Global Tensor Network Renormalization for 2D Quantum systems: A new window to probe universal data from thermal transitions

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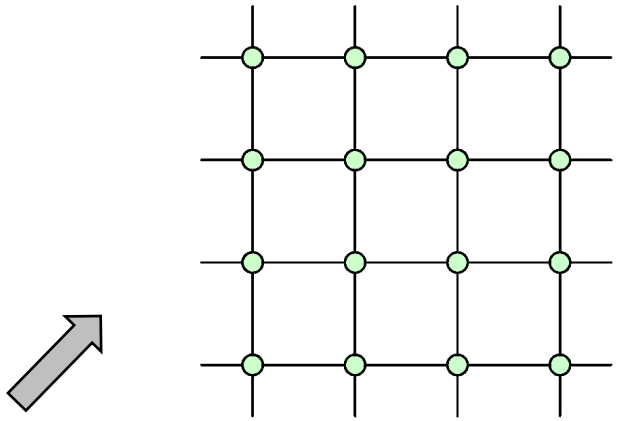
Abstract

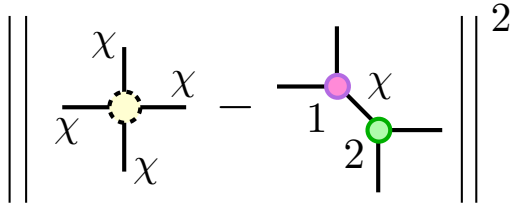
A new tensor network renormalization group scheme is proposed based on some global optimization. Thermal phase transition in the (2+1)D model is investigated via the CFT data. Their method seems useful to locate not only the thermal transition points, but also the quantum critical point at vanishing temperature

Tensor renormalization group (TRG)

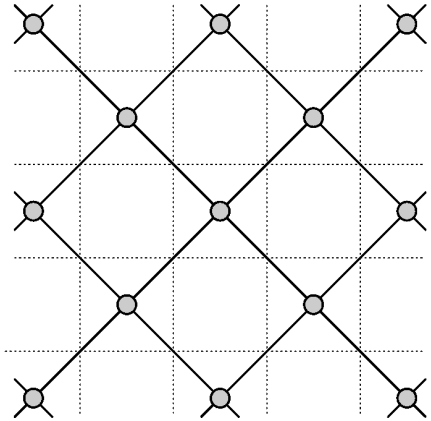
Levin-Nave, PRL99(2007)120601

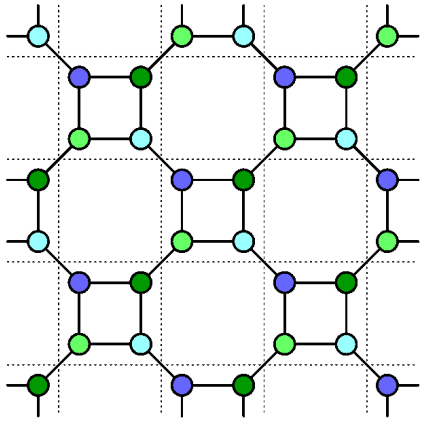
- TRG allows us to approximately compute partition functions of lattice theories
  - Based on local approximations





