Lattice quantum electrodynamics in (3+1)-dimensions at finite density with tensor networks

G. Magnifico, T. Felser, P. Silvi, and S. Montangero,

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Abstract

The first tensor network simulation of

(3+1)D LGT with dynamical matter in the Hamiltonian formalism

Tensor Network (TN) methods

Tensor network = A contraction of many # of tensors

TN methods are free from the sign problem

Lagrangian formalism

$$Z = \int D\phi \mathrm{e}^{-S[\phi]}$$

Express Z as a TN

Coarse-graining method on TN

Ex. TRG

Hamiltonian formalism

$$E_0 \leq \frac{\langle\psi|\hat{H}|\psi\rangle}{\langle\psi|\psi\rangle}$$

Prepare $|\psi\rangle$ as a TN

Variational method w/ TN

Ex. DMRG

For applications of TN methods to lattice field theory,
see Bañuls-Cichy, Rep. Prog. Phys. 83(2020)024401 (review)

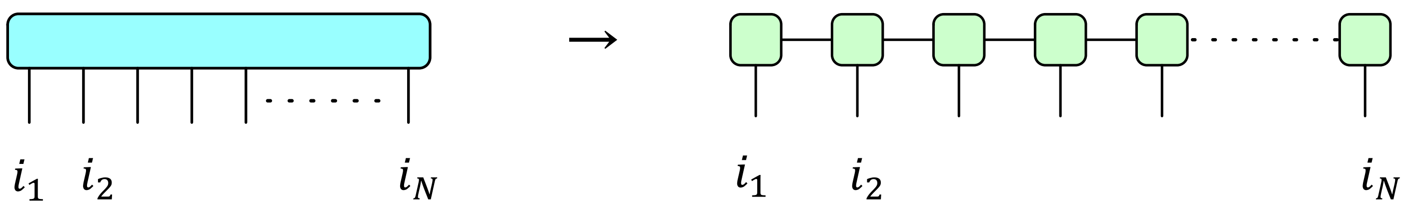
TN method based on the Hamiltonian formalism

The goal is to obtain the ground state, $|\Psi\rangle = \sum_{i_1,\cdots,i_N=1}^{m} \Psi_{i_1\cdots i_N} \, |i_1,\cdots,i_N\rangle$

We assume a TN form of $|\Psi\rangle$ and variationally tune it

Ex) Matrix Product State (MPS) ansatz

$|\Psi\rangle = \sum_{i_1,\cdots,i_N=1}^{m} \mathrm{Tr}\left[A^{(i_1)}A^{(i_2)}\cdots A^{(i_N)}\right] |i_1,\cdots,i_N\rangle$ $\quad A^{(i_k)}$: $\chi\times\chi$ matrix



m^N $\qquad$ $Nm\chi^2 \sim \mathrm{poly}(N)$

The bond dimension χ determines # of variational parameters

Examples of TN ansatz

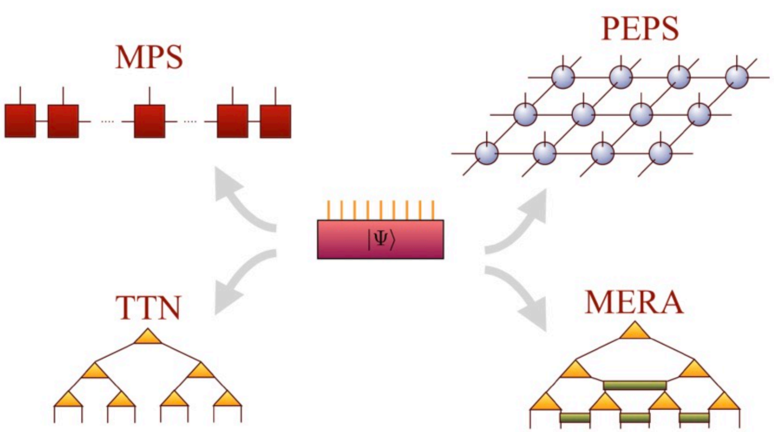


Figure from Bañuls-Cichy, Rep. Prog. Phys. 83(2020)024401

Type of ansatz	Cost
MPS	$O(\chi^3)$
PEPS	$O(\chi^{10})$
MERA	$O(\chi^8)$
Tree Tensor Network (TTN)	$\boldsymbol{O}(\chi^4)$

