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On the exactness of soft theorems

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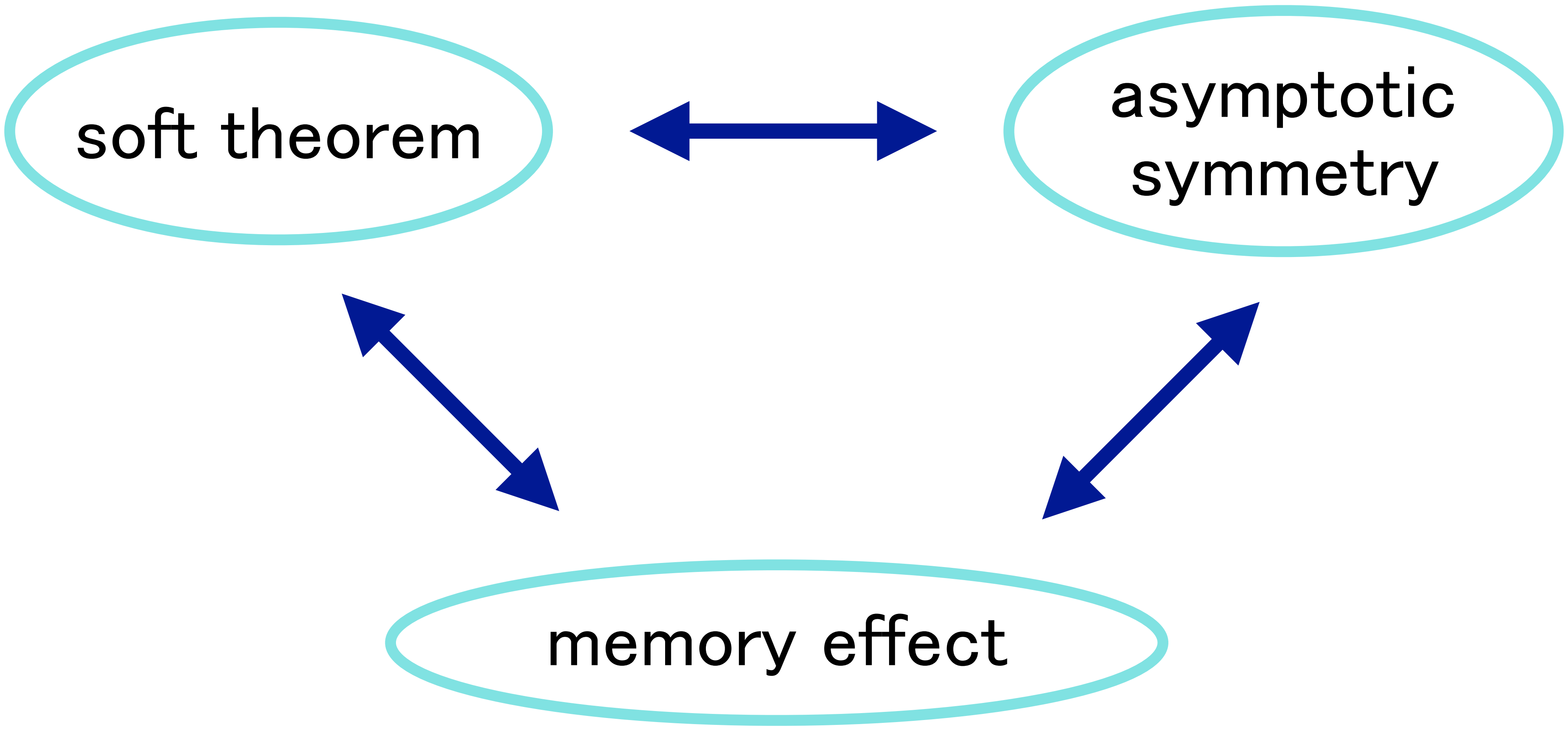
Backgrounds

- Developments in studies of amplitudes from general considerations [w/o Feynman diagrams]
  - S-matrix theory, string theory ･･･
  - series of conferences : Amplitudes **** (2009 –)
  - amplitudes of maximally susy Yang–Mills up to 5 loops (in agreement w/ finite-coupling amplitudes from string)
  - strong constraints on (tree) amplitudes: scattering equations
    [Cachazo-He-Yuan '13; Gross-Mende '88; Fairlie-Roberts '72]

⋮

- Developments in studies of soft theorems

"Infrared triangles" [Strominger '13 –]



- Importance of understanding low energy physics even for something new at high energy

