Localised black hole solution for the BFSS model

Oscar J.C. Dias and Jorge E. Santos,
“The low energy limit of BFSS quantum mechanics,” arXiv: 2407.15921
Oscar J.C. Dias and Jorge E. Santos,
“Localized states of BFSS super quantum mechanics,” arXiv: 2510.07379

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Introduction

BFSS Matrix model

The BFSS model is proposed as a non-perturbative formulation of M-theory on the flat background with a light-like S^1 compactification.

[Banks, Fishler, Shenker, Susskind '96; Seiberg '97; Sen '97; …]

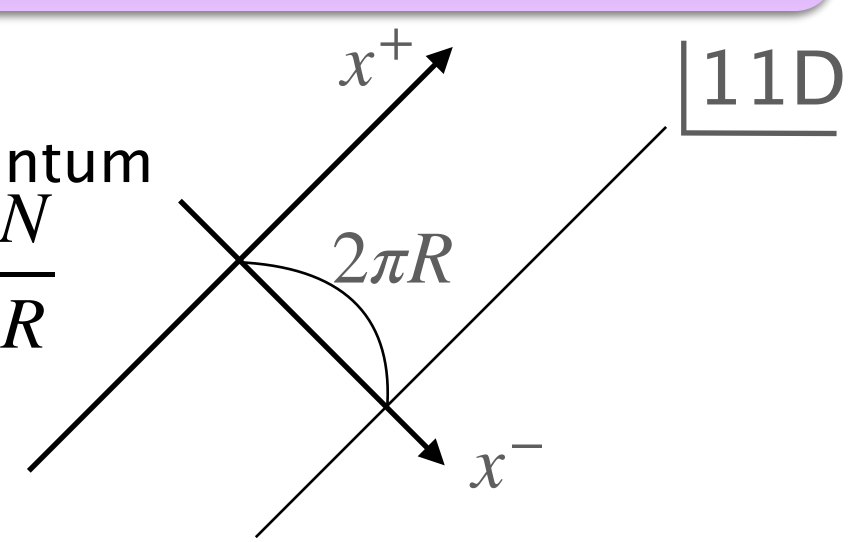
Weak coupling limit of type IIA string theory ⟷ **M-theory w/ light-like S^1 compact.**

string coupling $g_s \to 0$

D0-brane mass $\propto \dfrac{N}{R}$

KK momentum

$$p^+ = \frac{N}{R}$$



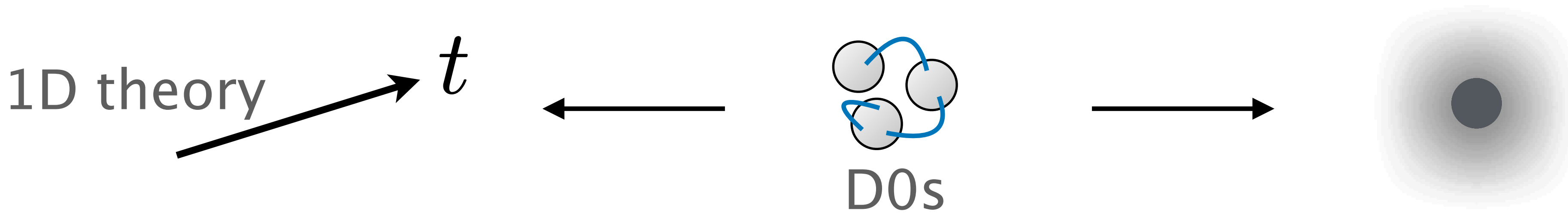
➡ The only non-trivial degrees of freedom in the limit are open strings attaching D0-branes

➡ The D0-brane theory describes M-theory

$$S_{\mathrm{BFSS}} = \frac{1}{g^2}\int dt\, \mathrm{tr}\left[\frac{1}{2}(D_t X^i)^2 + \frac{1}{4}[X^i, X^j]^2 + (\text{fermions})\right]$$

Introduction

The BFSS is also conjectured to have gauge/gravity duality for D0s.



Thermal state
BFSS matrix model

Black brane solution
SUGRA

[Itzhaki, Maldacena, Sonnenschein, Yankielowicz '98]

Deconfined phase ($\langle P \rangle \neq 0$)
… colour d.o.f. N^2 are "visible"

(Type IIA) non-extremal black 0-brane solution

$$\text{Entropy} \quad S = -\frac{\partial F}{\partial T} \quad \sim \quad O(N^2) \quad \sim \quad S = \frac{\text{Area}}{4}$$