Cf. Nishino san's lecture (11/17, 22, 24)

The following TN simulation is based on three-dimensional TTN

☺️ **Computationally economic & Straightforward extension to higher dim.**

😕 Area law in d dimensions ($d \geq 2$)

The model

✔ Lattice QED based on the KS formalism

fermions on even (odd) sites are (anti-)particle w/ positive (negative) charge

$$H = -t \sum_{x,\mu} \left(\psi_x^\dagger U_{x,\mu} \psi_{x+\mu} + \mathrm{h.c.}\right) + m \sum_x (-1)^x \, \psi_x^\dagger \psi_x$$
$$+ \frac{g_\mathrm{e}^2}{2} \sum_{x,\mu} E_{x,\mu}^2 - \frac{g_\mathrm{m}^2}{2} \sum_x \left(\Box_{\mu_x \mu_y} + \Box_{\mu_y \mu_z} + \Box_{\mu_x \mu_z} + \mathrm{h.c.}\right)$$

with

$$\Box_{\mu_\alpha \mu_\beta} = U_{x,\mu_\alpha} U_{x+\mu_\alpha,\mu_\beta} U_{x+\mu_\beta,\mu_\alpha}^\dagger U_{x,\mu_\beta}^\dagger$$

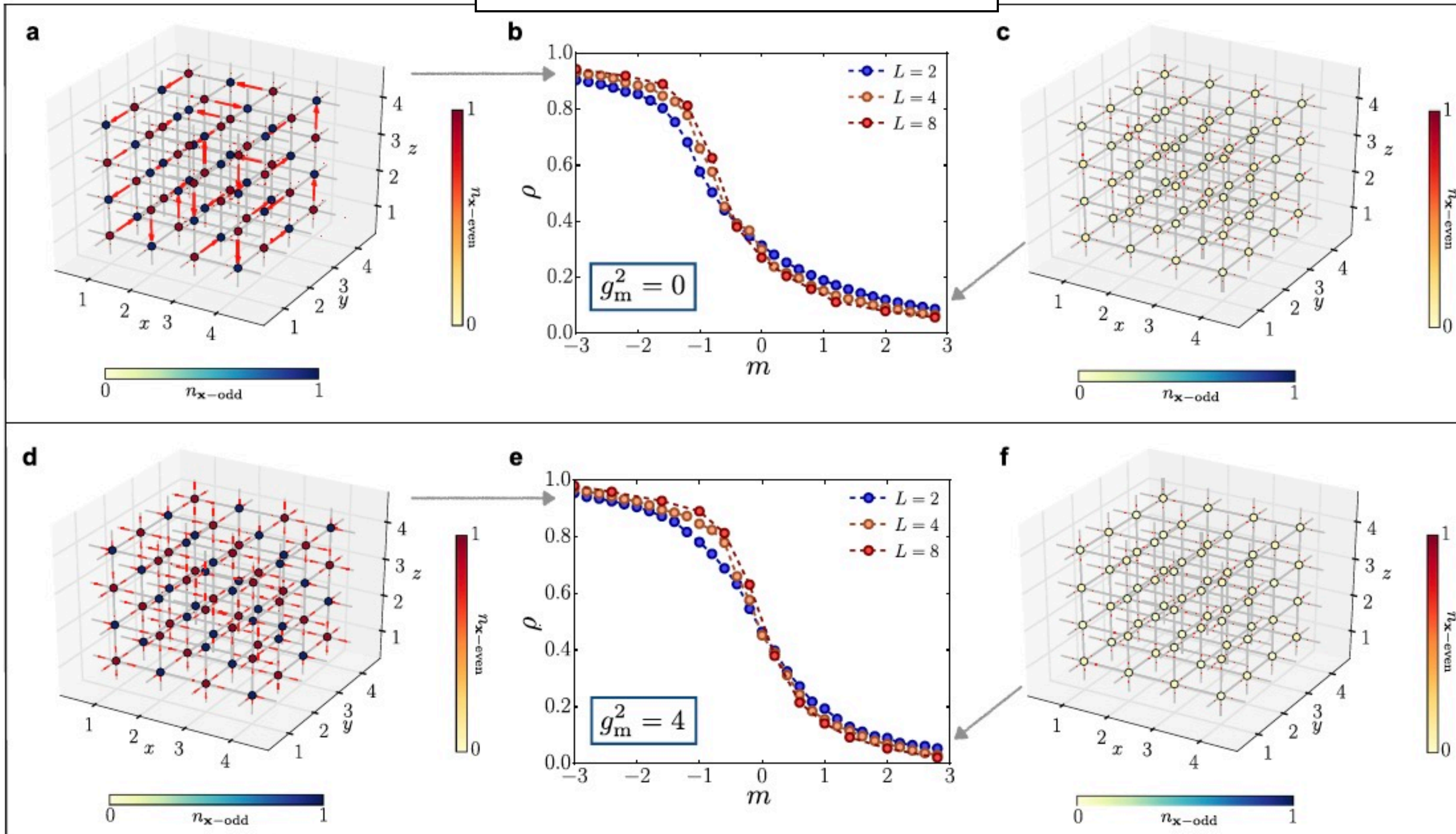
✔ **U(1) electromagnetic fields are truncated up to a spin-s representation**