$$||\mathbf{T}_A - \mathbf{S}_1 \cdot \mathbf{S}_2||^2$$

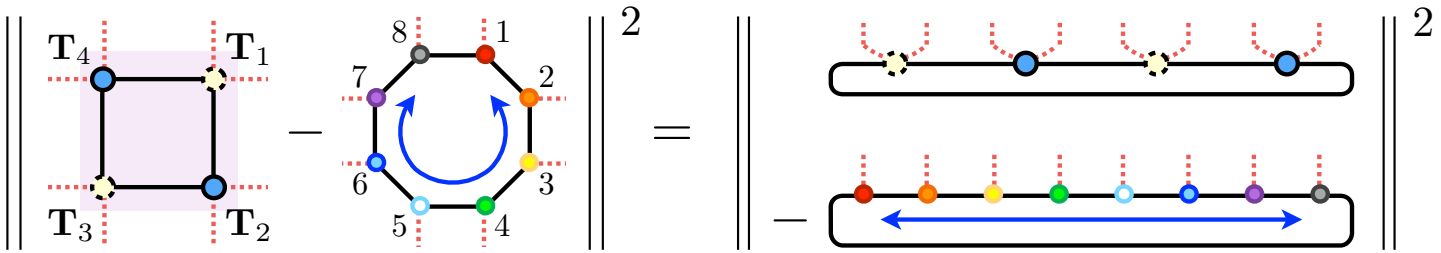


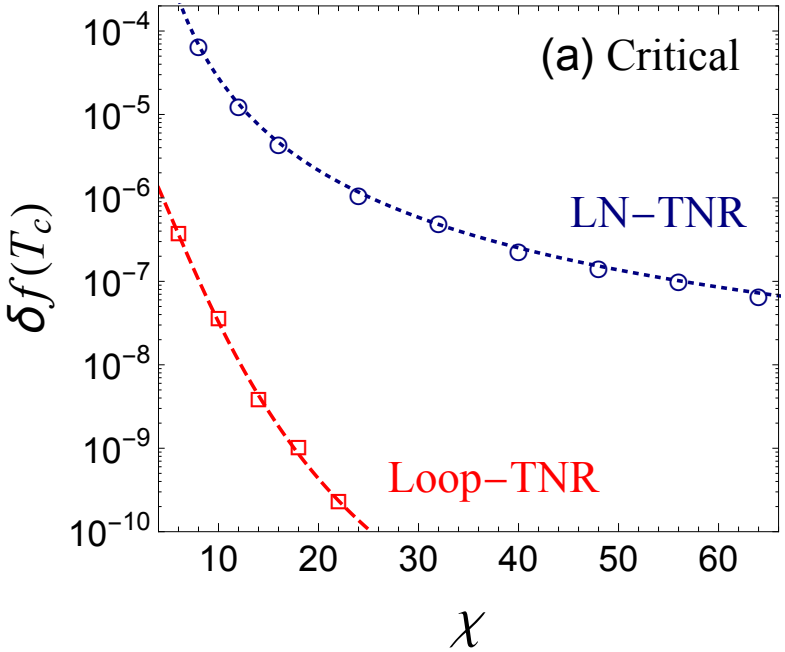


Tensor network renormalization (TNR)

Evenbly-Vidal, PRL115(2015)180405
Yang+, PRL118(2017)110504

- TNR also computes partition functions, removing some redundancy in TN rep.
  - A bit larger cost function than that in TRG