- observational data from CMB, GW, ⋯

  ⇒ imprints of
  new physics, UV completion of gravity, ⋯

  e.g.) "Cosmological collider"
  probe new particles from non-gaussianities
  [Arkani Hamed –Maldacena '15, Noumi–Yamaguchi–Yokoyama '12, ⋯]

Soft theorems

- leading-order soft theorems are (usually) derived from symmetries enforcing massless-ness [universal]

- beyond leading order
  some are universal, some are not

- useful to know what is universal/non-universal

This paper

- observes a criteria for universality/non-universality

- derives various soft theorems

- checks the criteria
  by concrete examples and loop computations

- an observation on open string amplitudes

Plan

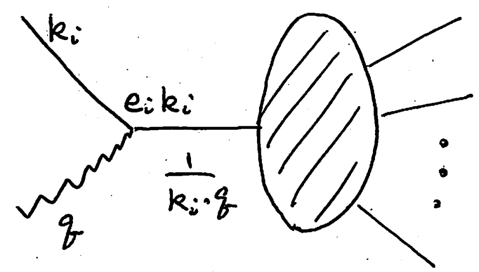
1. Introduction/backgrounds
2. Two types of soft theorems
3. Protected or unprotected
4. Examples
5. Dimensionally reduced string amplitudes
6. Summary

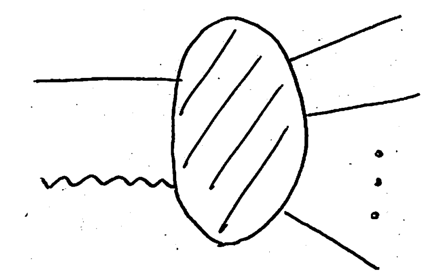
Soft theorem (I)

- consider scalar theory coupled w/ photon [Low '58]

- on shell,

$$A_n^\mu(q;k_1,...,k_n) = \sum_{i=1}^{n} \frac{e_i k_i^\mu}{k_i \cdot q} T_n(k_1,...,k_i+1,...,k_n) + N_n^\mu(q;k_1,...,k_n)$$





- expand around q=0, use gauge invariance $q_\mu A_n^\mu = 0$

$$\Rightarrow \qquad N_n^\mu = -\sum_i e_i \frac{\partial}{\partial k_{i\mu}} T_n$$

Soft theorem (I) :

$$A_n^\mu(q; k_1, \dots, k_n) = \sum_i \frac{e_i}{k_i \cdot q} \left[k_i^\mu - i q_\nu J_i^{\mu\nu} \right] T_n(k_1, \dots, k_n) + \mathcal{O}(q)$$

leading subleading

$$J_i^{\mu\nu} = i \left(k_i^\mu \frac{\partial}{\partial k_{i\nu}} - k_i^\nu \frac{\partial}{\partial k_{i\mu}} \right)$$

- similarly for gravitons [Weinberg '65]
- at leading order, it is understood as due to asymptotic symm.

[Strominger '13]

Soft theorem (II)

- consider soft theorem for SSB

- Ward–Takahashi Id. :

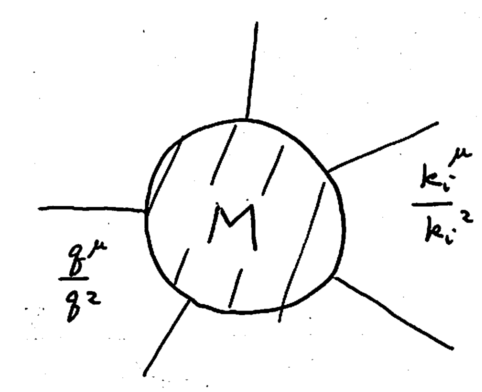
$$\partial^x_\mu \left\langle \, j^\mu(x)\, j^{\mu_1}(x_1)\, j^{\mu_2}(x_2) \cdots \right\rangle = \sum_i \delta(x - x_i) \left\langle \, j^{\mu_1}(x_1) \cdots \delta j^{\mu_i}(x_i) \cdots \right\rangle$$

- act w/ $\displaystyle \int dx\, e^{iqx} \prod_i \int dx_i e^{ik_i x_i} \partial^2_{\mu_i}$ [LSZ]

- note $j_\mu \to f_\pi \partial_\mu \pi^{\mathrm{as}}$ [NG boson] , $\displaystyle \mathrm{F.T.} \left\langle \, j^\mu(x) \cdots \right\rangle = \frac{iq^\mu}{q^2} \tilde{M} + N^\nu$