The original model is restored by the limit $s \to \infty$

✔ TN simulation is implemented selecting $s = 1$ (smallest but nontrivial one)

Transition at zero total charge w/ PBC

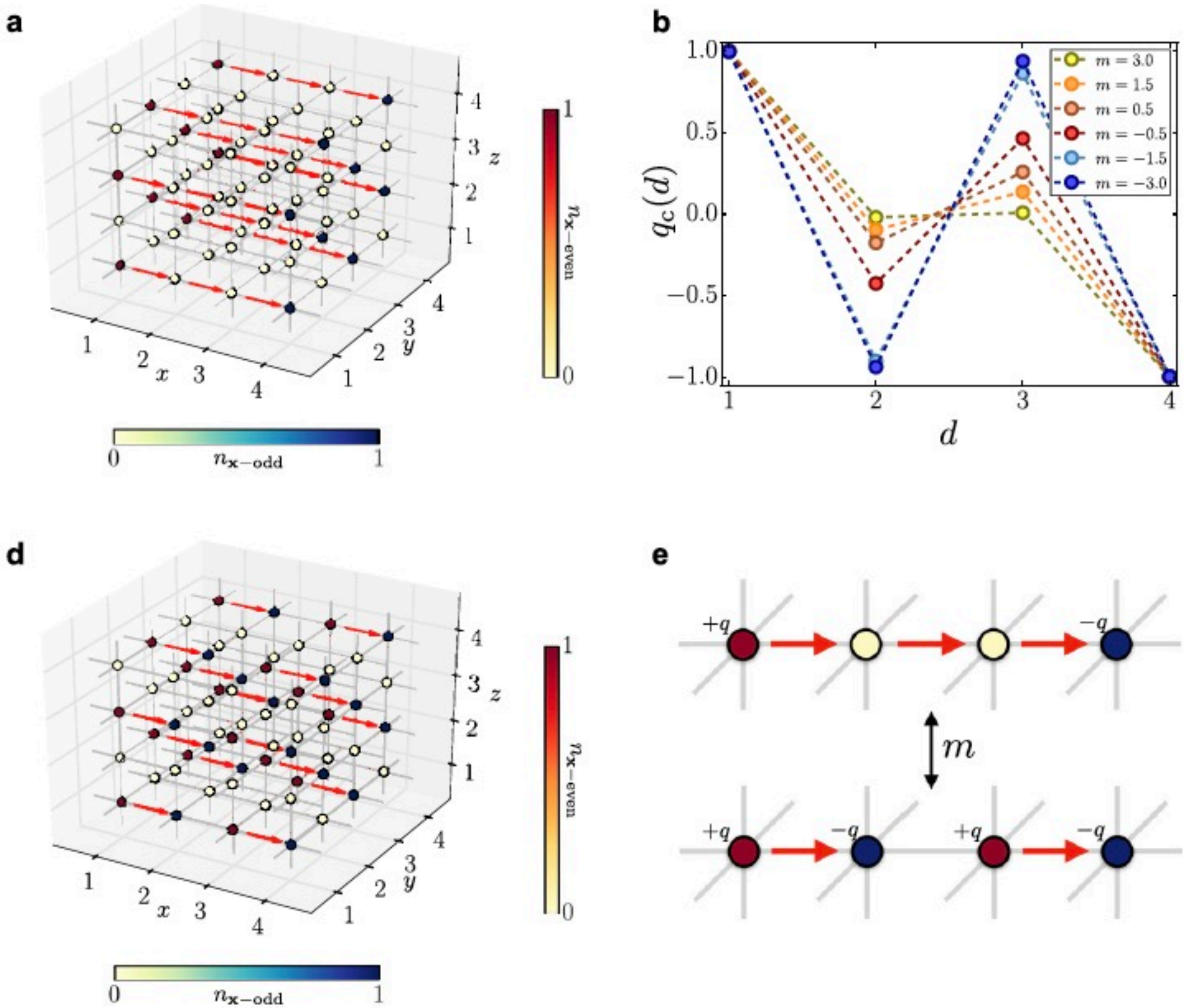
Average matter density

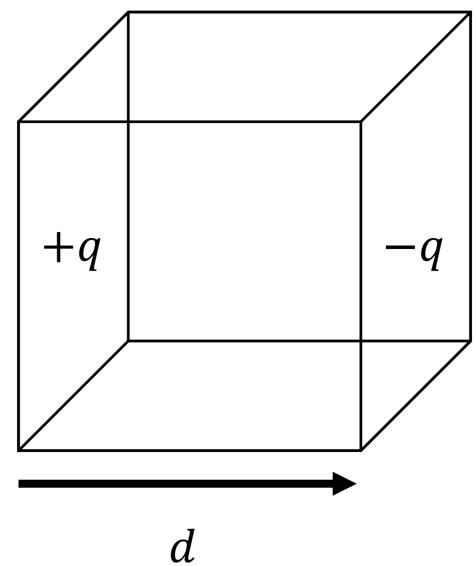


