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Born-Again Braneworld

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S. Kanno, M. Sasaki & J. Soda, [hep-th/0210250]
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§1. Introduction

Historical Notes

Superstring Theory ··· 10 Dimensions
Our Universe ··· 4 Dimensions

$$\Downarrow$$

Before 1996
Compactification : Kalza-Klein Compactification
- **Kalza-Klein Cosmology**

After 1996
- **Hořava-Witten Braneworld!**
- **Randall-Sundrum (RS) Model ··· Hierarchy Problem**

$$\Downarrow$$

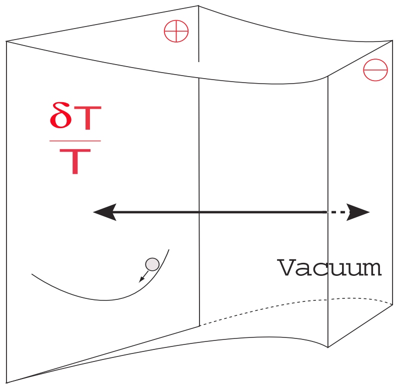
Braneworld Cosmology

$$H^2 = \frac{8\pi G}{3}\rho + \underbrace{\frac{\kappa^4}{36}\rho^2}_{\text{HighEnyergy}} + \underbrace{\frac{C}{a^4}}_{\text{DarkRadiation}}$$

Can we observe the effects of extra dimensions?

RS Two-Brane Cosmological Scenario

Assumption



Inflation occurs on the brane, when the energy scale,

$$\sigma \gtrsim (E_{\text{inflation}})^4$$

σ**: brane tension**

⇒Effective Action Approach

- **Large-angle CMB anisotropy :** $\dfrac{\delta T}{T}$
- **Primordial gravitational wave (GW) :** h_{ij}^{TT}

Strategy

1. **Derive the brane effective action**

   ★ **slow-roll era ≃ vacuum brane**

2. **Consider nonlinear dynamics of branes with vacuum energy**
   ⇒ **Born-Again Braneworld Scenario**

3. **Consider the cosmological perturbation**
   ⇒ **Analyze the CMB anisotropy and GW**
   - **Consistent with CMB observation**
   - **Blue spectrum could be detected by Advanced LIGO or LISA**

§2. Effective Action

5D Action

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left(\mathcal{R} + \frac{12}{l^2}\right)$$
$$- \sum_{i=\oplus,\ominus}\sigma_i\int d^4x\sqrt{-g^i_{\rm brane}} + \sum_{i=\oplus,\ominus}\int d^4x\sqrt{-g^i_{\rm brane}}\,\mathcal{L}^i_{\rm matter}$$

Goal : Brane Effective Theory for $\dfrac{\rho}{\sigma} \sim \ell^2 R \ll 1$

Valid Regime :

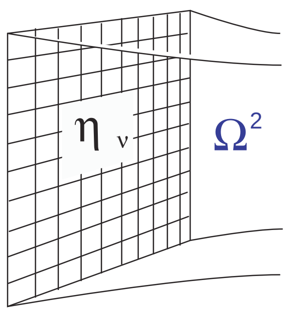
* **Energy Scale** $\leq 10^{15}{\rm GeV}\left(\dfrac{10^{-24}{\rm cm}}{\ell}\right)^{1/2}$
* **Gravitational Radius** $\geq 10^{-29}{\rm km}\left(\dfrac{\ell}{10^{-24}{\rm cm}}\right)$

Geometry

Vacuum Brane:

$$ds^2 = dy^2 + \Omega^2(y)\eta_{\mu\nu}dx^\mu dx^\nu\,.$$

$\Omega^2 = \exp[-2\frac{y}{\ell}]$: Warp Factor

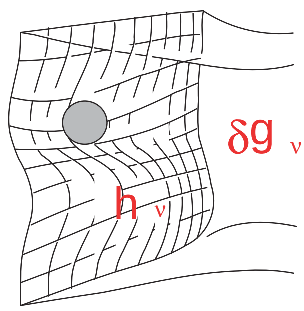


Brane with Matter:

$$ds^2 = dy^2 + \left(\Omega^2(y)h_{\mu\nu}(x) + \delta g_{\mu\nu}(y, x^\mu)\right)dx^\mu dx^\nu\,.$$

$h_{\mu\nu}$: **induced metric on the brane**

$\delta g_{\mu\nu}(y = 0, x^\mu) = 0$



General Formalism

5D Einstein Equations:

$$G^{(5)}_{AB} = \frac{6}{\ell^2} g_{AB} + \delta(y) 8\pi G_N \ell (-\sigma g_{\mu\nu} + T_{\mu\nu}) \delta^\mu_A \delta^\nu_B \ , \quad A = (y, \mu)$$

