Regge behavior saves String Theory from causality violations

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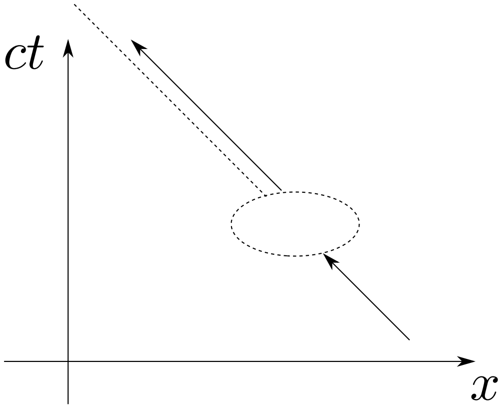
Shapiro time delay



- A time advance means that causality is violated.
- No time advance in Einstein gravity.

Camanho et. al.

If the gravity action is of the form

$$\frac{1}{l_P^{d-2}} \int d^d x \sqrt{-g} \left(R + l_2^2 (R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2) + l_4^4 (R)^3 + \cdots \right) ,$$

with $l_P \ll l_2, l_4$

- There is a situation in which we get a time advance.
- Only conceivable way to cure this is to consider string theory.

Camanho, Edelstein, Maldacena and Zhiboedov

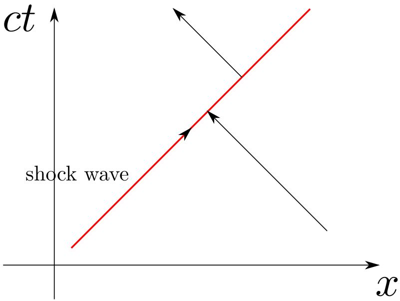
(arXiv:1407.5597[hep-th])

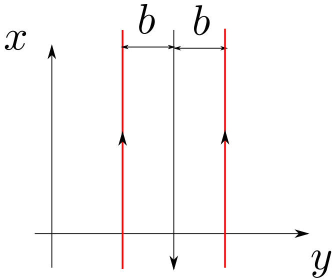
In this paper

it is checked that there is no causality problem in string theory.

1. Setup
2. String theory
3. Causality

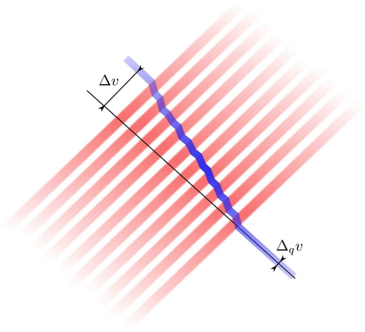
§1 Setup

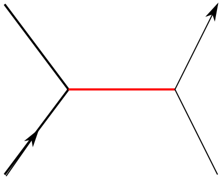




- We need to solve the equation of motion to get the shock wave solution.

Setup



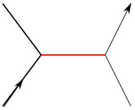


- It is possible to calculate the time delay or advance from the perturbative amplitudes.

Setup

For high energy scattering, only the phase of the wave function changes

$$\psi \to (1 + i\delta)\psi$$



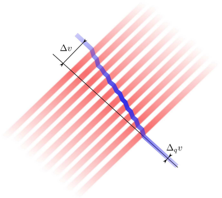
$$\frac{1}{l_P^{d-2}} \int d^d x \sqrt{-g} \left(R + l_2^2 (R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2) + l_4^4 (R)^3 + \cdots \right)$$

For the energy scale $l_P < \frac{1}{E} < l_2, l_4$, δ can be calculated perturbatively.

Setup

Repeating the same process $N \gg 1$ times

$$\psi \to e^{iN\delta}\psi = e^{\Delta t \partial_t}\psi$$



Taking ψ to be a wave packet localized in (t, E) space, Δt gives the time delay or advance.

§2 String theory

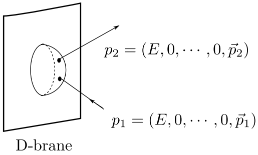
- The effective action of bosonic string theory

$$\frac{1}{g_s^2\left(\alpha'\right)^{\frac{D-2}{2}}}\int d^D x\sqrt{-g}\left(R+\alpha'(R_{\mu\nu\rho\sigma}^2-4R_{\mu\nu}^2+R^2)+\left(\alpha'\right)^2\left(R\right)^3+\cdots\right)$$

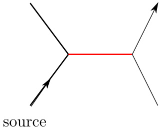
- For $g_s \ll 1$, we have $l_P \ll l_2, l_4$.
- Since only tree amplitudes matter, the tachyon does not cause any problem.

