Tensor Network Renormalization

PhysRevLett115,180405(2014), G.Evenbly,G.Vidal

Algorithms for tensor network renormalization

PhysRevB95,045117(2015), G.Evenbly

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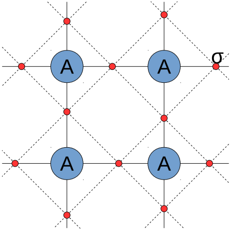
- Ising model

$$Z = \sum_{\{\sigma\}} e^{-H[\sigma]/T},$$

$$H[\sigma] = -\sum_{<i,j>} \sigma_i \sigma_j, \quad \sigma_i \in \{+1, -1\}$$



- Tensor network representation

$$Z = \sum_{ijk\cdots} A_{ijkl} A_{mnoj} A_{krst} A_{opqr} \cdots$$

$$A_{ijkl} = e^{(\sigma_i\sigma_j + \sigma_j\sigma_k + \sigma_k\sigma_l + \sigma_l\sigma_i)/T}$$

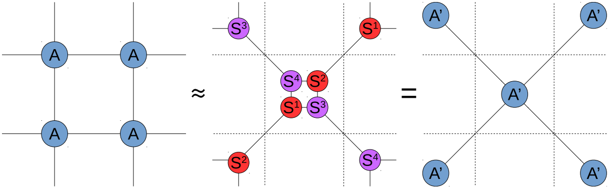
- テンソルの一つの足の自由度を χ で表す。

- Singular Value Decomposition(SVD)

$$
\begin{aligned}
A_{ijkl} &= \sum_{m}^{\chi^2} U_{ijm}\sigma_m V_{mkl}, \quad U, V : \text{unitary matrix}, \sigma_m \geq 0 \\
&\approx \sum_{m}^{\chi} S^1_{ijm} S^2_{mkl}, \quad S^1_{ijm} = U_{ijm}\sqrt{\sigma_m}, S^2_{mkl'} = \sqrt{\sigma_m} V_{mkl} \\
&\approx \sum_{m}^{\chi} S^3_{kjm} S^4_{mil}
\end{aligned}
$$

- contraction

$$
A'_{i'j'k'l'} = \sum_{ijkl} S^1_{iji'} S^3_{ilj'} S^2_{k'kl} S^4_{l'kj}
$$



- contraction

$$M_{X_1 y_1 X_2 y_2} = \sum_y A_{x_1 y_1 x_2 y} A_{x_3 y x_4 y_2}, \quad X_1 = (x_1, x_3), X_2 = (x_2, x_4)$$

- higher order svd

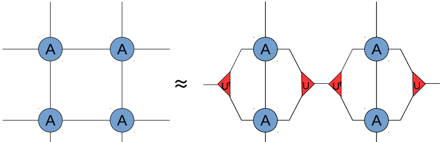
$$M_{X_1 y_1 X_2 y_2} = \sum_{i,j,k,l}^{\chi^2,\chi,\chi^2,\chi} S_{ijkl} U^1_{X_1 i} U^2_{y_1 j} U^3_{X_2 k} U^4_{y_2 l}, \quad U^{1,2,3,4} : \text{unitary matrix},$$

$$S : \text{core tensor}, \quad \sum_{jkl} S_{ijkl} S_{i'jkl} \propto \delta_{i,i'}$$

$$\sum_{y_1 X_2 y_2} M_{X_1' y_1 X_2 y_2} M^*_{X_1' y_1 X_2 y_2} = \sum_m^{\chi^2} U^1_{X_1 m} \lambda_m U^{1*}_{X_1' m} \approx \sum_m^{\chi} U^1_{X_1 m} \lambda_m U^{1*}_{X_1' m}$$

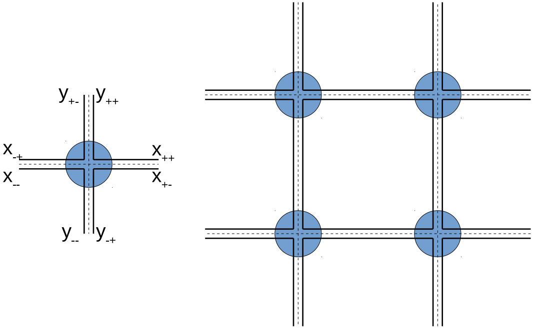
- truncation

$$A'_{x_1' y_1 x_2' y_2} = \sum_{X_1, X_2} M_{X_1 y_1 X_2 y_2} U^{1*}_{X_1 x_1'} U^1_{X_2 x_2}$$