- **Bulk equation:**

$$\delta K^\mu{}_\nu = -\frac{1}{2}\delta(g^{\mu\alpha} g_{\alpha\nu,y}) \equiv \delta\Sigma^\mu{}_\nu + \frac{1}{4}\delta^\mu{}_\nu \delta K \ , \quad \delta\Sigma^\mu{}_\mu = 0$$

Hamiltonian Constraint:

$$\delta K = -\frac{\ell}{6}\left[\frac{3}{4}\delta K^2 - \delta\Sigma^\mu{}_\nu \delta\Sigma^\nu{}_\mu - \overset{(4)}{R}\right]$$

Momentum Constraint:

$$\nabla_\lambda \delta\Sigma^\lambda{}_\mu - \frac{3}{4}\nabla_\mu \delta K = 0$$

Evolution Equation:

$$\frac{1}{\Omega^4}\left[\Omega^4 \delta\Sigma^\mu{}_\nu\right]_{,y} = \delta K \delta\Sigma^\mu{}_\nu - \left[\overset{(4)}{R}{}^\mu{}_\nu\right]_{\text{traceless}}$$

- **Junction condition:**

$$\frac{2}{\ell}\left[\delta\Sigma^{\mu}{}_{\nu}-\frac{3}{4}\delta^{\mu}_{\nu}\delta K\right]\Bigg|_{y=0}=8\pi G_N T^{\mu}_{\nu}$$

$$\begin{aligned}\frac{2}{\ell}\left[\delta\Sigma^{\mu}{}_{\nu}-\frac{3}{4}\delta^{\mu}_{\nu}\delta K\right] &= -\frac{\chi^{\mu}{}_{\nu}}{\Omega^4}-\frac{2}{\ell\Omega^4}\int_{\infty}^{y}dy\Omega^4\left[\overset{(4)}{R}{}^{\mu}{}_{\nu}-\frac{1}{4}\delta^{\mu}_{\nu}\overset{(4)}{R}-\delta K\delta\Sigma^{\mu}{}_{\nu}\right]\\ &\quad -\frac{1}{4}\delta^{\mu}_{\nu}\overset{(4)}{R}+\frac{1}{4}\delta^{\mu}_{\nu}\left[\frac{3}{4}\delta K^2-\delta\Sigma^{\alpha}{}_{\beta}\delta\Sigma^{\beta}{}_{\alpha}\right]\\ &= -\frac{\chi^{\mu}{}_{\nu}(x)}{\Omega^4(y)}+\overset{(4)}{G}{}^{\mu}{}_{\nu}\left(\Omega^2(y)h_{\mu\nu}(x)\right)+\mathcal{O}(\ell^4R^2)\end{aligned}$$

$$\boxed{\overset{(4)}{G}{}^{\mu}{}_{\nu}(\Omega^2_{y=0}h_{\mu\nu})=8\pi G_N T^{\mu}{}_{\nu}+\frac{\chi^{\mu}{}_{\nu}}{\Omega^4_{y=0}}}$$

$\cdots$ Brane Effective Theory

$\chi^{\mu}{}_{\nu}\cdots$ **Integration constant.** $\chi^{\mu}{}_{\mu}=\chi^{\mu}{}_{\nu|\mu}=0$

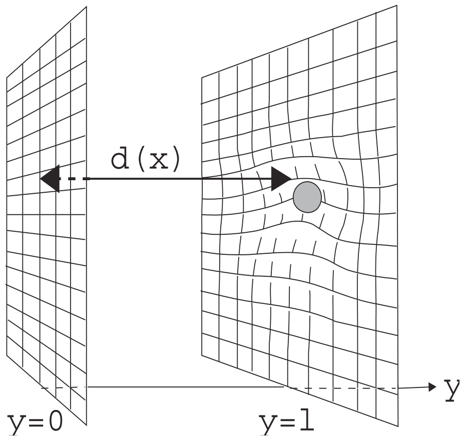


FIG. 1: S^1/Z_2 orbifold spacetime with positive tension brane and negative tension brane at the fixed points.

Radion

$$d(x) = e^{\phi(x)}\ell \Longrightarrow \Omega^2 = \exp[-2\frac{d(x)}{\ell}]$$

$\cdots$ Warp Factor

Effective Equation

- **Positive tension brane**

$$\overset{(4)}{G^{\mu}}_{\nu}\,(h_{\mu\nu}) = \frac{\kappa^2}{\ell}\,\overset{\oplus}{T^{\mu}}_{\nu} + \ell^2 \chi^{\mu}{}_{\nu}$$

- **Negative tension brane**

$$\overset{(4)}{G^{\mu}}_{\nu}\,(\Omega^2 h_{\mu\nu}) = -\frac{\kappa^2}{\ell}\,\overset{\ominus}{T^{\mu}}_{\nu} + \frac{\ell^2}{\Omega^4}\chi^{\mu}{}_{\nu}$$

Each Eq. holds irrespective of the existence of the other brane.
We can eliminate $\chi^{\mu}{}_{\nu}$ or $G^{\mu}{}_{\nu}(h)$ from the above two equations.