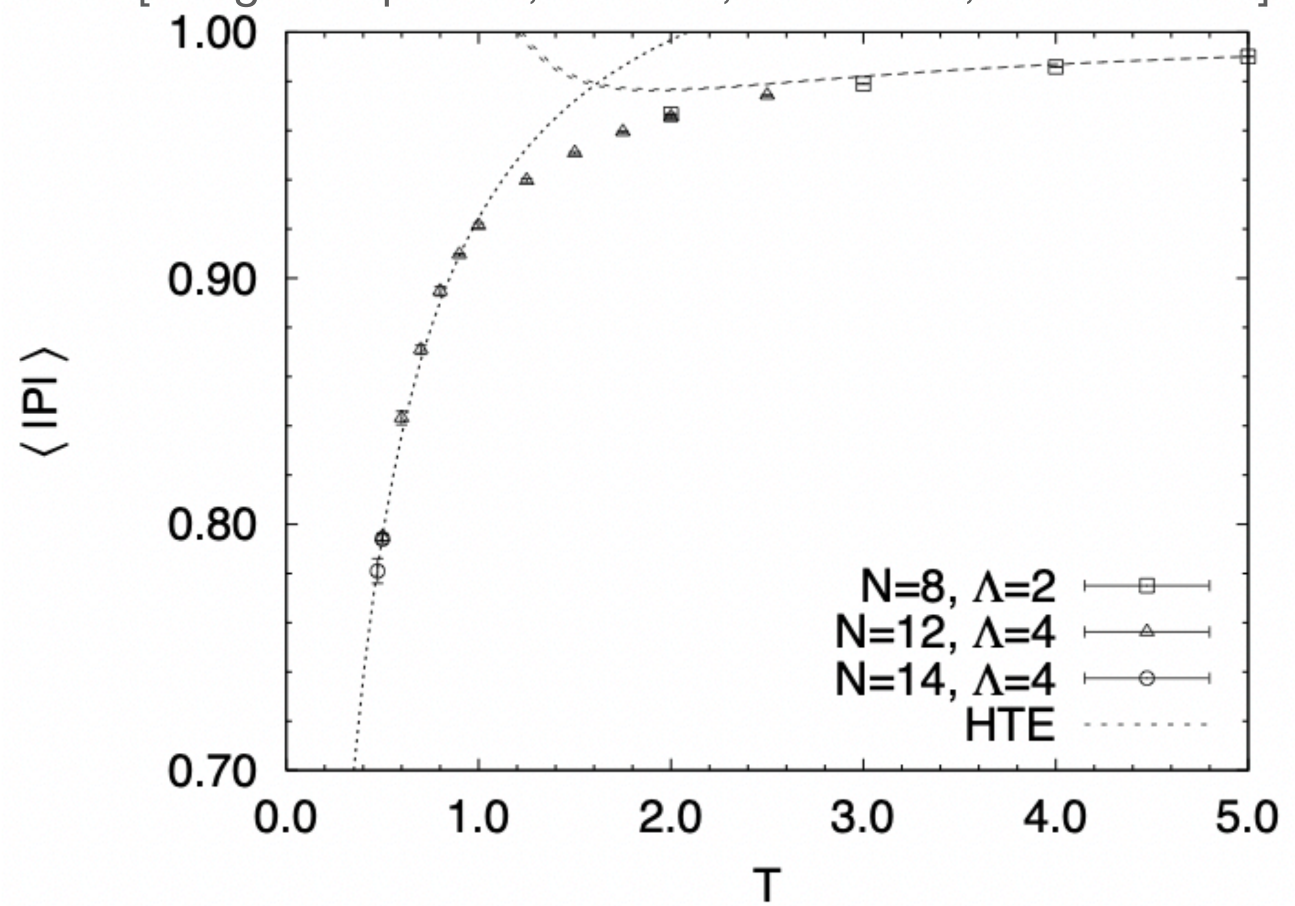
This correspondence is valid for $N^{-\frac{10}{21}} \ll \dfrac{T}{(g^2N)^{1/3}} \ll 1.$

Today, we consider temperatures lower than this validity region.