m_c is shifted by the effect of magnetic coupling

Quantum capacitor w/ OBC

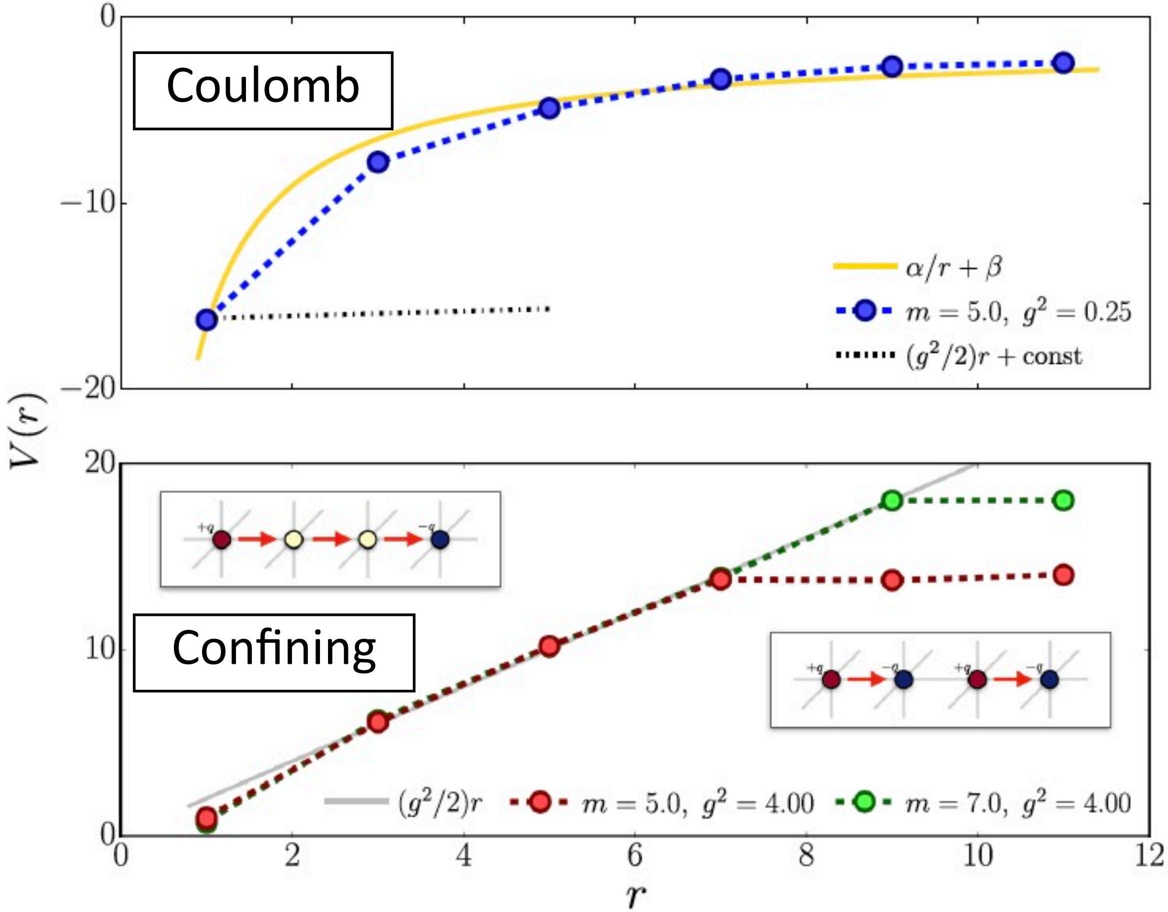
Charge density along w/ μ_x direction





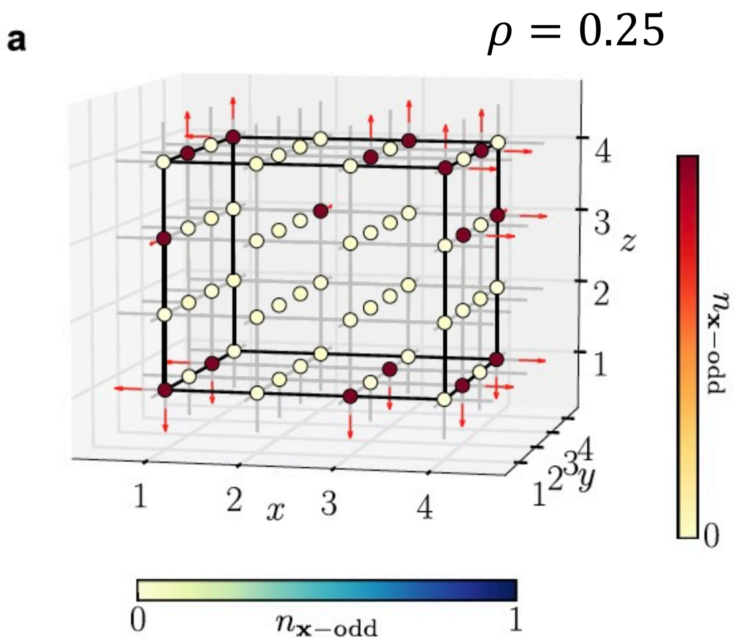
Confinement properties

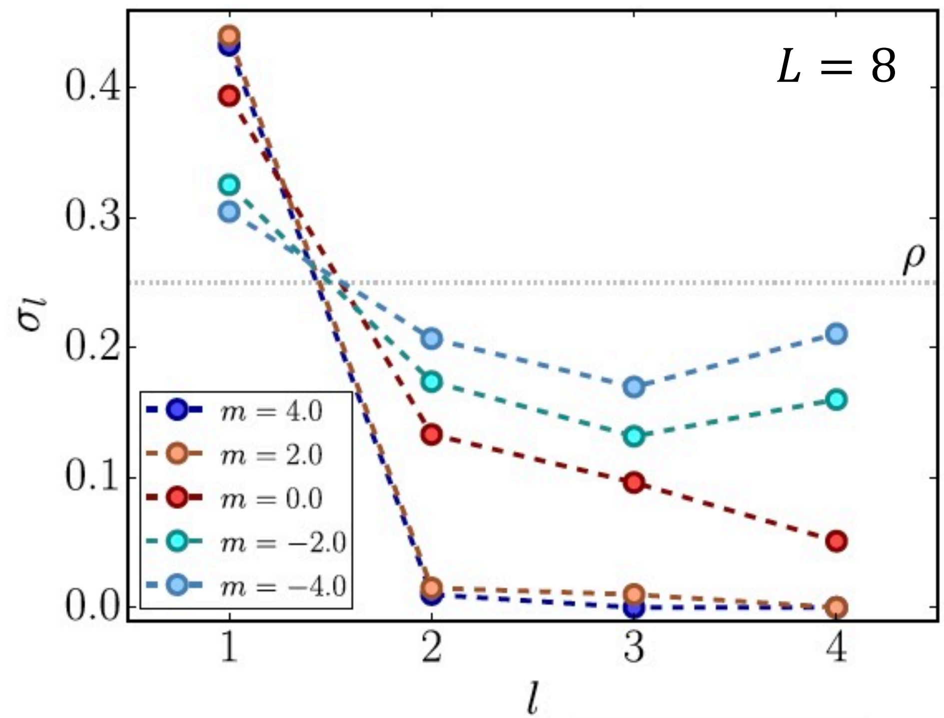
$V = 16 \times 4 \times 4$

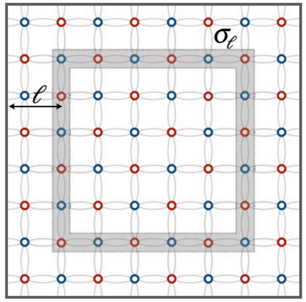


String-breaking by dynamical matter

Surface charge density at finite density







2D expression of σ_l

the spontaneous creation of charge-anticharge pairs determines a finite charge density on the bulk