Eliminating $\chi_{\mu\nu}$ and introducing a new field $\Psi = 1 - \Omega^2$,

$$G^\mu{}_\nu(h) = \frac{\kappa^2}{l\Psi} T^{\oplus\mu}{}_\nu + \frac{\kappa^2(1-\Psi)^2}{l\Psi} T^{\ominus\mu}{}_\nu + \frac{1}{\Psi}\left(\Psi^{|\mu}{}_{|\nu} - \delta^\mu_\nu \Psi^{|\alpha}{}_{|\alpha}\right) + \frac{\omega(\Psi)}{\Psi^2}\left(\Psi^{|\mu}\Psi_{|\nu} - \frac{1}{2}\delta^\mu_\nu \Psi^{|\alpha}\Psi_{|\alpha}\right)$$

Coupling Function

$$\omega(\Psi) = \frac{3}{2}\frac{\Psi}{1-\Psi}$$

Eliminating $G^\mu_\nu(h)$,

$$\chi^\mu{}_\nu = -\frac{\kappa^2(1-\Psi)}{2\Psi}\left(T^{\oplus\mu}{}_\nu + (1-\Psi)T^{\ominus\mu}{}_\nu\right) - \frac{l}{2\Psi}\left[\left(\Psi^{|\mu}{}_{|\nu} - \delta^\mu_\nu \Psi^{|\alpha}{}_{|\alpha}\right) + \frac{\omega(\Psi)}{\Psi}\left(\Psi^{|\mu}\Psi_{|\nu} - \frac{1}{2}\delta^\mu_\nu \Psi^{|\alpha}\Psi_{|\alpha}\right)\right]$$

The traceless condition $\chi^\mu{}_\mu = 0$ gives

$$\Box\Psi = \frac{\kappa^2}{l}\frac{T^\oplus + (1-\Psi)T^\ominus}{2\omega+3} - \frac{1}{2\omega+3}\frac{d\omega}{d\Psi}\Psi^{|\mu}\Psi_{|\mu}$$

$\cdots$ Quasi-Scalar-Tensor gravity
(from $\oplus$-tension brane's point of view)

· $\chi_{\mu\nu}$ is an auxiliary field which is determined only after Ψ is solved.
· Effective theory from $\ominus$-tension brane's point of view is also obtained.

Effective Action on the Positive Tension Brane

$$S_\oplus = \frac{l}{2\kappa^2}\int d^4x\sqrt{-h}\left[\Psi R(h) - \frac{3}{2(1-\Psi)}\Psi^{|\alpha}\Psi_{|\alpha}\right] + \int d^4x\sqrt{-h}\mathcal{L}^\oplus + \int d^4x\sqrt{-h}\,(1-\Psi)^2\,\mathcal{L}^\ominus$$

Intrducing a new field: $\Phi = \frac{1}{\Omega^2} - 1$

Effective Action on the Negative Tension Brane

$$S_\ominus = \frac{l}{2\kappa^2}\int d^4x\sqrt{-f}\left[\Phi R(f) + \frac{3}{2(1+\Phi)}\Phi^{;\alpha}\Phi_{;\alpha}\right] + \int d^4x\sqrt{-f}\mathcal{L}^\ominus + \int d^4x\sqrt{-f}(1+\Phi)^2\mathcal{L}^\oplus$$

where $f_{\mu\nu} = \Omega^2 h_{\mu\nu}$.

Coupling Function

$$\omega(\Phi) = -\frac{3}{2}\frac{\Phi}{1+\Phi}$$

§3. Born-Again Braneworld

Jordan-Frame Effective Action ($\mathcal{L}^{\oplus} = -\delta\sigma^{\oplus}, \quad \mathcal{L}^{\ominus} = -\delta\sigma^{\ominus}$)

$$S_{\oplus} = \frac{l}{2\kappa^2}\int d^4x\sqrt{-h}\left[\Psi R - \frac{\omega(\Psi)}{\Psi}\Psi^{|\alpha}\Psi_{|\alpha}\right] -\delta\sigma^{\oplus}\int d^4x\sqrt{-h} - \delta\sigma^{\ominus}\int d^4x\sqrt{-h}\,(1-\Psi)^2$$