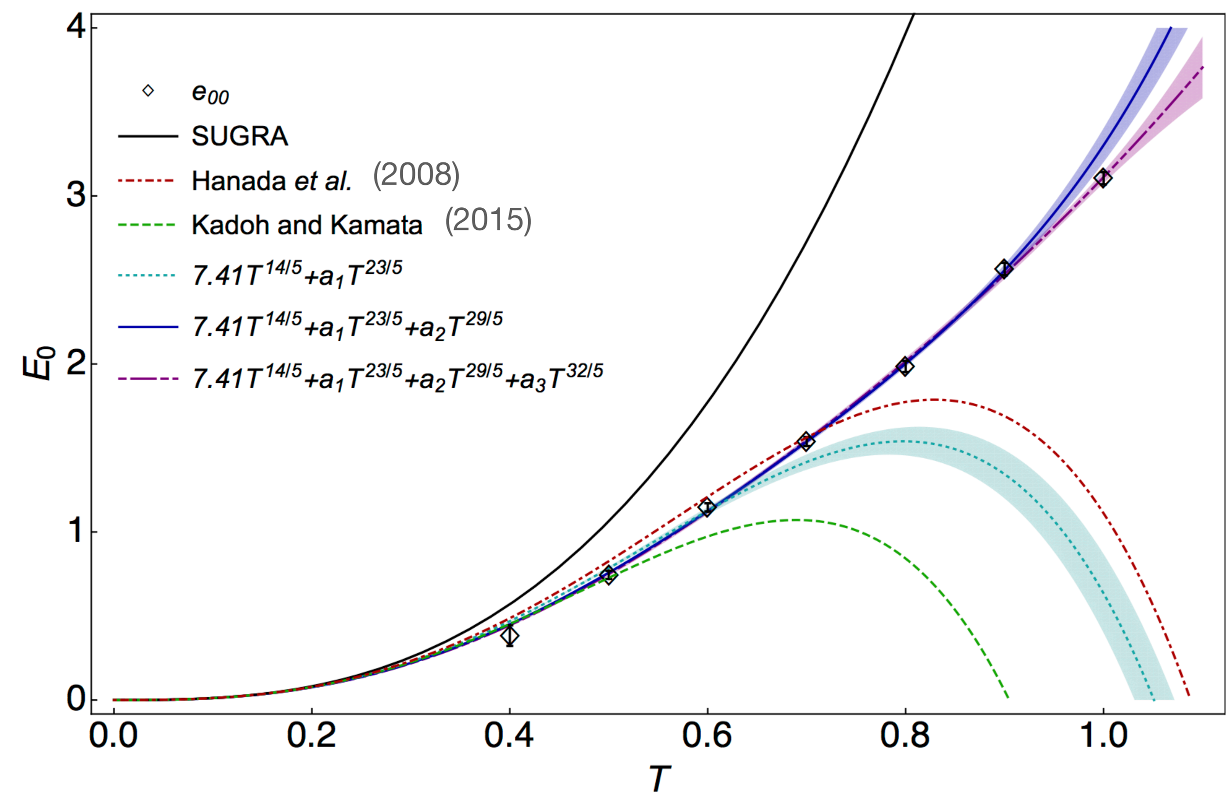
Introduction

Numerical simulations of the BFSS model:

[Anagnostopoulos, Hanada, Nishimura, Takeuchi ’07]



[Berkowitz, Rinaldi, Hanada, Ishiki, Shimasaki, Vranas ’16]



No confinement transition has been observed in the BFSS model, as predicted from the gravity side.

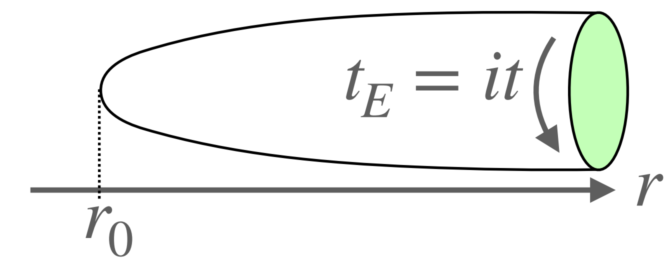
[Barbon, Kogan, Rabinovici ’98; Aharony, Marsano, Minwalla, Papadodimas, Raamsdonk, Wiseman ’05]

The numerical results agree with the gauge/gravity prediction.

Introduction

There are 2 types of black brane solution in type IIA 10D SUGRA.

- Non-extremal black 0-brane solution



$$ds^2 = -\frac{f(r)}{H^{\frac{1}{2}}(r)}dt^2 + \frac{H^{\frac{1}{2}}(r)}{f(r)}dr^2 + H^{\frac{1}{2}}(r)\, r^2 d\Omega_8^2$$

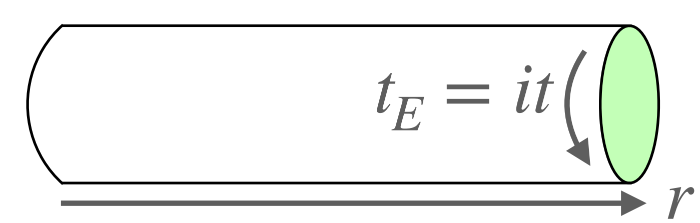
Effective string coupling: $e^{\Phi} = g_s H^{\frac{3}{4}}(r)$ $\quad f(r) = 1 - \frac{r_0^7}{r^7} \quad H(r) = 1 + \frac{r_-^7}{r^7}$

$\cdots$ Event horizon: $r = r_0$ (Black hole in 10D)

Hawking temperature: $T \propto \frac{r_0^{\frac{5}{2}}}{\sqrt{g_s N}}$ (near extremal)

Entropy $S \propto N^2 \left(\frac{T}{(g_s N)^{1/3}}\right)^{\frac{9}{5}}$ Free energy $F \propto -N^2 \left(\frac{T}{(g_s N)^{1/3}}\right)^{\frac{14}{5}}$

- Thermal 0-brane solution



It is "extremal" ($r_0 = 0$) but has a non-contractible S^1.