$\Rightarrow$ F.T. (LHS) = [LSZ] $\tilde{M}$ $(q \to 0)$

$$= M(\pi_q, \pi_i, \ldots) \prod k_i^\mu + \mathcal{O}(q)$$



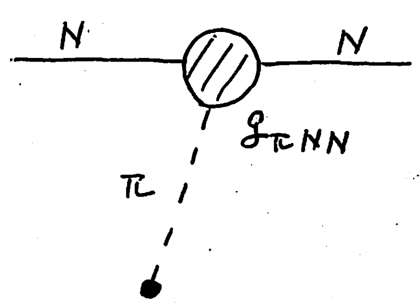
Soft theorem (II) [leading]:

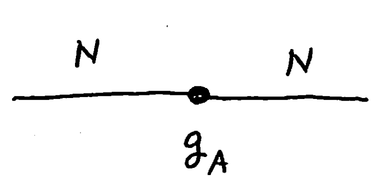
$$M(\pi_q, \pi_i, \dots) \prod k_i^{\mu} = \text{ F.T. (RHS) } + \mathcal{O}(q)$$

- RHS = 0 if $\delta j^{\mu_i}(x_i)$ does not produce asymptotic states

  [Adler's zero]

- Goldberger–Treiman ['58]:
  - $\partial_\mu^x \Big\langle\, j^\mu(x)\, j_{\mu_1}(x_1)\, j_{\mu_2}(x_2) \Big\rangle \to \partial_\mu^x \Big\langle\, j^\mu_{\text{axial}}(x)\, N(x_1)\, N(x_2) \Big\rangle$
  - (LHS) = (RHS)





Soft theorem (II) [subleading]:

- consider two generators of SSB related by translation

$$[P, G_1] \sim G_2$$

e.g.) G1: rotation

G2: translation

- in terms of currents

$$J_1 \sim xJ_2 \qquad \text{or} \qquad \tilde{J}_1 \sim \frac{\partial}{\partial p}\tilde{J}_2$$

- soft theorem of type (II) for J1 via that for J2

$$\text{(LHS)} = \frac{\partial}{\partial q} M(\pi_q, \pi_i, \ldots) \prod k_i^{\mu} + \mathcal{O}(q)$$

Observation in this paper

- soft theorem like type (II) : uses only symmetry
  survives quantum corrections; remains universal
  [irrespectively of UV completion]

- soft theorem of type (I) : uses concrete form of vertices
  may not survive quantum corrections

This paper checks this

(and identifies some existing theorems as type (I))

Double soft theorems for SSB

- this paper gives
  leading and subleading double soft theorems
  for spontaneously broken

  (1) translation + rotation [leading, subleading]

  (2) dilatation + special conformal [leading, subleading]

  (3) susy, conformal susy [leading]

e.g.)

$$M_{n+2}\big(\pi(q)\pi(p)\pi(k_1)\cdots\pi(k_n)\big)\Big|_{p,q\to 0} = \sum_i \big(S_i^{(0)}+S_i^{(1)}\big)M_n\big(\pi(k_1)\cdots\pi(k_n)\big)$$

$$S_i^{(0)} = \frac{1}{4}\frac{\big(k_i\cdot(p-q)\big)^2}{k_i\cdot(p+q)}\,,\quad S_i^{(1)} = \frac{1}{2}\left(-\frac{(k_i\cdot p)^2+(k_i\cdot q)^2}{\big(k_i\cdot(p-q)\big)^2}(p\cdot q)+\frac{k_i\cdot(p-q)}{k_i\cdot(p+q)}(p_\mu q_\nu J_i^{\mu\nu})\right)$$

- checks by explicit computation that leading/subleading single/double soft theorems are indeed protected at 1 loop for

  (1) DBI theory [4, 6-dim.]

  (2) conformal DBI theory [4, 6-dim.]

  (3) Akulov-Volkov theory [4-dim.]

  -effective theories for Goldstone modes-

- some soft theorems in literature are not protected
  ⇒ identified them as type (I)

Dimensionally reduced string amplitudes

- consider super/bosonic open string tree amplitudes

- extract U(1) part
  dimensionally reduce it by setting $\epsilon_i \cdot k_j = 0$

- the resultant amplitudes satisfy
  leading/subleading single/double soft theorems
  for spontaneously broken translational symmetry

- perturbative strings do not have to know it
  ⇒ perturbative strings knew D-branes!

[non-perturbative, inducing SSB of translation]

Summary

- observed a criteria of universality of soft theorems
- checked it by concrete examples and computations
- it seems that perturbative strings did know D-branes
- soft theorem of type (I) may be more useful
  to probe new physics   cf. Laddha-Sen