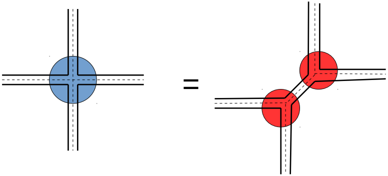


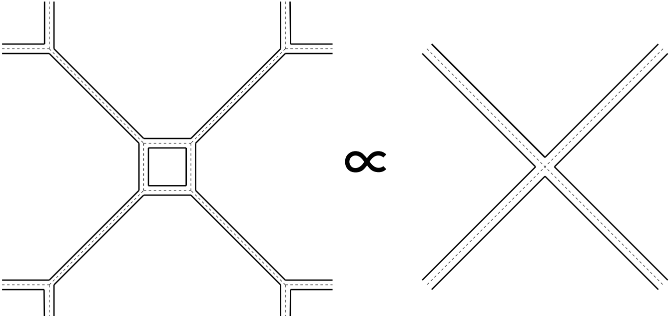
- Corner Double Line Tensor(CDLT) は短距離の相関だけを持つようなテンソル

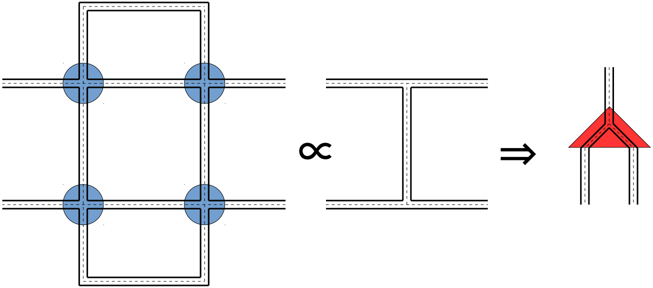
$$A^{\mathrm{CDL}}_{x_{++}x_{+-}y_{++}y_{+-}x_{-+}x_{--}y_{-+}y_{--}} = \delta_{x_{++}y_{++}}\delta_{x_{-+}y_{+-}}\delta_{x_{--}y_{--}}\delta_{x_{+-}y_{-+}} \tag{1}$$

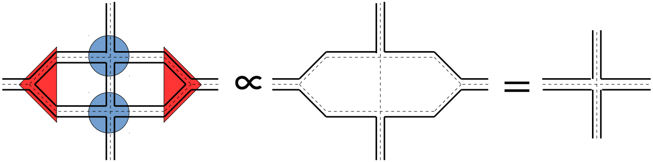


- TRG,HOTRG の固定点になっており、これらのくりこみ変換で短距離相関が取り除き切れていないことを意味する。



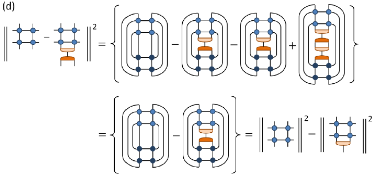






- あるネットワーク $\mathcal{F}$ に射影 $P = ww^\dagger (w^\dagger w = I)$ をかけて近似された $\tilde{\mathcal{F}}$ をつくる。

$$\tilde{\mathcal{F}} = \mathcal{F}P, PP^\dagger = P^2 = P$$



- $\mathcal{F}$ と $\tilde{\mathcal{F}}$ の誤差 ϵ

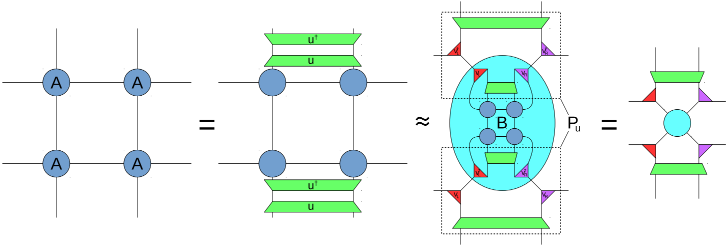
$$\epsilon^2 = ||\mathcal{F} - \tilde{\mathcal{F}}||^2 = ||\mathcal{F}||^2 + ||\tilde{\mathcal{F}}||^2 - \mathrm{tTr}\left[\mathcal{F} \otimes \tilde{\mathcal{F}}^*\right] - \mathrm{tTr}\left[\mathcal{F}^* \otimes \tilde{\mathcal{F}}\right]$$
$$= ||\mathcal{F}||^2 - ||\tilde{\mathcal{F}}||^2, \quad \left(||\tilde{\mathcal{F}}||^2 = \mathrm{tTr}\left[\mathcal{F} \otimes \tilde{\mathcal{F}}^*\right] = \mathrm{tTr}\left[\mathcal{F}^* \otimes \tilde{\mathcal{F}}\right]\right)$$

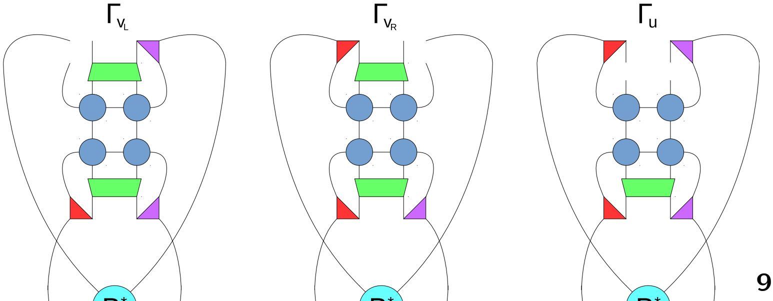
- ϵ の最小化は $||\tilde{\mathcal{F}}||^2$ の最大化になる。

$$||\tilde{\mathcal{F}}|| = ||\mathcal{F}w||^2 = \mathrm{tTr}\left[\Gamma_w w\right], \quad \Gamma_w = w^\dagger \mathcal{F}^* \otimes F$$