Felser+, PRX10(2020)041040

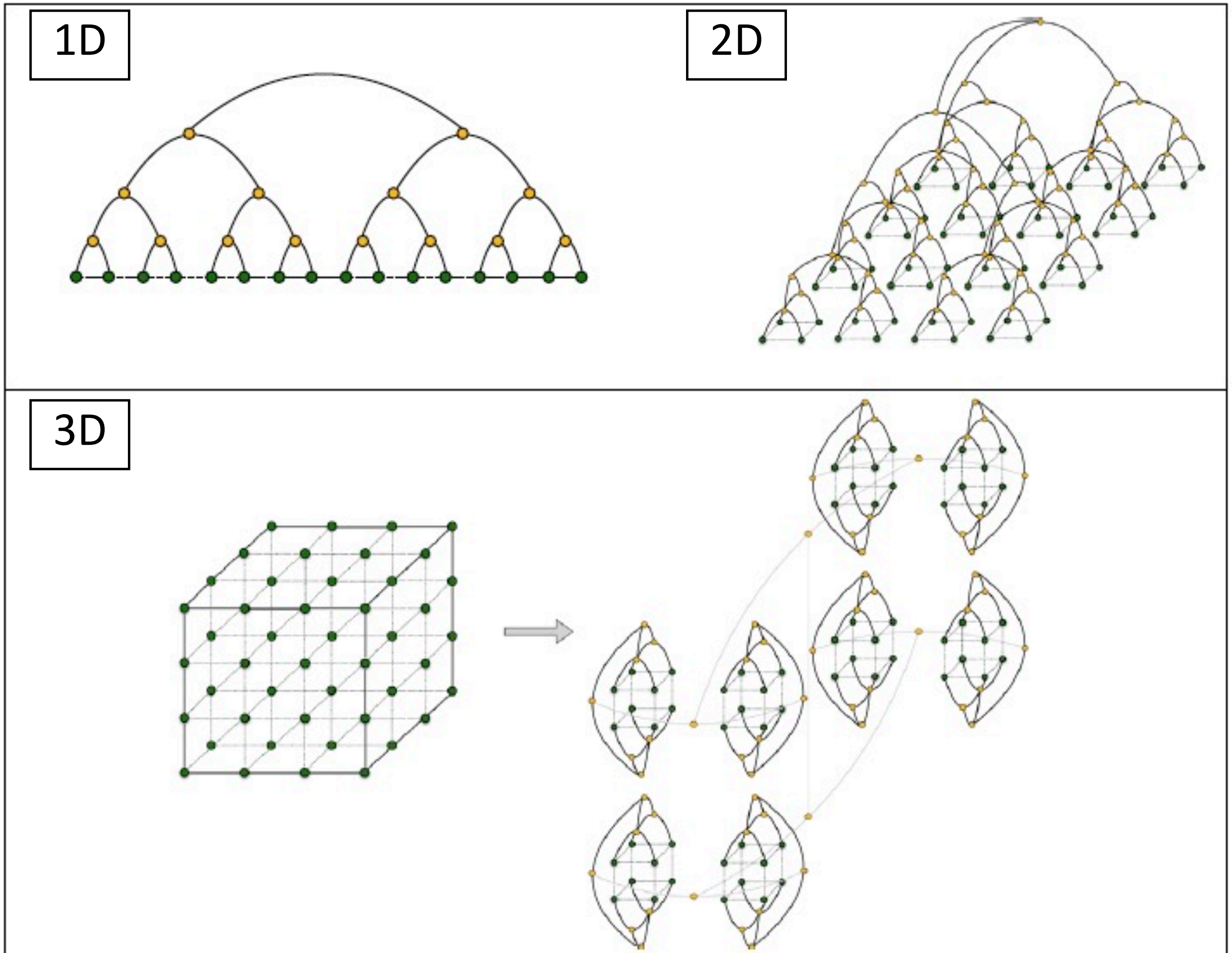
Summary

- ✔ Frist study of (3+1)D LGT with dynamical matter using the TN method based on the Hamiltonian formalism
- ✔ Confinement property has been confirmed
- ✔ TTN works efficiently in this study