$$\omega(\Psi) = \frac{3}{2}\frac{\Psi}{1-\Psi}$$

$\cdots$ Quasi-Scalar-Tensor gravity

Einstein-frame Effective Action

Conformal transformation : $h_{\mu\nu} = \frac{1}{\Psi}g_{\mu\nu}$

Introducing a new field :

$$\eta = -\log\left|\frac{\sqrt{1-\Psi}-1}{\sqrt{1-\Psi}+1}\right|$$

$$S_{\oplus} = \frac{l}{2\kappa^2}\int d^4x\sqrt{-g}\left[R(g) - \frac{3}{2}\eta^{;\alpha}\eta_{;\alpha}\right] - \int d^4x\sqrt{-g}\;V(\eta)$$

This action is also obtained starting from the effective action on the $\ominus$-tension brane

Conformal transformation : $h_{\mu\nu} = \frac{1}{\Phi}g_{\mu\nu}$

A new field :

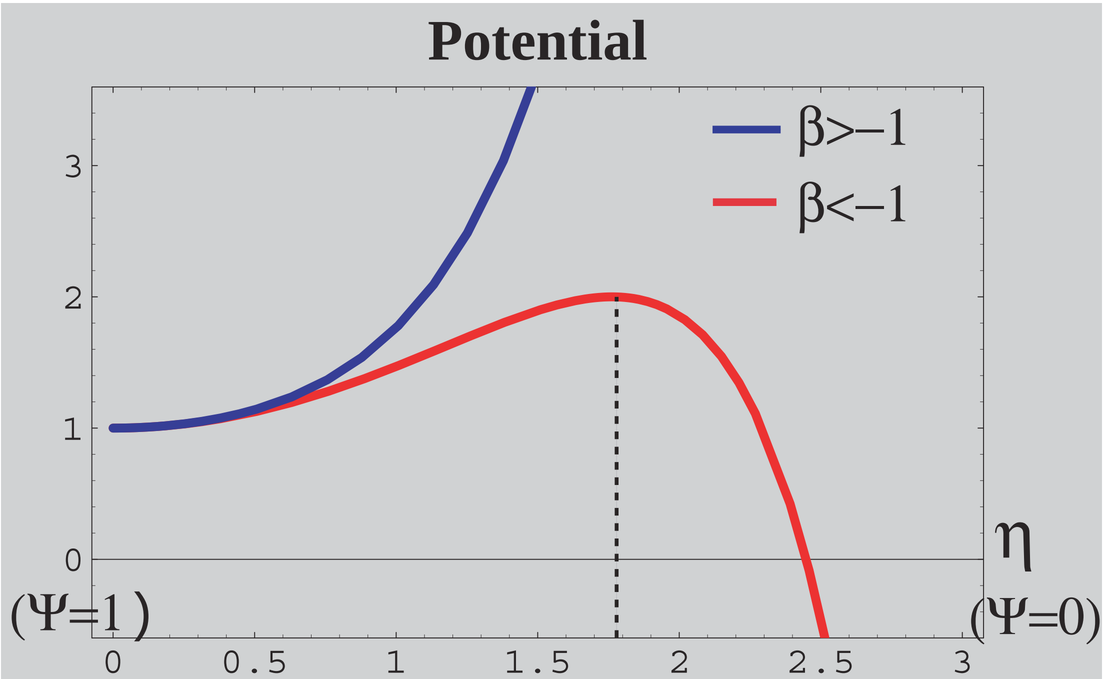
$$\eta = -\log\left|\frac{\sqrt{\Phi+1}-1}{\sqrt{\Phi+1}+1}\right|$$

Potential

Ψ	$-\infty$	$\cdots$	0	$\cdots$	1
η	0	$\cdots$	∞	$\cdots$	0

In the case of $0 < \Psi < 1$

$$V(\eta) = \delta\sigma^{\oplus}\left[\ \cosh^4\frac{\eta}{2} + \beta\sinh^4\frac{\eta}{2}\ \right], \qquad \beta = \frac{\delta\sigma^{\ominus}}{\delta\sigma^{\oplus}}$$



When $\beta < -1(\delta\sigma^{\oplus} + \delta\sigma^{\ominus} < 0)$, maximum at $\Psi_c = 1 + \frac{1}{\beta}$.
Two branes would collide, provided that

$$\Psi < \Psi_c$$

What happens to us if the two branes collide?