String theory amplitudes

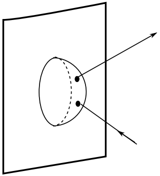
Let us consider the scattering of a graviton by a D-brane in bosonic string theory,



in order to realize



String theory amplitudes



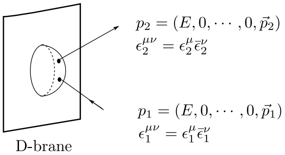
$$p_2 = (E, 0, \cdots, 0, \vec{p}_2)$$
$$\epsilon_2^{\mu\nu} = \epsilon_2^{\mu}\bar{\epsilon}_2^{\nu}$$

$$p_1 = (E, 0, \cdots, 0, \vec{p}_1)$$
$$\epsilon_1^{\mu\nu} = \epsilon_1^{\mu}\bar{\epsilon}_1^{\nu}$$

$$q \equiv p_2 - p_1$$
$$t \equiv -q^2$$
$$s \equiv E^2$$

$$\begin{aligned} A \quad &= \quad \frac{\kappa T}{2} \frac{\Gamma(-\frac{\alpha'}{4}t - 1)\Gamma(-\alpha' s + 1)}{\Gamma(-\frac{\alpha'}{4}t - \alpha' s)} \\ &\qquad \times \left[\epsilon_1 \cdot \epsilon_2 - \frac{\alpha'}{2}\epsilon_1 \cdot q \epsilon_2 \cdot q\right]\left[\bar{\epsilon}_1 \cdot \bar{\epsilon}_2 - \frac{\alpha'}{2}\bar{\epsilon}_1 \cdot q \bar{\epsilon}_2 \cdot q\right] + \cdots \end{aligned}$$

§3 Causality



$q \equiv p_2 - p_1$

$t \equiv -q^2$

$s \equiv E^2$

- High energy scattering:

$$E = |\vec{p}_1| = |\vec{p}_2| \gg |\vec{p}_2 - \vec{p}_1| \longrightarrow s \gg t$$

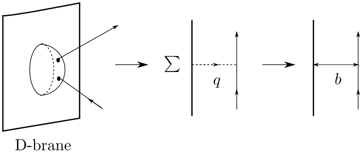
- In order to make R^2, R^3 relevant $\alpha' t \sim 1$

$$S \sim \frac{1}{g_s^2 \left(\alpha'\right)^{\frac{D-2}{2}}} \int d^D x \sqrt{-g} \left(R + \alpha' R^2 + \left(\alpha'\right)^2 R^3 + \cdots\right)$$

$\alpha' s \gg \alpha' t \sim 1$

$$
\begin{aligned}
A &= \frac{\kappa T}{2}\frac{\Gamma(-\frac{\alpha'}{4}t-1)\Gamma(-\alpha' s+1)}{\Gamma(-\frac{\alpha'}{4}t-\alpha' s)} \\
&\quad \times \left[\epsilon_1\cdot\epsilon_2-\frac{\alpha'}{2}\epsilon_1\cdot q\epsilon_2\cdot q\right]\left[\bar{\epsilon}_1\cdot\bar{\epsilon}_2-\frac{\alpha'}{2}\bar{\epsilon}_1\cdot q\bar{\epsilon}_2\cdot q\right]+\cdots \\
&\sim \frac{\kappa T}{2}e^{-i\pi\frac{\alpha' t}{4}}(\alpha' s)^{1+\frac{\alpha' t}{4}}\Gamma\left(-1-\frac{\alpha' t}{4}\right) \\
&\quad \times \left[\epsilon_1\cdot\epsilon_2-\frac{\alpha'}{2}\epsilon_1\cdot q\epsilon_2\cdot q\right]\left[\bar{\epsilon}_1\cdot\bar{\epsilon}_2-\frac{\alpha'}{2}\bar{\epsilon}_1\cdot q\bar{\epsilon}_2\cdot q\right]
\end{aligned}
$$

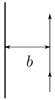
Contributions from the massless particles



$$
\begin{aligned}
A &\sim \frac{\kappa T}{2} e^{-i\pi\frac{\alpha' t}{4}} (\alpha' s)^{1+\frac{\alpha' t}{4}} \Gamma\left(-1-\frac{\alpha' t}{4}\right) \\
&\quad \times \left[\epsilon_1 \cdot \epsilon_2 - \frac{\alpha'}{2}\epsilon_1 \cdot q \epsilon_2 \cdot q\right] \left[\bar{\epsilon}_1 \cdot \bar{\epsilon}_2 - \frac{\alpha'}{2}\bar{\epsilon}_1 \cdot q \bar{\epsilon}_2 \cdot q\right] \\
&\to -\kappa T \frac{2s}{q^2} \left[\epsilon_1 \cdot \epsilon_2 - \frac{\alpha'}{2}\epsilon_1 \cdot q \epsilon_2 \cdot q\right] \left[\bar{\epsilon}_1 \cdot \bar{\epsilon}_2 - \frac{\alpha'}{2}\bar{\epsilon}_1 \cdot q \bar{\epsilon}_2 \cdot q\right] \\
&\to -2\kappa T s \left[\epsilon_1 \cdot \epsilon_2 + \frac{\alpha'}{2}\epsilon_1 \cdot \partial_b \epsilon_2 \cdot \partial_b\right] \left[\bar{\epsilon}_1 \cdot \bar{\epsilon}_2 + \frac{\alpha'}{2}\bar{\epsilon}_1 \cdot \partial_b \bar{\epsilon}_2 \cdot \partial_b\right] \frac{\Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}} b^{D-2}}
\end{aligned}
$$