- ✔ How about the large-volume calculation w/ TN in the Hamiltonian formalism? $s > 1$?

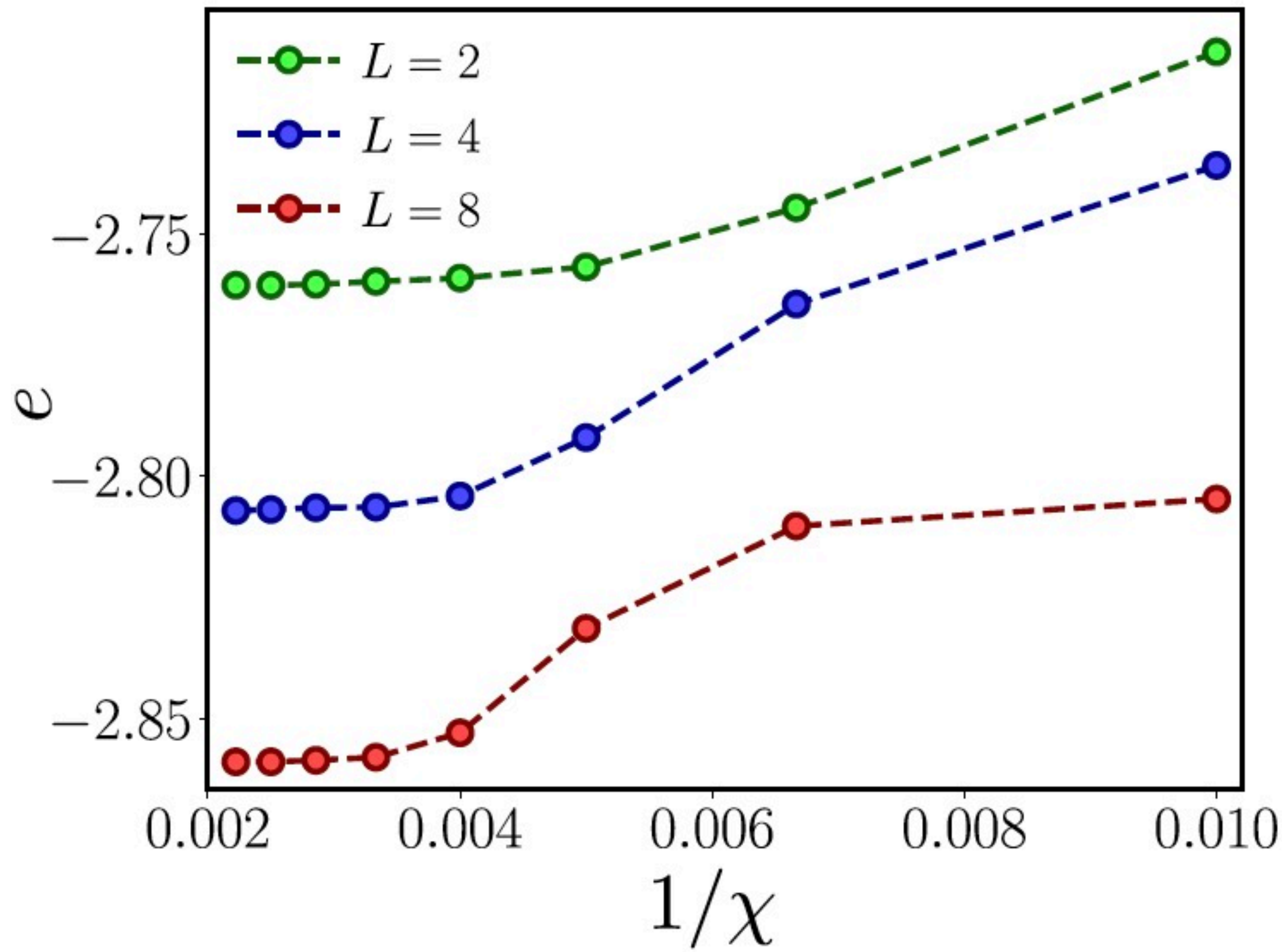
> “a single simulation for the maximum size that we reached, an 8×8×8 lattice, can last up to five weeks until final convergence, depending on the different regimes of the model and the control parameters of the algorithms”