After Collision

Effective Action on the $\oplus$-tension brane after collision .
(replacing Ψ as $\Psi \longrightarrow -\tilde{\Psi}$)

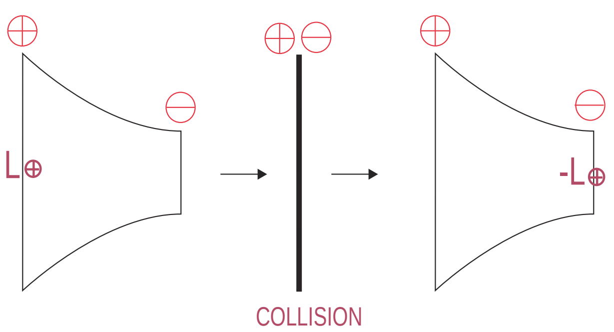
$$-S_{\oplus} = \frac{\ell}{2\kappa^2}\int d^4x\sqrt{-h}\left[\ \tilde{\Psi}R(h) + \frac{3}{2}\frac{1}{1+\tilde{\Psi}}\tilde{\Psi}^{|\alpha}\tilde{\Psi}_{|\alpha}\ \right] + \int d^4x\sqrt{-h}(-\mathcal{L}^{\oplus}) + \int d^4x\sqrt{-h}\left(1+\tilde{\Psi}\right)^2(-\mathcal{L}^{\ominus})$$

Comparing with the action on the $\ominus$-tension brane

$$S_{\ominus} = \frac{\ell}{2\kappa^2}\int d^4x\sqrt{-f}\left[\ \Phi R(f) + \frac{3}{2}\frac{1}{1+\Phi}\Phi^{;\alpha}\Phi_{;\alpha}\ \right] + \int d^4x\sqrt{-h}\ \mathcal{L}^{\ominus} + \int d^4x\sqrt{-h}\left(1+\tilde{\Psi}\right)^2\mathcal{L}^{\oplus}$$

$\oplus$-tension brane$\Longrightarrow$ $\ominus$-tension brane
The sign in front of the matter Lagrangians also changes

We were initially on the $\ominus$-tension brane before the collision!?



Cosmological Evolution

EOM for a Vacuum Brane

Spatially isotropic and homogeneous metric (K=0 flat)

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

Inflation: $p = -\rho, \quad \rho^{\oplus} = \delta\sigma^{\oplus}, \quad \rho^{\ominus} = \delta\sigma^{\ominus}$

$$-3H^2 = -\frac{\kappa^2}{\ell}\frac{1}{\Phi}\delta\sigma^{\ominus} - \frac{\kappa^2}{\ell}\frac{(1+\Phi)^2}{\Phi}\delta\sigma^{\oplus} + 3H\frac{\dot{\Phi}}{\Phi} + \frac{3}{4}\frac{\dot{\Phi}^2}{\Phi(1+\Phi)},$$

$$-2\dot{H} - 3H^2 = -\frac{\kappa^2}{\ell}\frac{1}{\Phi}\delta\sigma^{\ominus} - \frac{\kappa^2}{\ell}\frac{(1+\Phi)^2}{\Phi}\delta\sigma^{\oplus} + \frac{\ddot{\Phi}}{\Phi} + 2H\frac{\dot{\Phi}}{\Phi} - \frac{3}{4}\frac{\dot{\Phi}^2}{\Phi(1+\Phi)} \ ,$$

$$\ddot{\Phi} + 3H\dot{\Phi} = \frac{4\kappa^2}{3\ell}(1+\Phi)\left[\delta\sigma^{\ominus} + (1+\Phi)\delta\sigma^{\oplus}\right] + \frac{1}{2}\frac{1}{1+\Phi}\dot{\Phi}^2 \ .$$

Friedmann equation with the dark radiation

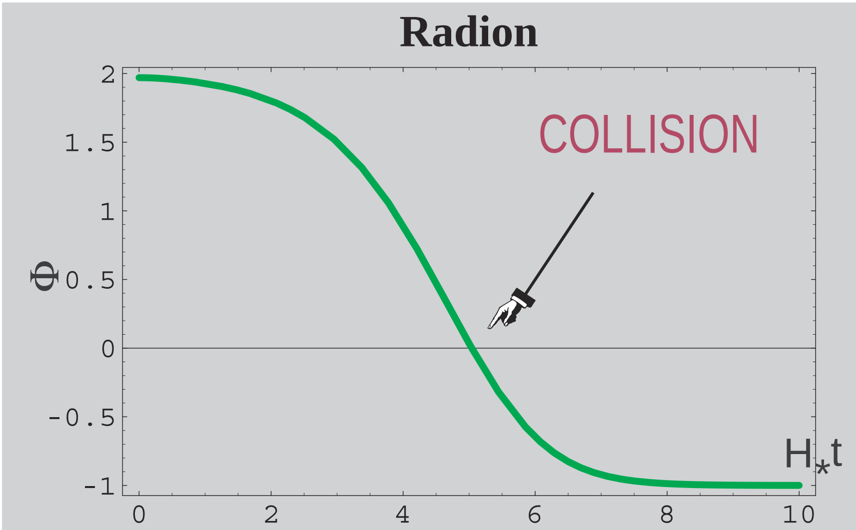
$$H^2 + \frac{K}{a^2} = -\frac{\kappa^2}{3\ell}\delta\sigma^{\ominus} + \frac{C}{a^4} \ .$$