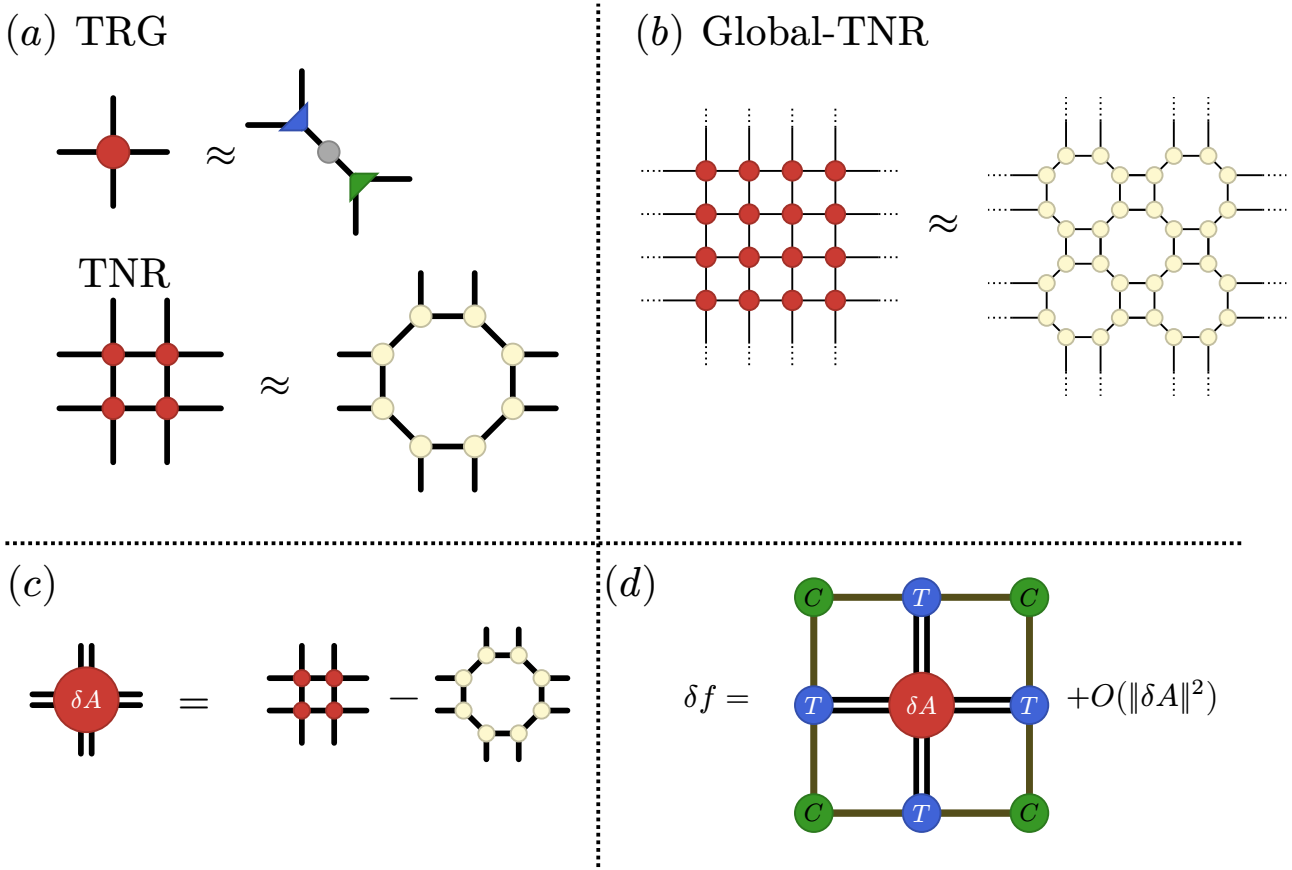




The global TNR scheme

- **Their algorithm takes into account the environment effects**

$$C(A) = \|\delta A\|^2 + \alpha \|\Gamma_{\mathrm{env}} \delta A\|^2,$$

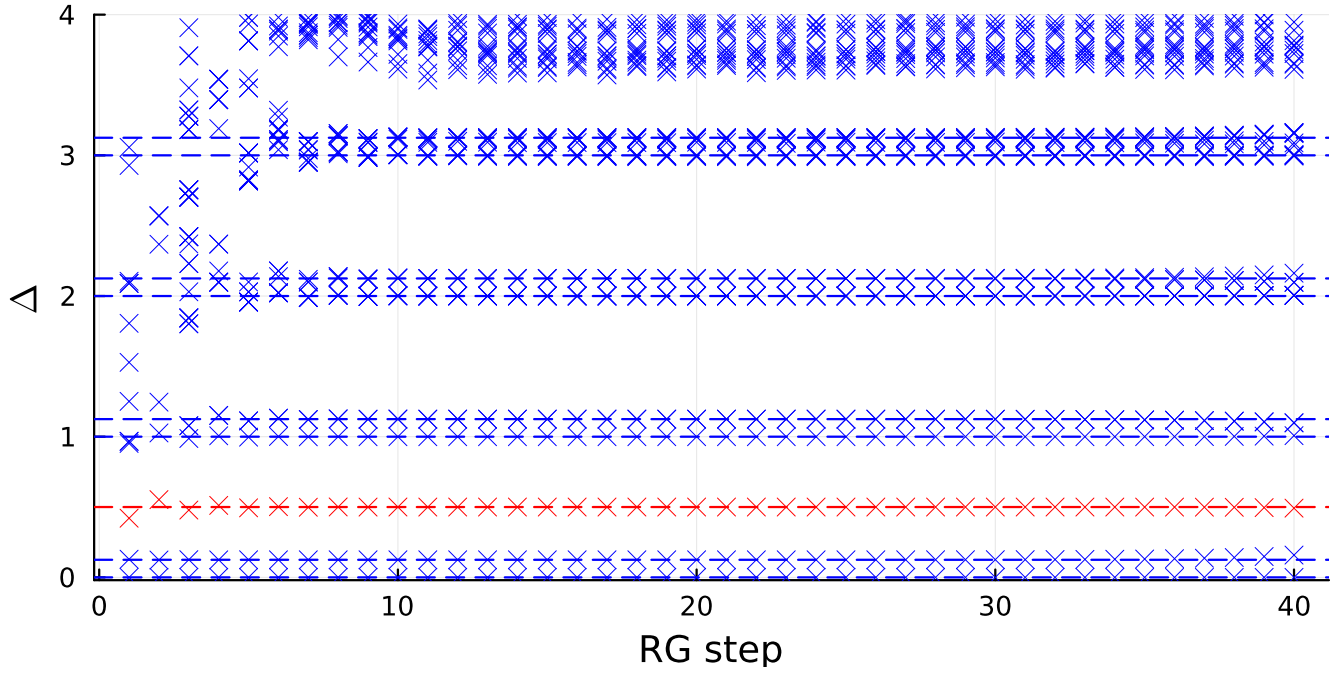


Validation in the 2D model