Backup

Tree tensor network (TTN)



Convergence



The relative error of the energy is in the range of $[10^{-2}, 10^{-4}]$

The model (1/2)

$$H = -t\sum_{x,\mu}\left(\psi_x^\dagger U_{x,\mu}\psi_{x+\mu} + \mathrm{h.c.}\right) + m\sum_x (-1)^x\,\psi_x^\dagger\psi_x$$
$$+\frac{g_\mathrm{e}^2}{2}\sum_{x,\mu}E_{x,\mu}^2 - \frac{g_\mathrm{m}^2}{2}\sum_x\left(\square_{\mu_x\mu_y} + \square_{\mu_y\mu_z} + \square_{\mu_x\mu_z} + \mathrm{h.c.}\right)$$

Relation btw the model parameters and a, m_0, g:

$$t = a^{-1}, m = m_0, g_\mathrm{e}^2 = g^2a^{-1}, g_\mathrm{m}^2 = 8/(g^2a)$$

Physical regime of QED: $g_\mathrm{e}g_\mathrm{m} = 2\sqrt{2}t$

The physical state $|\Phi\rangle$ satisfies $G_x|\Phi\rangle = 0$ w/

$$G_x = \psi_x^\dagger\psi_x - \frac{1-(-1)^x}{2} - \sum_\mu E_{x,\mu}$$

The model (2/2)

Charge operator

$$Q = \sum_x (\psi_x^\dagger \psi_x - \frac{1-(-1)^x}{2})$$

Matter occupation operator

$$n_x = \frac{1-(-1)^x}{2} - (-1)^x \psi_x^\dagger \psi_x$$

Charge density along w/ μ_x direction

$$q_c(d) = \frac{2}{L^2} \sum_{j,k=1}^{L} \langle \mathrm{GS} | (-1)^x \psi_x^\dagger \psi_x | \mathrm{GS} \rangle \quad \text{w/} \quad x = (d, j, k)$$

Average matter density

$$\rho = \frac{1}{L^3} \sum_x \langle \mathrm{GS} | n_x | \mathrm{GS} \rangle \quad \text{w/} \quad n_x = \frac{1-(-1)^x}{2} - (-1)^x \psi_x^\dagger \psi_x$$