Relation between the radion and the dark radiation

$$\frac{\kappa^2\delta\sigma^{\ominus}}{3\ell}\frac{1+\Phi}{\Phi}\left[1 + \frac{(1+\Phi)}{\beta}\right] - H\frac{\dot{\Phi}}{\Phi} - \frac{1}{4}\frac{1}{1+\Phi}\frac{\dot{\Phi}^2}{\Phi} = \frac{C}{a^4} \ .$$

Does the Born-Again Braneworld scenario realize?

Numerical solution of Φ



Φ passes through zero smoothly and approaches -1

Analitic solution around the time of collision

$$\Phi = -2(1-\sqrt{\gamma})H_c(t-t_c)\,; \quad \gamma = 1 - \frac{H_*^2}{H_c^2}\left(1+\frac{1}{\beta}\right)\,.$$

$H_* = \kappa^2/3\ell(-\delta\sigma^B)$
H_c : Hubble constant at the time of collision $t = t_c$.

Φ behaves perfectly smoothly around the time of collision

Einstein frame

The relation: Einstein frame $\Leftrightarrow$ Jordan frame

$$\begin{aligned} ds_E^2 &= -dt_E^2 + b^2(t_E)\delta_{ij}dx^i dx^j \\ &= |\Phi|\left[-dt_J^2 + a(t_J)^2\delta_{ij}dx^i dx^j\right], \end{aligned}$$

$$\Longrightarrow b = \sqrt{|\Phi|}\,a, \quad dt_E = \sqrt{|\Phi|}\,dt_J$$

Hubble parameter in the Einstein frame around the collision

$$\frac{\dot{b}(t_E)}{b(t_E)} = \frac{1}{3t_E} + \frac{H_c}{\left(3(1-\sqrt{\gamma})H_c|t_E|\right)^{1/3}}\,.$$

$t_E = 0$: collision time in the Einstein frame

Einstein frame : Big-Bang Singularity.
Jordan frame : Pre-Big-Bang phase $\Leftrightarrow$ Post-Big-Bang phase

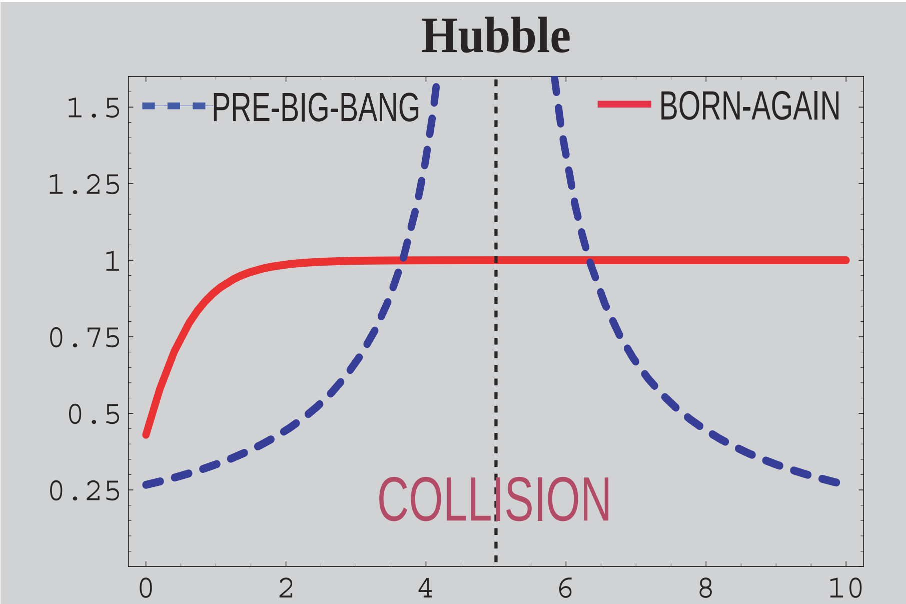


FIG. 2: The evolution of the Hubble constant in the Jordan frame. The solution rapidly approaches to the de-Sitter spacetime. We also ploted the pre-big-bang solution in the Einstein frame.

