Tensor renormalization group in bosonic field theory

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Introduction

Tensor network あ S

$$Z = \sum_{\{x,y\}} \prod_n T_{x_n y_n x_{n-\hat{1}} y_{n-\hat{2}}}$$

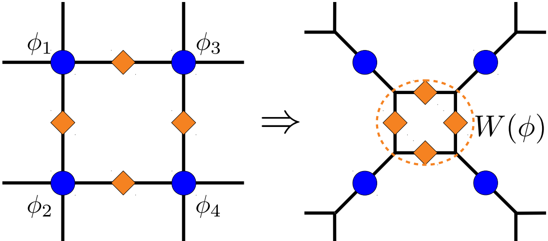
The model

A free scalar of mass m

$$Z = \int \mathcal{D}\phi \exp\left\{-\frac{1}{2}\sum_n \left[(\phi_n - \phi_{n+\hat{1}})^2 + (\phi_n - \phi_{n+\hat{2}})^2 + m^2\phi_n^2\right]\right\}$$

On the dual lattice: $\phi = (\phi_1, \phi_2, \phi_3, \phi_4)$,

$$W(\phi) = \exp\left[-\frac{1}{2}\phi^T M \phi\right], \quad M = \frac{m^2}{2} + \begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{pmatrix}.$$



Gaussian SVD

Splitting: $\phi_L = (\phi_1, \phi_2), \phi_R = (\phi_3, \phi_4)$

$$M = \begin{pmatrix} A & -B \\ -B & A \end{pmatrix}, \quad A = \frac{m^2}{2} + \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}, B = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$W(\phi_L, \phi_R) = \exp\left(-\frac{1}{2}\phi_L^T A \phi_L\right) \exp\left(\phi_L^T B \phi_R\right) \exp\left(-\frac{1}{2}\phi_R^T A \phi_R\right)$$

Decomposition: $B \simeq UDU^T$

$$\exp\left(\phi_L^T B \phi_R\right) = \int d\pi \; e^{i\phi_L^T U \pi} S(\pi) e^{-i\pi^T U^T \phi_R}$$

Gaussian SVD

$B = UDU^T$

$$D = \begin{pmatrix} d_+ & 0 \\ 0 & d_- \end{pmatrix}, \quad U = \frac{1}{\sqrt{2}} \begin{pmatrix} u_+ & u_- \\ u_+ & -u_- \end{pmatrix}$$

$$W(\varphi_L, \varphi_R) = \int d\tilde{\varphi} V(\varphi_L, \tilde{\varphi}) V^\dagger(\tilde{\varphi}, \varphi_R)$$

$$V(\varphi_L, \tilde{\varphi}) = G(\varphi_L) e^{i\varphi_L^T U \tilde{\varphi}} S^{1/2}(\tilde{\varphi})$$

$$\tilde{W}(\tilde{\varphi}) = \int \prod_{i=1}^{4} d\varphi_i V(\varphi_1, \varphi_2; \tilde{\varphi}_1) V^\dagger(\tilde{\varphi}_2, \varphi_2, \varphi_3)$$
$$\cdot V(\varphi_3, \varphi_4; \tilde{\varphi}_4) V^\dagger(\tilde{\varphi}_3; \varphi_4, \varphi_1)$$
$$= \tilde{\rho} \exp\left(-\frac{1}{2} \tilde{\varphi}^T \tilde{M} \tilde{\varphi}\right)$$

$$\tilde{M} = \frac{1}{2} I \otimes D^{-1} + C^T Q^{-1} C, \quad \tilde{\rho} = \rho^2 \frac{(2\pi)^{2\chi - \tilde{\chi}}}{\det D \det Q^{1/2}}$$

$$Q = I \otimes s + K \otimes a, C_L = \begin{pmatrix} U & 0 \\ 0 & 0 \end{pmatrix} - S \begin{pmatrix} 0 & 0 \\ 0 & U' \end{pmatrix}, C_R = \begin{pmatrix} 0 & 0 \\ 0 & U \end{pmatrix} - S$$