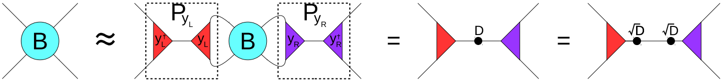
Γ_w を SVD した結果を $\Gamma_w = usv^\dagger$ とすると、$w = vu^\dagger$ とすると最大化できる。

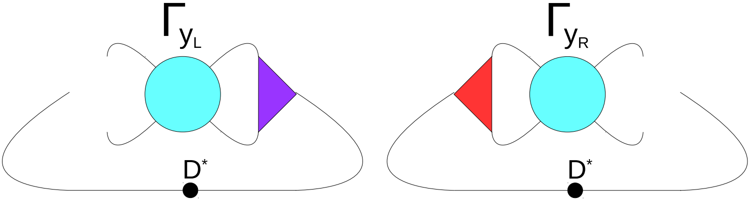
- u:disentangler,$\chi^2 \times \chi^2$ のユニタリー行列
- v_L, v_R:isometry,$\chi^2 \times \chi$ の行列,$v_L^\dagger v_L = I^{\chi\otimes\chi}$



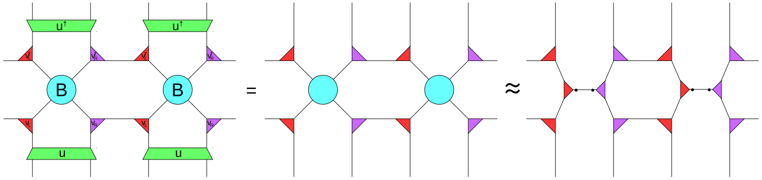


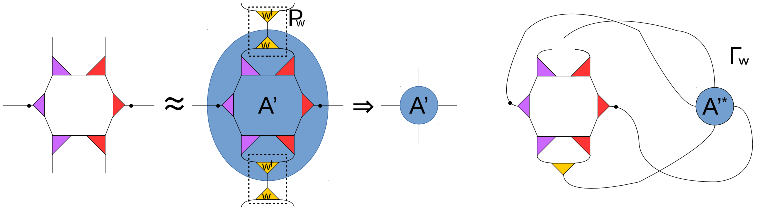
TNR: 2nd truncation



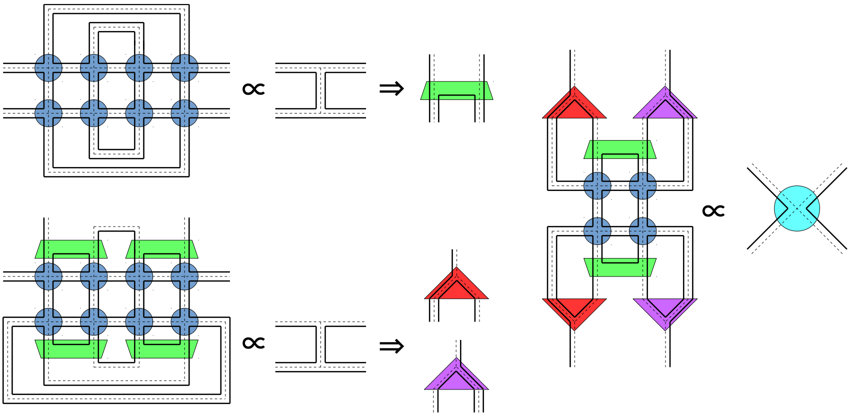


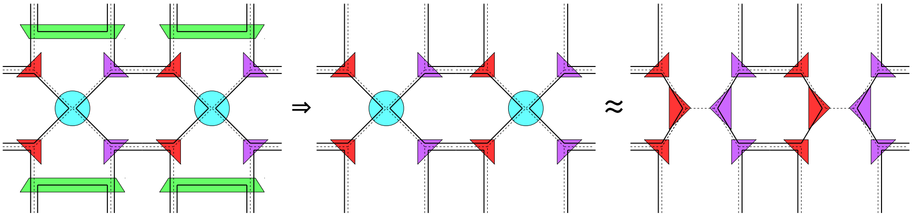
TNR: 3rd truncation

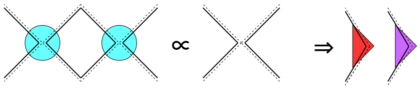




初期テンソルは HOSVD で与える。







CDLT: 3rd truncation