Free energy $F = 0$

Non-extremal black 0-brane is always favoured.

Introduction

The validity region for the type IIA description:

Effective string coupling: $e^{\Phi} \sim \frac{1}{N}\tau^{-\frac{21}{10}} \ll 1$

Dimensionless curvature: $\alpha' \boldsymbol{R} \sim \tau^{\frac{3}{5}} \ll 1$

Dim.less temperature

$$\tau := \frac{T}{(g^2N)^{1/3}} \sim \frac{T}{(g_s N)^{1/3}}$$

$$\Rightarrow \quad N^{-\frac{10}{21}} \ll \tau \ll 1$$

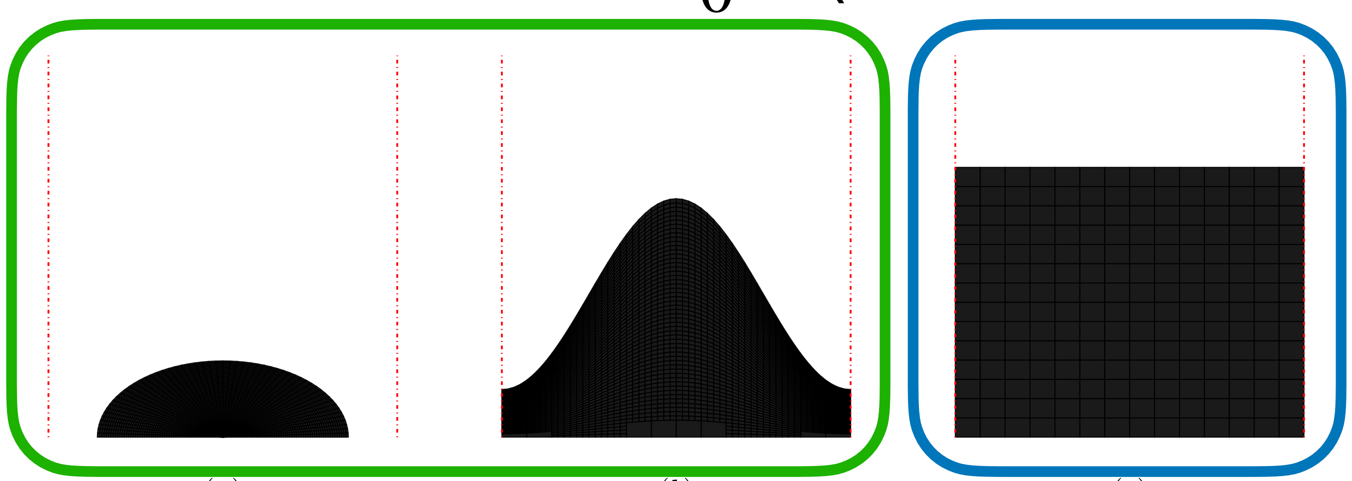
The non-extremal black 0-brane is actually a “black string” in **11D**.

For $\tau \ll 1$ and $r \sim r_0 \sim O(1)$ (near horizon),
the dimensional “oxidisation” from type IIA 10D to 11D yields

$$ds_{11}^2 = -f(r)dt'^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_8^2 + \underline{dz'^2} \qquad f(r) = 1 - \frac{r_0^7}{r^7}$$

$\cdots$ Event horizon: $r = r_0$ (**Uniform** black string)