§4. Observational implication

Cosmological perturbations are generated from quantum (vacuum) fluctuations of the inflaton ϕ and the metric $g_{\mu\nu}$

Results of Standard Inflation

FRW Metric (K=0):

$$
\begin{aligned}
ds^2 &= -dt^2 + a^2(t)\delta_{ij}dx^i dx^j \\
&= a^2(\eta)\left[-d\eta^2 + \delta_{ij}dx^i dx^j\right] .
\end{aligned}
$$

η: conformal time

- Canonical quantization

Canonical Action:

$$
S = \int d^3x d\eta \left[\frac{1}{2}\psi'^2 - \frac{1}{2}\partial_i\psi\partial^i\psi + \frac{a''}{2a}\psi^2\right]
$$

$\psi = a\delta\phi$: canonical variable

Equation of Motion

$$
\psi'' - \frac{a''}{a}\psi - \partial^i\partial_i\psi = 0
$$

Canonical momentum & commutation relation :

$$
\pi(\vec{x}, \eta) = \psi' , \qquad [\psi(\vec{x}, \eta), \pi(\vec{x}', \eta)] = i\delta(\vec{x} - \vec{x}')
$$

Mode Expansion :

$$
\psi(\vec{x}, \eta) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \left[u_{\vec{k}}(\eta)e^{i\vec{k}\cdot\vec{x}}a_{\vec{k}} + u^*_{\vec{k}}(\eta)e^{-i\vec{k}\cdot\vec{x}}a^\dagger_{\vec{k}}\right]
$$

Creation Anihilation Operator & Normalization :

$$\left[a_{\vec{k}}, a^\dagger_{\vec{k}'}\right] = \delta(\vec{k} - \vec{k}') \,, \quad (u_{\vec{k}}, u_{\vec{k}}) = i\left(u^*_{\vec{k}}\partial_\eta u_{\vec{k}} - u_{\vec{k}}\partial_\eta u^*_{\vec{k}}\right) = 1$$

De-Sitter Inflation : $a(\eta) = -\dfrac{1}{H\eta}$

$$\Rightarrow u_k = \frac{\sqrt{\pi}}{2}\sqrt{-\eta}H^{(1)}_{3/2}(-k\eta) \begin{cases} \overset{-k\eta\to\infty}{\sim} & \dfrac{1}{\sqrt{2k}}e^{-ik\eta} \\ \overset{-k\eta\to 0}{\sim} & \dfrac{H}{\sqrt{2k^3}}a(\eta) \end{cases}$$

↑

Bunch-Davies vacuum

Power spectrum : $P(k)$

$$\begin{aligned} &< 0|\left(\delta\phi(\vec{x},\eta)\right)^2|0> \\ &= \frac{1}{(2\pi)^3 a^2}\int d^3k \int d^3k' < 0|\left\{u_k(\eta)a_k\right\}\left\{u^*_{k'}(\eta)a^\dagger_{k'}\right\}|0> \\ &\equiv \int \frac{dk}{k}P(k) \end{aligned}$$

$$\Rightarrow P(k) = \frac{k^3}{2\pi^2 a^2}|u_k|^2 \to \left(\frac{H}{2\pi}\right)^2 \quad \text{for} \quad k\eta \to 0 \,(\text{super-horizon})$$

Spectrum Index : $P(k) \propto k^{n-1}$

$$n = 1 + \frac{d\log P(k)}{d\log k} = 1$$

$\cdots$ **scale-invariant (Harrison-Zeldovich) spectrum**

Large-angle CMB anisotropy

$$< \frac{\delta T}{T}(\vec{n})\frac{\delta T}{T}(\vec{n}') >= \frac{1}{4\pi}\sum_{\ell}(2\ell+1)C_\ell P_\ell(\vec{n}\cdot\vec{n}')$$

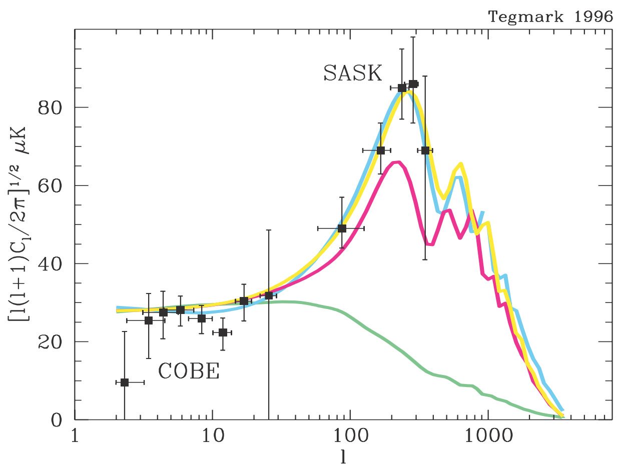
$\frac{\delta T}{T} = \frac{1}{3}\Phi \sim \mathcal{H}\frac{\delta\phi}{\phi'}$ **valid for** $\ell \ll 100$ (**Sachs–Wolfe formula**)

