappa''} + \delta^{\mathrm{g}}_{\mu''}, \qquad [\delta^{\mathrm{g}}_{\mu_1}, \delta^{\mathrm{g}}_{\mu_2}] = \delta^{\mathrm{g}}_{\mu_3'},$$

[Y.A. to appear]

➡ BRST quantisation w/o ghosts of ghosts

※ Note the algebra is closed even if we take only the area-preserving diffeo. part $\delta^{\mathrm{b}'} X^\mu = -\,\varepsilon^{ab}\partial_b \xi \partial_a X^\mu$ instead of δ^{b}. ⋯ SU(N) trf. after the matrix regularisation

Schild-type action

Gauge-fixing for obtaining the IKKT action

Fixing the gauge by $e_g = \hat{e}_g(\sigma) = 1 \qquad \varphi = 0$

one obtains

$$S_{\text{Schild}} = \frac{1}{2\pi}\int d^2\sigma \left[\frac{1}{4}\underline{\{X^\mu, X^\nu\}^2_{\hat{\mathrm{P}}}} - \frac{1}{2} + 2i\psi^T\Gamma_\mu\{X^\mu, \psi\}_{\hat{\mathrm{P}}} + \text{ghosts}\right]$$

$$h = \det(\partial_a X^\mu \partial_b X_\mu) = \frac{1}{2}(\underbrace{\varepsilon^{ab}\partial_a X^\mu \partial_b X^\nu}_{=:\ \{X^\mu, X^\nu\}_{\hat{\mathrm{P}}}})^2$$

Matrix regularisation of this action would give the Lorentzian IKKT.

However, this Poisson bracket is defined on (1+1)D worldsheet…

(matrix reg. wouldn't be well-defined)

“Derivation” of the IKKT model

Wick rotation

Unlike the Polyakov-type action, we can find a Wick rotation that rigorously connects the Lorentzian and Euclidean for the Schild-type action:

$$\sigma^0 = e^{-i\theta}\sigma^2, \qquad X^0 = e^{-i\theta}X^{10}, \qquad \psi = e^{i\theta/2}\psi^{(\mathrm{E})}$$

Then, $$\{f_1, f_2\}_{\mathrm{P}} = -e^{i\theta}\sum_{a,b=1}^{2}\frac{\varepsilon^{ab}}{e_g}\partial_a f_1 \partial_b f_2 =: -e^{i\theta}\{f_1, f_2\}_{\mathrm{P}}^{(\mathrm{E})}$$

$$\exp[iS_{\mathrm{Schild}}] = \exp\left[\frac{i}{2\pi}\int d\sigma^0 d\sigma^1\left(\frac{1}{4}\{X^i, X^j\}_{\hat{\mathrm{P}}}^2 - \frac{1}{2}\{X^0, X^i\}_{\hat{\mathrm{P}}}^2 - \frac{1}{2} + 2i\psi^T\Gamma_i\{X^i, \psi\}_{\hat{\mathrm{P}}} + 2i\psi^T\Gamma_0\{X^0, \psi\}_{\hat{\mathrm{P}}}\right)\right]$$

$$= \exp\left[-\frac{1}{2\pi}\int d\sigma^1 d\sigma^2\left(\frac{-ie^{i\theta}}{4}(\{X^i, X^j\}_{\hat{\mathrm{P}}}^{(\mathrm{E})})^2 + \frac{ie^{-i\theta}}{2}(\{X^{10}, X^i\}_{\hat{\mathrm{P}}}^{(\mathrm{E})})^2 \right.\right.$$
$$\left.\left. -2e^{2i\theta}\psi^{(\mathrm{E})T}\Gamma_i\{X^i, \psi^{(\mathrm{E})}\}_{\hat{\mathrm{P}}}^{(\mathrm{E})} - 2ie^{i\theta}\psi^{(\mathrm{E})T}\Gamma_{10}\{X^{10}, \psi^{(\mathrm{E})}\}_{\hat{\mathrm{P}}}^{(\mathrm{E})} + \frac{ie^{-i\theta}}{2}\right)\right]$$

$$\xrightarrow{\theta \to \frac{\pi}{2}} \exp\left[-\frac{1}{2\pi}\int d\sigma^1 d\sigma^2\left(\frac{1}{4}\{X^m, X^n\}_{\hat{\mathrm{P}}}^{(\mathrm{E})\,2} + 2\psi^{(\mathrm{E})T}\Gamma_m\{X^m, \psi^{(\mathrm{E})}\}_{\hat{\mathrm{P}}} + \frac{1}{2}\right)\right]$$

$(m = 1, \cdots, 9, 10)$

We’ve derived the Euclidean path integral by a change of contour.