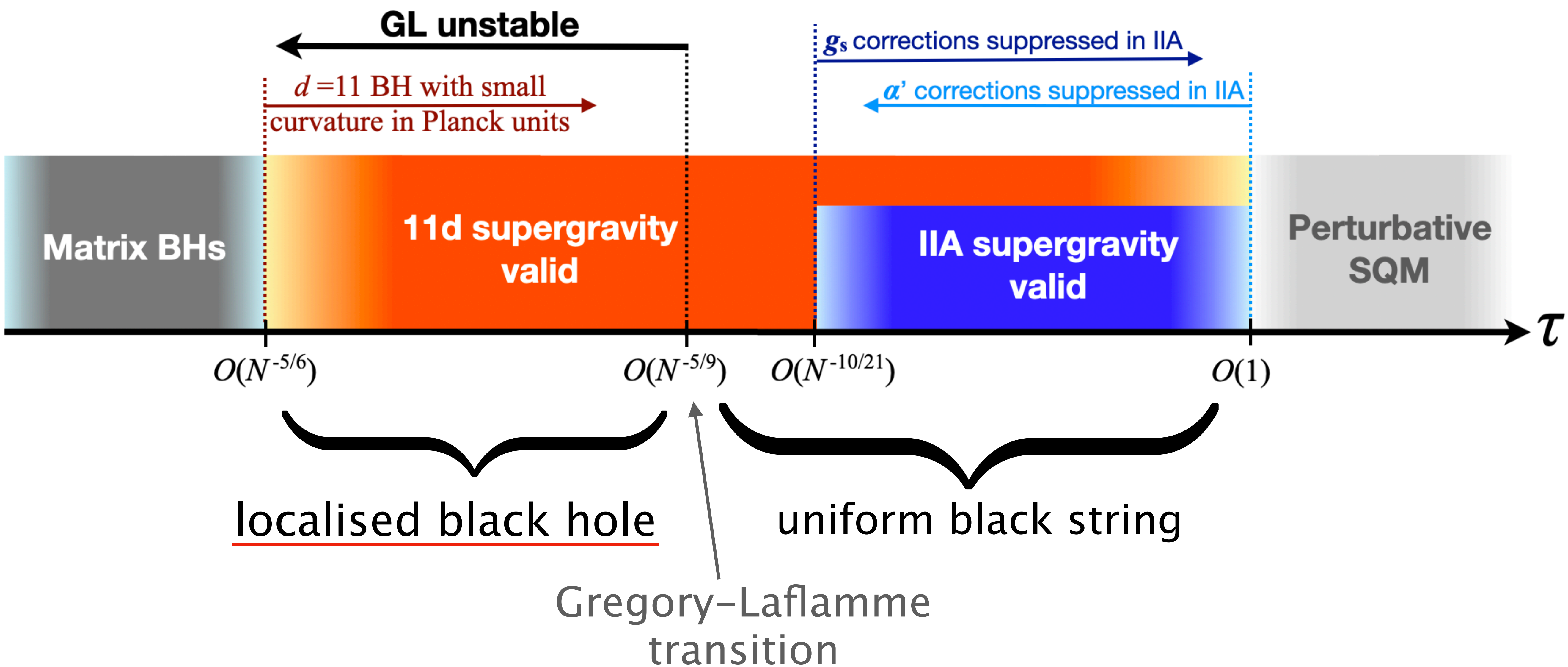
We search for **localised** black hole & **non-uniform** black string solutions.



(a) (b) (c)

Introduction

Result:



There is a phase transition from the uniform black string sol. to the localised black hole sol., in which the non-uniform one is subdominant. On the other hand, the non-uniform black string sol. becomes dominant in the microcanonical ensemble.

Localised & non-uniform solutions

The asymptotic behaviour (in 11D) of the gravity solutions should be the same as that of the uniform black string solution:

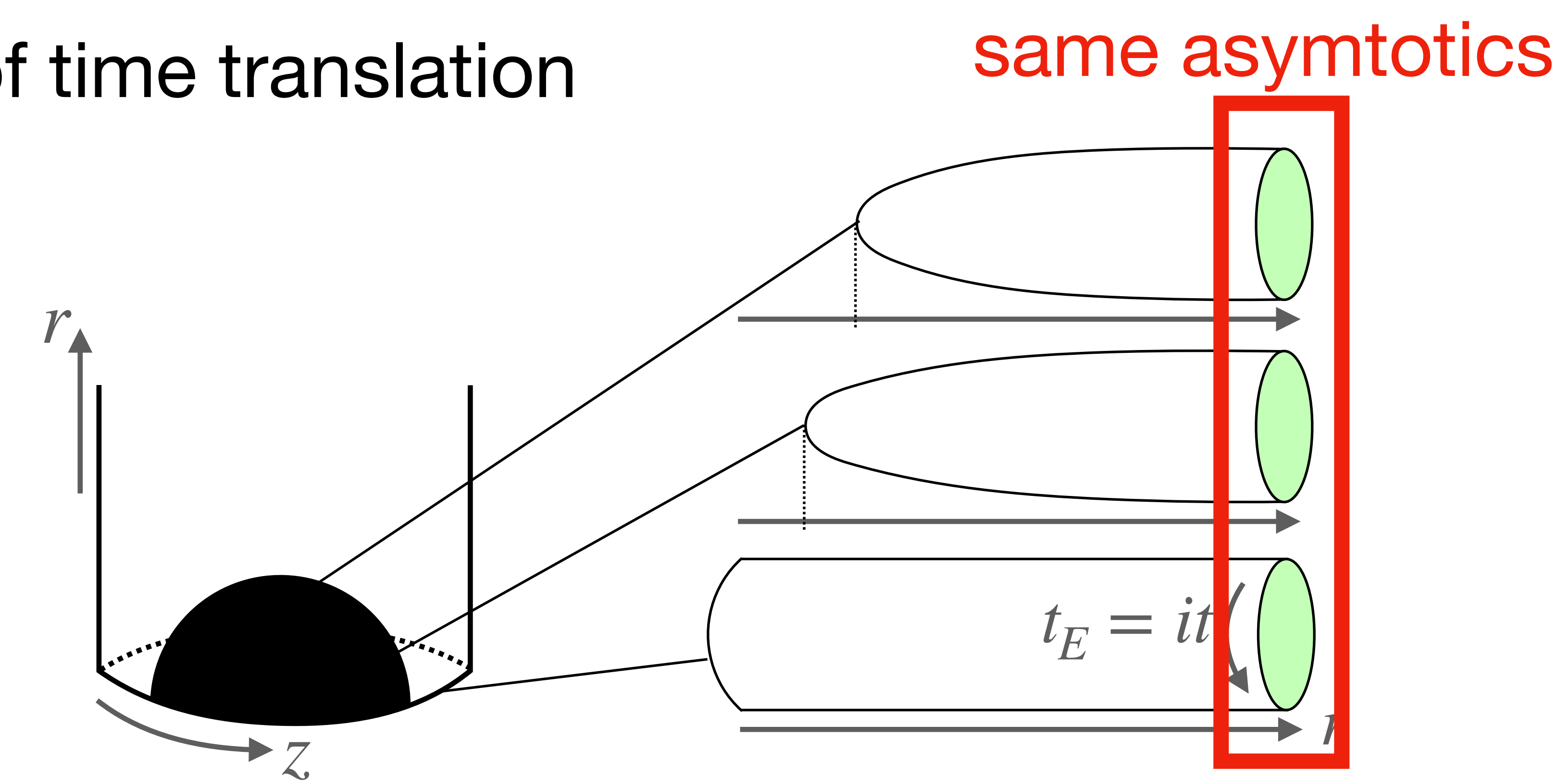
$$ds_{11}^2 \to -\frac{r^7}{r_-^7}dt^2 + dr^2 + r^2 d\Omega_8^2 + \frac{r_-^7}{r^7}\left(dz + \frac{r^7}{r_-^7}dt\right)^2 \qquad r_0 \ll r \ll r_-$$

$$= 2dt\,dz + \frac{r_-^7}{r^7}dz^2 + dr^2 + r^2 d\Omega_8^2 \qquad \cdots \text{ pp-wave background}$$

It ensures

- the same number of D0-branes, N
- the same compactification radius R $\quad \left(z \sim z + 2\pi R\right)$
- the same generator of time translation

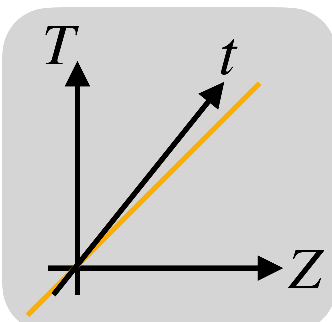
Localised black hole:



Localised & non-uniform solutions
(technicalities)

The authors numerically constructed the localised black hole solution and the non-uniform black string solution which are asymptotically $\mathbb{R}^{1,9} \times S^1$, which can be mapped to those asymptotically pp-wave.

"Carrollian" transformation: [Carrollian limit: the speed of light $c \to 0$]



$$T = \frac{R^{7/2}}{r_-^{7/2}}\gamma t - \frac{1}{\gamma}\frac{r_-^{7/2}}{R^{7/2}}z, \qquad Z = \frac{1}{\gamma}\frac{r_-^{7/2}}{R^{7/2}}\frac{z}{R} \qquad \left(\gamma = \sqrt{a+c}\right)$$

$$ds_{11}^2 \to -\left(1 - a\frac{R^7}{r^7} + \cdots\right)dT^2 + \frac{dr^2}{B(r)} + r^2 d\Omega_8^2 + \left(1 + c\frac{R^7}{r^7} + \cdots\right)R^2 dZ^2 \quad \cdots\ \mathbb{R}^{1,9} \times S^1$$

$$\sim 2dt\,dz + \frac{r_-^7}{r^7}dz^2 + \frac{dr^2}{B(r)} + r^2 d\Omega_8^2 \quad \cdots\ \text{pp-wave}$$