- **Their method improves the accuracy of conventional TRG, TNRs**
- **Scale invariant behavior is indeed observed up to V=2^{40}**

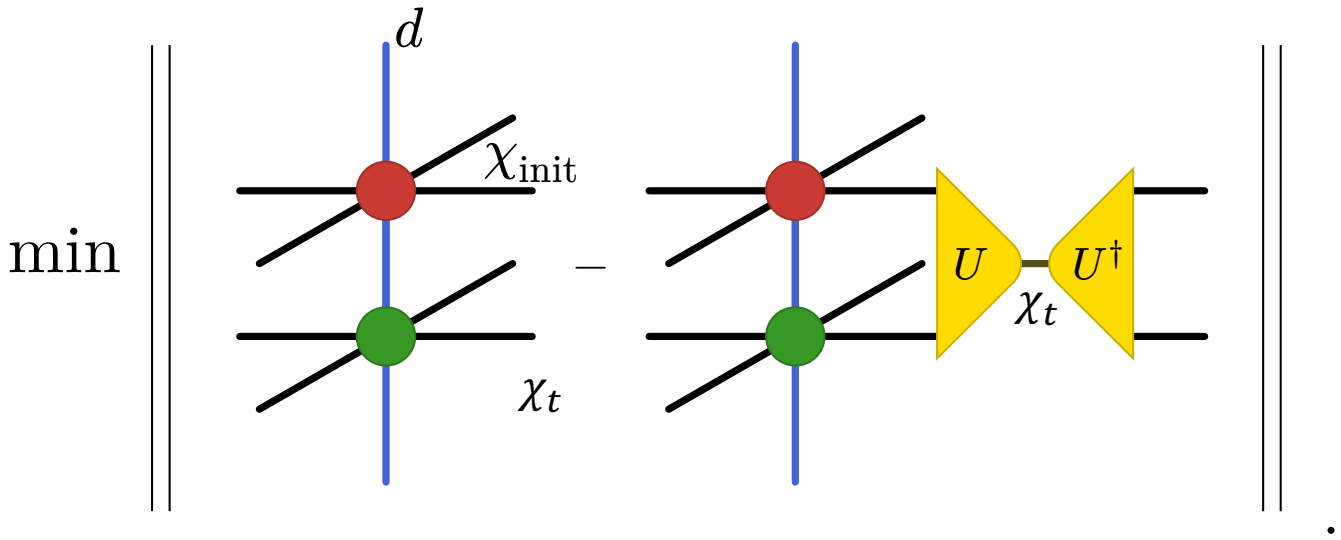
δf	TRG	GTRG	CTMRG	Loop TNR	Current work
Ising	$9.8 \cdot 10^{-6}$	$6.9 \cdot 10^{-7}$	$2.7 \cdot 10^{-7}$	$3.5 \cdot 10^{-8}$	$8.5 \cdot 10^{-10}$