“Derivation” of the IKKT model

Matrix regularisation [Hoppe ’82]

Regularisation by a map from a function to a matrix

$$f(\sigma) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} f_{lm} Y_{lm}(\sigma) \longrightarrow \sum_{l=0}^{N-1} \sum_{m=-l}^{l} f_{lm} (Y_{lm})_{ij} = f_{ij}$$

$Y_{lm}(\sigma)$: spherical harmonics; f_{ij}: matrix

This maps the Poisson bracket and worldsheet integral to

$$\{\,\cdot\,,\,\cdot\,\}^{(\mathrm{E})}_{\hat{\mathrm{P}}} \to \frac{2N}{i}[\,\cdot\,,\,\cdot\,], \qquad \frac{1}{\pi}\int d\sigma^1 d\sigma^2 \to \frac{1}{N}\,\mathrm{tr},$$

Then the action becomes, with rescaling of X^m and $\psi^{(\mathrm{E})}$,

$$\exp[-S^{(\mathrm{E})}] = \exp\left[-N\,\mathrm{tr}\left(-\frac{1}{4}[X^m, X^n]^2 - \frac{i}{2}\psi^{(\mathrm{E})T}\Gamma_m[X^m, \psi^{(\mathrm{E})}] + \frac{1}{4N}\right)\right]$$

We have “derived” the Euclidean weight w/ the Euclidean IKKT action from the perturbative superstring theory.

“Derivation” of the IKKT model

Wick-rotating back the theory

Actually, the Euclidean IKKT model w/ e^{-S} is equivalent to
a Lorentzian IKKT model w/ e^{iS} with natural regulators introduced

By a change of contour $X^i = e^{i\theta/4}\tilde{X}^i, \quad X^{10} = -e^{-3i\theta/4}\tilde{X}^0, \quad \psi^{(\mathrm{E})} = e^{3i\theta/8}\psi$

$$\exp\left[-N\,\mathrm{tr}\left(-\frac{1}{4}[X^m, X^n]^2 - \frac{i}{2}\psi^{(\mathrm{E})T}\Gamma_m[X^m, \psi^{(\mathrm{E})}]\right)\right]$$

$$= \exp\left[N\,\mathrm{tr}\left(\frac{e^{-i\theta}}{2}[\tilde{X}^0, \tilde{X}^i]^2 + \frac{e^{i\theta}}{4}[\tilde{X}^i, \tilde{X}^j]^2 - \frac{1}{2}\psi^T\Gamma_0[\tilde{X}^0, \psi] + \frac{ie^{i\theta}}{2}\psi^T\Gamma_i[\tilde{X}^i, \psi]\right)\right]$$

$$\xrightarrow{\theta \to \frac{\pi}{2}} \exp\left[iN\,\mathrm{tr}\left(\frac{1}{4}[\tilde{X}_\mu, \tilde{X}_\nu][\tilde{X}^\mu, \tilde{X}^\nu] + \frac{i}{2}\psi^T\Gamma_\mu[\tilde{X}^\mu, \psi] + i\varepsilon\left((X^i)^2 + (X^0)^2\right)\right)\right]_{\varepsilon\to 0}$$

So, we get

$$Z = \int [dX][d\psi^{(\mathrm{E})}]\, e^{-S[X,\psi^{(\mathrm{E})};\delta_{\mu\nu}]} \propto \int [d\tilde{X}][d\psi]\, e^{iS[\tilde{X},\psi;\eta_{\mu\nu}]}$$

This is a well-defined finite integral for finite N.

