Higher Flavor Symmetries in the Standard Model

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Symmetries in QFT

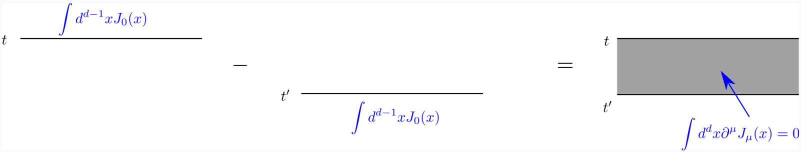
- Conserved current ($U(1)$)

$$\partial^\mu J_\mu = 0$$

- Unitary operator

$$U_\theta = e^{i\theta Q} = \exp\left[i\theta \int d^{d-1}x J_0(x)\right]$$

- Because of the conservation law, U_θ does not depend on time



Differential forms

$$J \equiv J_\mu dx^\mu$$

- The conservation law

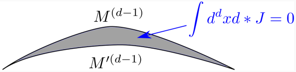
$$\partial^\mu J_\mu = 0 \quad \leftrightarrow \quad d * J = 0$$

$$*J = \frac{1}{(d-1)!}\epsilon_{\mu\mu_1\cdots\mu_{d-1}} J^\mu dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_{d-1}}$$

- Unitary operator

$$U_\theta = \exp\left[i\theta \int_{M^{(d-1)}} *J\right]$$

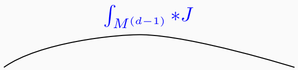
$$\int_{M^{(d-1)}} *J - \int_{M'^{(d-1)}} *J = 0$$



- Such objects are called "topological defects".

Generalized symmetries

ordinary symmetry $\rightarrow$ d-1dim. topological defects



- Let us generalize the notion of symmetry so that

$$\text{generalized symmetry} \quad \leftrightarrow \quad \text{topological defects}$$

- The following new types of "symmetries" are introduced as generalized symmetres:
    - Higher form symmetry

$$J_\mu \rightarrow J_{\mu_1 \cdots \mu_{p+1}}$$

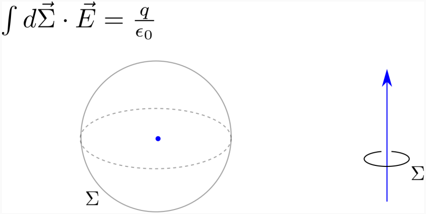
    - Noninvertible symmetry
        - topological defects can be defined by using topological field theories.

Higher form symmetry

- If J is a $p+1$ form, $*J$ is a $d-p-1$ form.

$$U_\theta = \exp\left[i\theta \int_{M^{(d-p-1)}} *J\right]$$

- For $p > 0$, the topological defect is the one with dimension lower than $d-1$.
- Actually, we are familiar with such an object.



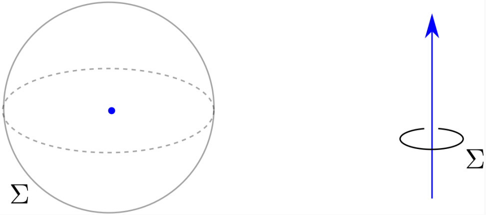
Electric and magnetic one form symmetries

In the presence of a $U(1)$ gauge field, we have two one form symmetries

- Electric $U(1)^{(1)}$:$J_{\mu\nu} = F_{\mu\nu}$
- Magnetic $U(1)^{(1)}$:$J_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$

$$\int d\vec{\Sigma} \cdot \vec{E} = \int_\Sigma *F$$

$$\int d\vec{\Sigma} \cdot \vec{B} = \int_\Sigma F$$



This paper

- For theories with a $U(1)$ gauge field, we have the magnetic one form symmetry.
- Together with a chiral current, it forms a so-called 2-group symmetry.
- The existence of such a symmetry restricts the possible UV completion of the theory.
- The authors apply this idea to the standard model.

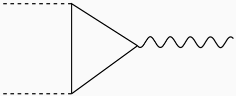
2-group symmetry

Ex. $U(1)$ gauge theory coupled to N_f massless fermions

$$J^a_\mu = \bar{\psi}\gamma_\mu \frac{1+\gamma^5}{2} t^a \psi$$

- J^a_μ satisfies the following OPE:

$$\partial^\mu J^a_\mu(x) J^b_\nu(y) \sim -\frac{e}{16\pi^2}\delta^{ab}\partial^\lambda \delta^4(x-y)\epsilon_{\lambda\nu\rho\sigma}F^{\rho\sigma}(y)$$



2-group symmetry

$$\partial^\mu J^a_\mu(x) J^b_\nu(y) \sim \kappa \delta^{ab} \partial^\lambda \delta^4(x-y) \epsilon_{\lambda\nu\rho\sigma} F^{\rho\sigma}(y)$$

- The chiral rotation symmetry (0-form symmetry) mixes with the magnetic 1-form symmetry.
- The symmetry is called the higher group symmetry.
    - If r is the largest integer for which an r-form symmetry participates in the higher group, the higher group is called an $r+1$-group.
- Hence we have a 2-group symmetry $SU(N_f)^{(0)} \times_\kappa U(1)^{(1)}$ (Baez-Lauda).

- If the 2-group symmetry with $\kappa \neq 0$ is broken, the following patterns are possible:
    - The symmetry is broken altogether.
    - The magnetic one form symmetry is unbroken.
- If the magnetic one form symmetry is broken the chiral symmetry is also broken.

The standard model

- Above the Higgs scale, we have $U_Y(1)$.
- The flavor symmetries of fermions and the magnetic $U_Y(1)^{(1)}$ form 2-group symmetry if we put the Yukawa couplings to be 0.

- Let us assume that $U_Y(1)$ is a part of a non-abelian group (c.f. GUTs) in UV.
  - Non-abelian groups do not have any magnetic one-form symmetry. (They have center symmetry instead.)
  - The magnetic one-form symmetry is broken in UV.
  - From the reasoning in the previous slide, the flavor symmetry should also be broken in UV if $\kappa \neq 0$.
  - The possible flavor symmetries for GUTs should be the one with $\kappa = 0$, if we put the Yukawa couplings to be 0.

The standard model

The possible flavor symmetries for GUTs should be the one with $\kappa = 0$, if we put the Yukawa couplings to be 0.

Table 9. Flavor subgroups not participating in the two-group with $U(1)_Y^{(1)}$, indexed by how many fermion species they act on.

One		None	
Two	$\{L, Q\}$	$\{L, \bar{d}\}$	$\{L, \bar{e}\}$
Three	$\{\bar{u}, \bar{d}, \bar{e}\}$	$\{\bar{u}, \bar{e}, Q\}$	$\{\bar{u}, \bar{d}, Q\}$
Four		None	
Five		$\{Q, \bar{u}, \bar{d}, L, \bar{e}\}$	

- The flavor symmetries with $\kappa = 0$ are given in the above table.
- The authors checked the flavor symmetries of familiar grand unified models.
- This table may be used as a starting point to explore exotic grand unified theories.