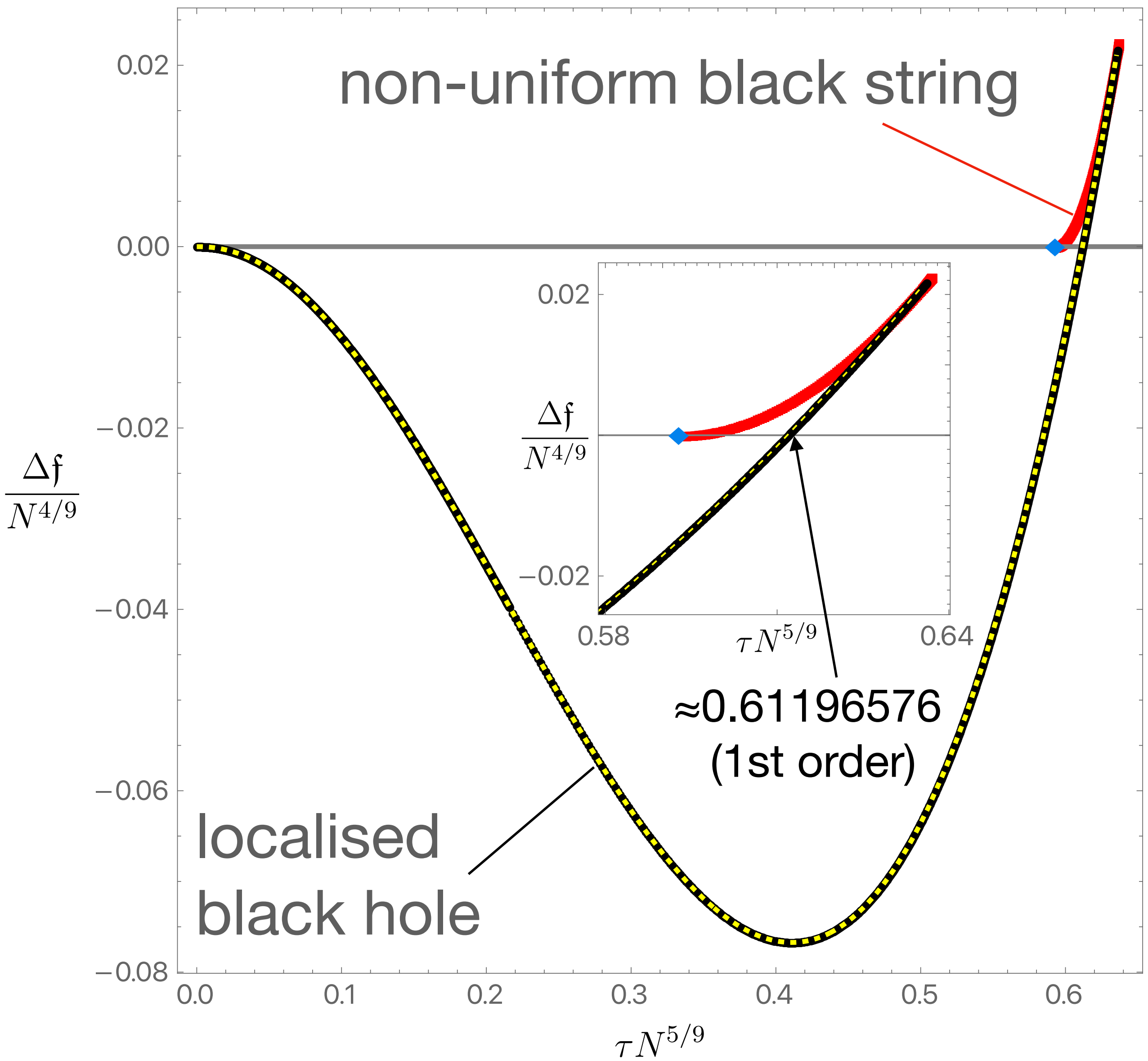
This transforms thermodynamic quantities as well: e.g. $S^{(pp)} = \dfrac{2\pi N S^{(0)}}{E^{(0)} - T^{(0)}}$

They also obtained a perturbative result for the localised BH:

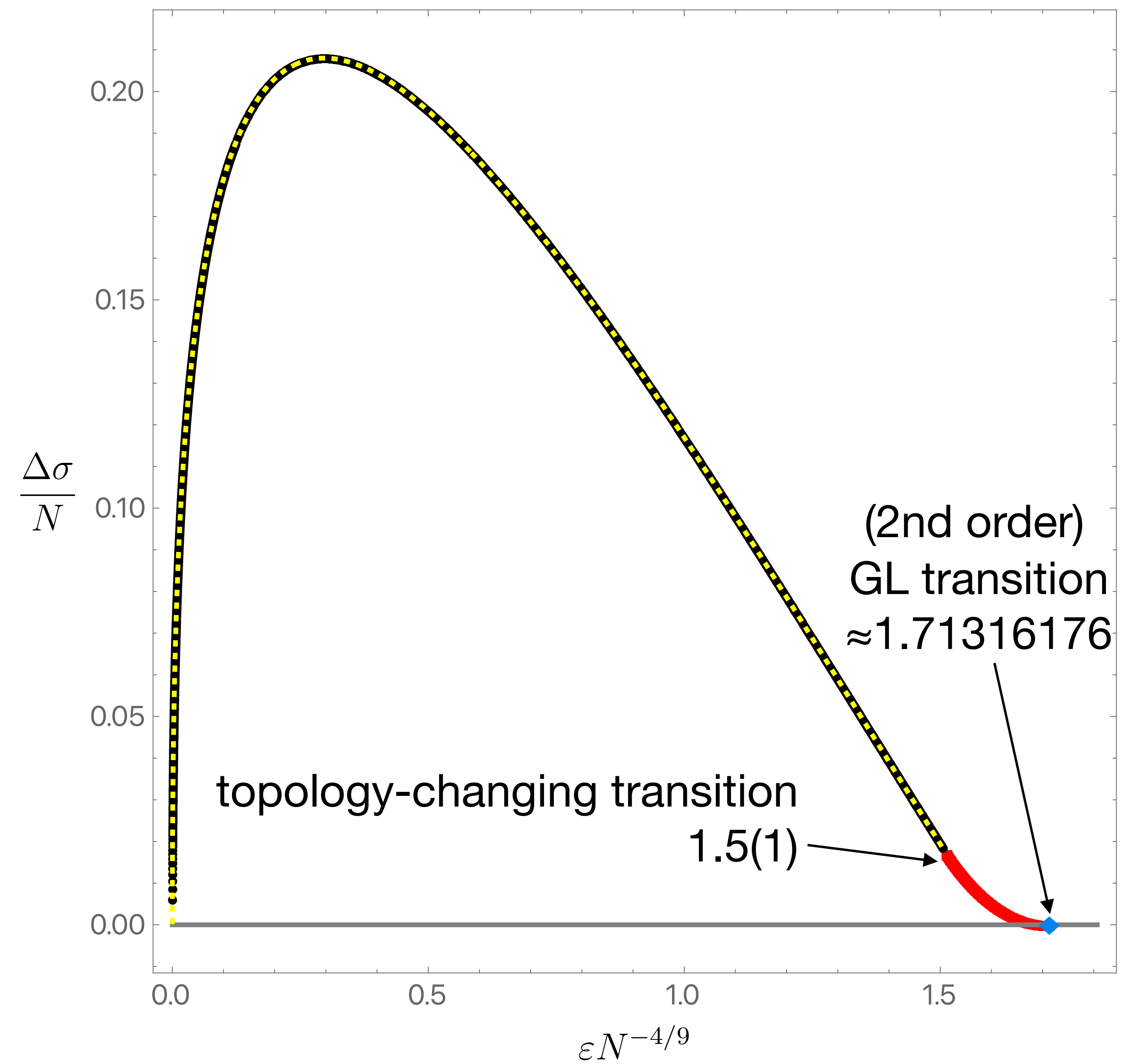
$$F^{(pp)} \propto -N^{\frac{4}{9}}(N^{\frac{5}{9}}\tau)^{\frac{16}{7}}\left[1 + \frac{\pi^{72/7}}{264600 \cdot 6^{2/7}}(N^{\frac{5}{9}}\tau)^{\frac{72}{7}} + O(\tau^{\frac{144}{7}})\right]$$

Localised & non-uniform solutions

Free energy $F^{(pp)}$



Entropy $S^{(pp)}$



These are thermodynamic quantities of the novel solutions minus those of the uniform black string sol.

We observe phase transitions from the uniform black string.

Summary

- In the canonical ensemble, we observe a 1st order phase transition at $\tau \approx 0.6119 N^{-\frac{5}{9}}$ from the uniform black string to the localised black hole.
- In the microcanonical ensemble, we observe a 2nd order phase transition at energy $\varepsilon \approx 1.713 N^{\frac{4}{9}}$ from the uniform black string to the non-uniform black string, unlike other gauge/gravity dualities.
- Therefore we expect a 1st order transition in BFSS simulations (e.g. of energy) at $\tau \approx 0.6119 N^{-\frac{5}{9}}$. But what is a good order parameter? We need to understand this transition on the BFSS side.

※ There may exist another solution that is more dominant.

※ This uniform/localised transition is observed in the bosonic BFSS model (high-T limit of the 2D SYM on $S^1 \times S^1$). In this case, the transition corresponds to the confined/deconfined transition, where the eigenvalue distribution of the holonomy (Polyakov loop) is uniform/localised.