Phase shift

$$A(E,b) = -2\kappa T s \left[\epsilon_1 \cdot \epsilon_2 + \frac{\alpha'}{2}\epsilon_1 \cdot \partial_b \epsilon_2 \cdot \partial_b\right]\left[\bar{\epsilon}_1 \cdot \bar{\epsilon}_2 + \frac{\alpha'}{2}\bar{\epsilon}_1 \cdot \partial_b \bar{\epsilon}_2 \cdot \partial_b\right]\frac{\Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}} b^{D-2}}$$



- phase shift

$$\begin{aligned}\delta(E,b) &= \frac{A(E,b)}{2E} \\ &= -\kappa T E \left[\epsilon_1 \cdot \epsilon_2 + \frac{\alpha'}{2}\epsilon_1 \cdot \partial_b \epsilon_2 \cdot \partial_b\right]\left[\bar{\epsilon}_1 \cdot \bar{\epsilon}_2 + \frac{\alpha'}{2}\bar{\epsilon}_1 \cdot \partial_b \bar{\epsilon}_2 \cdot \partial_b\right] \\ &\quad \times \frac{\Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}} b^{D-2}}\end{aligned}$$

Time delay or advance

- time delay or advance

$$\begin{aligned}\delta(E,b) &= \frac{A(E,b)}{2E} \\ &= -\kappa T E \left[\epsilon_1\cdot\epsilon_2 + \frac{\alpha'}{2}\epsilon_1\cdot\partial_b\epsilon_2\cdot\partial_b\right]\left[\bar\epsilon_1\cdot\bar\epsilon_2 + \frac{\alpha'}{2}\bar\epsilon_1\cdot\partial_b\bar\epsilon_2\cdot\partial_b\right] \\ &\quad\times\frac{\Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}}b^{D-2}}\end{aligned}$$

- $e^{iN\delta} = e^{\Delta t\partial_t}$ with

$$\Delta t = \kappa T N\left[\epsilon_1\cdot\epsilon_2 + \frac{\alpha'}{2}\epsilon_1\cdot\partial_b\epsilon_2\cdot\partial_b\right]\left[\bar\epsilon_1\cdot\bar\epsilon_2 + \frac{\alpha'}{2}\bar\epsilon_1\cdot\partial_b\bar\epsilon_2\cdot\partial_b\right]\frac{\Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}}b^{D-2}}$$

This can be positive or negative depending on the polarization.

String theory

$$
\begin{aligned}
A &\sim -\frac{\kappa T}{2} e^{-i\pi\frac{\alpha' t}{4}} (\alpha' s)^{1+\frac{\alpha' t}{4}} \Gamma\left(-1-\frac{\alpha' t}{4}\right) \\
&\quad \times \left[\epsilon_1 \cdot \epsilon_2 - \frac{\alpha'}{2}\epsilon_1 \cdot q \epsilon_2 \cdot q\right]\left[\bar{\epsilon}_1 \cdot \bar{\epsilon}_2 - \frac{\alpha'}{2}\bar{\epsilon}_1 \cdot q \bar{\epsilon}_2 \cdot q\right] \\
\delta(E,b) &\sim \kappa T E \left[\epsilon_1 \cdot \epsilon_2 + \frac{\alpha'}{2}\epsilon_1 \cdot \partial_b \epsilon_2 \cdot \partial_b\right]\left[\bar{\epsilon}_1 \cdot \bar{\epsilon}_2 + \frac{\alpha'}{2}\bar{\epsilon}_1 \cdot \partial_b \bar{\epsilon}_2 \cdot \partial_b\right] \\
&\quad \times \left[e^{-\frac{b^2}{\alpha' Y}} \cdots\right]
\end{aligned}
$$

with $Y = \ln(\alpha' s) \gg 1$. Therefor $\Delta t \propto \epsilon_1 \cdot \epsilon_2 \bar{\epsilon}_1 \cdot \bar{\epsilon}_2$ and it is always positive.