at the criticality, χ=16



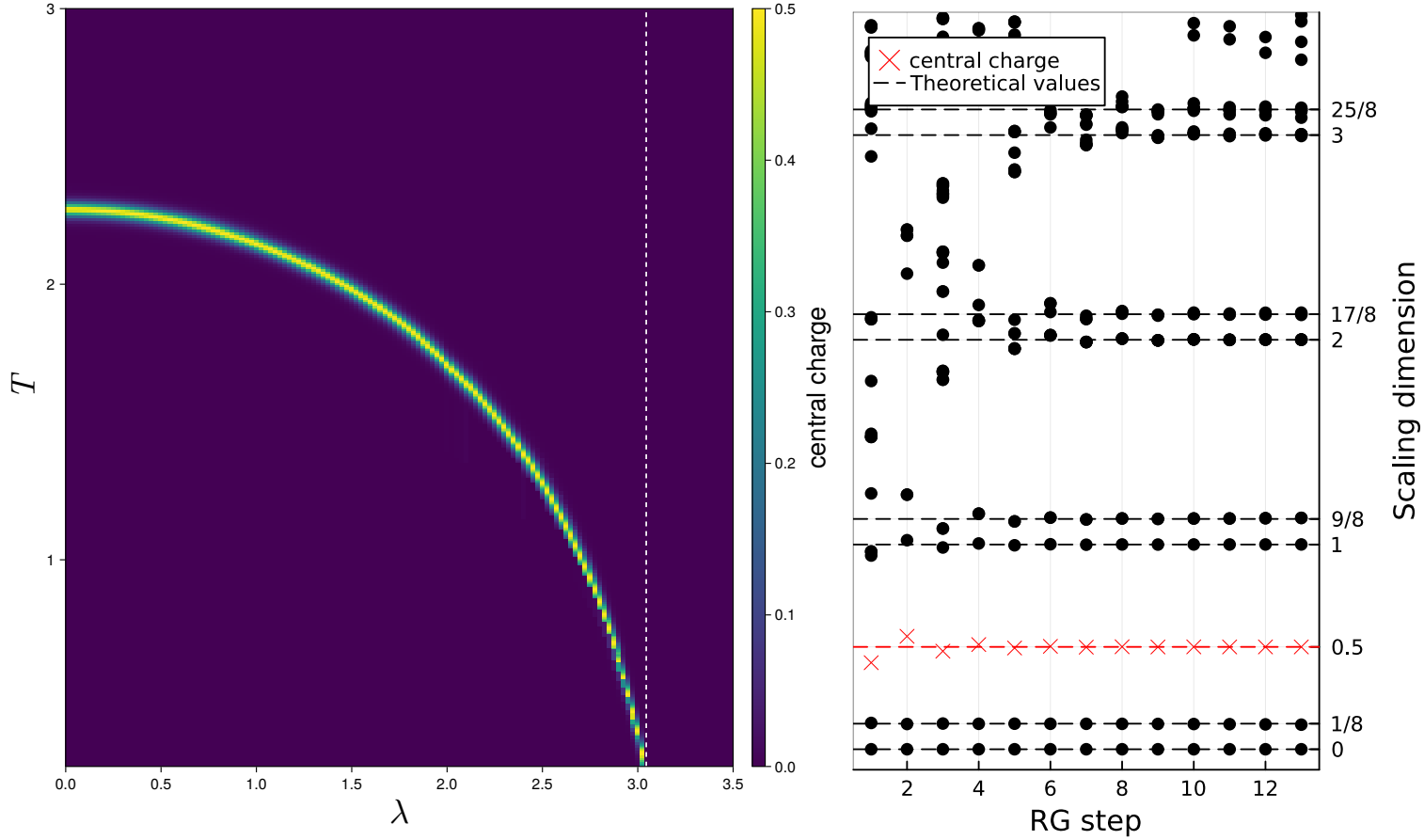
Extension to the (2+1)D systems

- Consider the system at finite temperature
  - Imaginary-time evolution is linearly carried out first
  - After the imaginary-time evolution, the (2+1)D TN can be reduced to be a 2D TN
  - The resulting 2D TN is contracted by the global TNR scheme



Transverse-field quantum Ising model

- **The method seems still working in the relatively lower temperature regime**
  - **Quantum critical point at zero temperature is known as λc=3.04438…**



at 1/T=0.472, χ_t=12, χ=10

Summary & Discussion

- Thermal phase transition in the (2+1)D model is investigated by the global TNR
  - Employing the CFT data, the phase diagram of the transverse-field quantum Ising model is determined
- Not only the thermal transition points, but also the quantum critical point at vanishing temperature might be located

- Conventional TNRG schemes are established as efficient algorithms to study the 2D CFT. How about the 3D CFT data?
  - So far, many attempts have been made to develop a 3D TNRG scheme
  - Global TNR w/ the reduction of the (2+1)D problem into the 2D one
  - A practical advantage over a possible 3D TNR scheme?

Central charge vs T