Relation between inflation and CMB

$$C_\ell = \frac{2}{\pi}\int\frac{dk}{k} < \frac{1}{9}|\Phi|^2 > k^3 j_\ell^2(k(\eta_0-\eta))$$

COBE v.s. Harrison-Zeldovich spectrum (n=1)

$$\frac{\ell(\ell+1)C_\ell}{2\pi} \propto \ell(\ell+1)\frac{\Gamma\left(\ell+\frac{n-1}{2}\right)}{\Gamma\left(\ell+\frac{5-n}{2}\right)} = \text{const.}$$



Consistent with scale invariant spectrum (n=1)

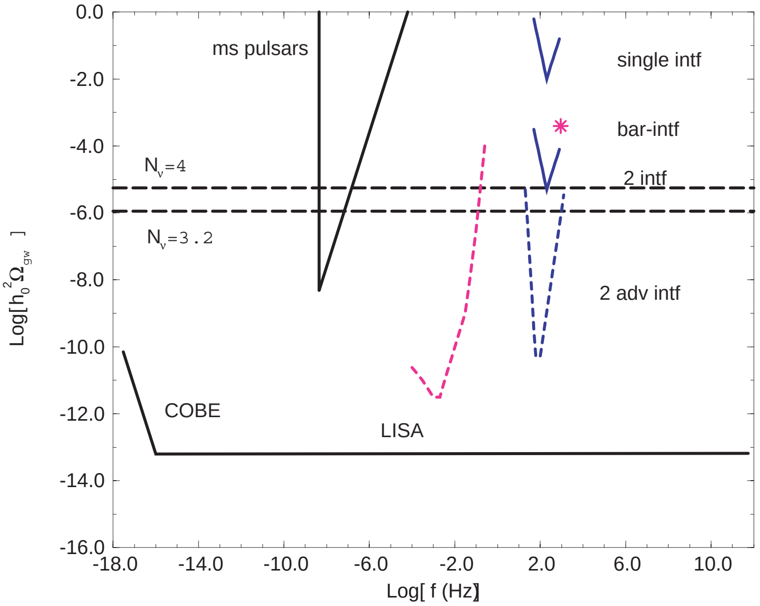
Primordial Gravitational Wave

Action for GW:

$$S_{GW} = \frac{1}{8\pi G}\int d^3x d\eta a^2(\eta) \sum_{A=\oplus,\otimes}\left[\frac{1}{2}h_A'^2 - \frac{1}{2}\partial_i h_A \partial^i h_A\right]$$

$$\frac{h_A}{\sqrt{8\pi G}} = \text{massless scalar}$$

$$\Rightarrow h_A \sim \frac{H}{\mathrm{M_{Pl}}} \cdots n = 1 \ : \ \text{Harrison—Zeldovich spectrum}$$



Born-Again Braneworld Scenario

Primordial Gravitational Wave

$$ds^2 = b^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right], \quad h_{ij}{}^{,j} = h^i{}_i = 0$$

$\cdots$ metric perturbation in the Einstein frame

Gravitational tensor perturbations (Amplitude)

$$h_k'' + 2\frac{b'}{b}h_k' + k^2 h_k = 0 \ ,$$

where $\mathcal{H} = b'/b\,, \quad b = |H_* \tau|^{1/2}$.

Positive frequency modes

$$h_k = \frac{\pi\kappa^2}{6H_*\ell} H_0^{(1)}(-k\tau)\,,$$

Spectrum : $P_{h_k}(k) = \dfrac{k^3}{2\pi^2}|h_k|^2 \sim \dfrac{k^3}{H_* M_{pl}^2}\,,$

where $M_{pl}^2 = \kappa^2/\ell$.

Spectrum is very blue

Spectral index : $n = 4$

Observationally testable in near future!!

Wait for Advanced LIGO or LISA

Inflaton perturbation

Inflaton does not couple with the radion field,
$\Rightarrow$ the spectrum is the conventional flat spectrum.
$\therefore$ Harrison-Zeldovich spectrum n=1

Consistent with COBE results

§5. Summary

- **4D Effective Action**
- **Born-Again Braneworl**
- **Inflation + Pre-Big-Bang**
  - ★ **Possible Blue spectrum for Primordial GW ⇒ LISA**
  - ★ **Standard CMB fluctuations**

Future Work

- **Inflaton dynamics**
- **Evolution of fluctuations** $\Rightarrow \dfrac{\delta T}{T}$ **(all scales)**