[Krauth, Nicolai, Staudacher ’98; Austing, Wheater ’01]

“Derivation” of the IKKT model

Caveat about the Lorentzian IKKT model

However there is another definition of the Lorentzian IKKT model w/ e^{iS}:

$$Z_\gamma = \int [d\tilde{X}][d\psi] \exp \left[iN \operatorname{tr} \left(\frac{1}{4}[\tilde{X}^\mu, \tilde{X}^\nu]^2 + \underline{\frac{\gamma}{2}(e^{-i\varepsilon}\tilde{X}^i\tilde{X}^i - e^{i\varepsilon}\tilde{X}^0\tilde{X}^0)} + \frac{i}{2}\psi^T\Gamma^\mu[\tilde{X}_\mu, \psi] \right) \right]$$

$\varepsilon \to 0^+$ for $\gamma > 0$, $\varepsilon \to 0^-$ for $\gamma < 0$

$$\to \int [dX][d\psi^{(\mathrm{E})}] \exp\left[-N \operatorname{tr}\left(-\frac{1}{4}[X^m, X^n]^2 - \frac{e^{\frac{\pi i}{4}}\gamma}{2}(e^{-i\varepsilon}X^iX^i + e^{i\varepsilon}X^{10}X^{10}) - \frac{i}{2}\psi^{(\mathrm{E})T}\Gamma_m[X^m, \psi^{(\mathrm{E})}] \right) \right]$$

The model w/ $\gamma \to 0^-$ is equivalent to the Euclidean model w/ e^{-S} because Cauchy’s thm. connects them via the change of contour.

But not for $\gamma > 0$; thus the IKKT model w/ $\gamma \to 0^+$ is different!

[Y.A., Nishimura, Piensuk, Yamamori;
Y.A., Chou, Nishimura, Piensuk, Tripathi, Yamamori, to appear]

This difference would be interpreted as “different definitions of the vacuum.”

Vertex operators

The BRST transformation on the worldsheet is

$$\delta^{\mathrm{BRST}} X^\mu = -2i\epsilon\gamma^T\Gamma^\mu\psi + \epsilon'\{c, X^\mu\}_{\hat{\mathrm{P}}}, \quad \delta^{\mathrm{BRST}}\psi = \epsilon'\{c,\psi\}_{\hat{\mathrm{P}}}, \quad \delta^{\mathrm{BRST}}\varphi = \epsilon\gamma + \epsilon'\{c,\varphi\}_{\hat{\mathrm{P}}},$$

A BRST inv. vertex: $\int d^2\sigma\, e^{ik_\mu(X^\mu + 2i\varphi^T\Gamma^\mu\psi)} \to \int d^2\sigma\, e^{ik_\mu X^\mu}$ (momentum-k_μ mode)

➡ In the matrix model, $V^\Phi = \mathrm{tr}\, e^{ik_\mu X^\mu}$

This forms a massless multiplet of type IIB SUGRA by acting the supercharge operator Q onto this vertex.

$$e^{\lambda^T Q}\,\mathrm{tr}\, e^{ik_\mu X^\mu} e^{-\lambda^T Q} = \mathrm{tr}\, e^{ik_\mu X^\mu + \psi^T k_\mu \Gamma^\mu \lambda + \cdots}$$

$$= V^\Phi + V^{\tilde{\Phi}} k_\mu \Gamma^\mu \lambda + V^B_{\mu\nu} k_\rho \lambda^T \Gamma^{\mu\nu\rho}\lambda + \cdots$$

λ^0	λ^1	λ^2	λ^3	λ^4	…
dilaton Φ	dilatino $\tilde{\Phi}$	$B_{\mu\nu}$	gravitino Ψ	graviton $h_{\mu\nu}$ & $A_{\mu\nu\rho\sigma}$	…

[Kitazawa ’02; Iso, Terachi, Umetsu ’04; Kitazawa, Mizoguchi, Saito ’07]

Summary

- We find there is a set of gauge trf. that closes its algebra for the string action in the Green-Schwarz formalism by rewriting the action to the Schild type. It allows us to quantise the theory without an infinite tower of ghosts.

- If we assume the IKKT matrix model is derived as a pure matrix regularisation of the Schild-type action, it is the Euclidean action w/ e^{-S}.

- There is a Lorentzian IKKT model w/ e^{iS} equiv. to the Euclidean one w/ e^{-S}. However, since the Lorentzian IKKT partition fn. is conditionally convergent, there is another Lorentzian IKKT model inequivalent to the Euclidean one. Probably they share the same structure of the $1/N$ expansion, but the "derivation" cannot tell the correct "definition of the vacuum."

- A BRST-inv. massless-mode vertex is consistent with the suggested matrix-model vertex operator $\mathrm{tr}\, e^{ik_\mu X^\mu}$ of a string (or D1), which forms a massless multiplet of type IIB SUGRA by acting the supercharge operator. Further analysis should clarify how to construct massive states in the IKKT.