Magnetic monopole and confinement/deconfinement phase transition in SU(3) Yang-Mills theory

文献紹介（鈴木遊）

2015/06/12

K. Kondo, S. Kato, A. Shibata, T. Shinohara, arXiv:1501.06271
K. Kondo, S. Kato, A. Shibata, T. Shinohara, arXiv:1409.1599
K. Kondo, S. Kato, A. Shibata, Phys. Rev. D91, 034506 (2015)
Y. Nambu, Phys. Rev. D10, 4262-4268 (1974).

超伝導

[電磁気学]

電荷 ⟷ 磁荷

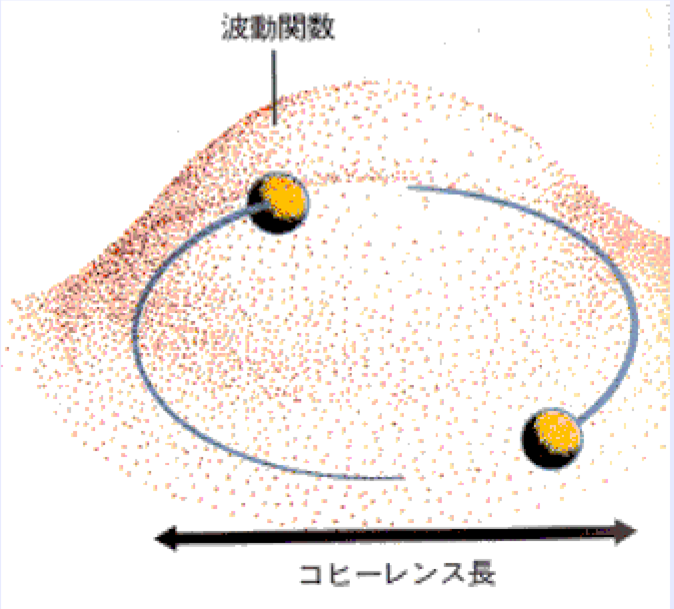
双対性

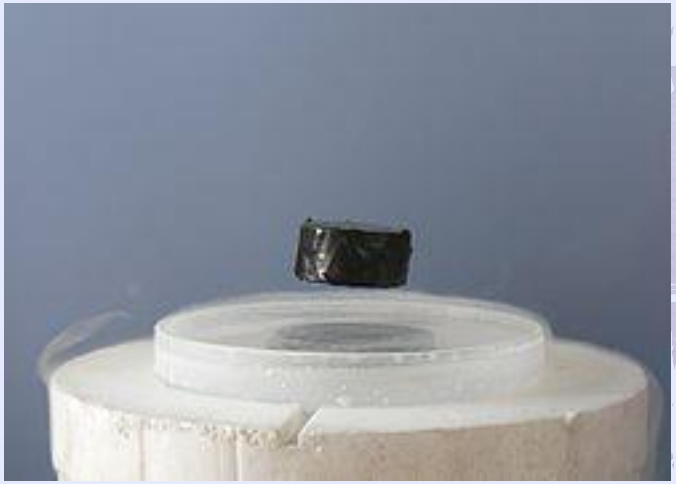
凝縮

クーパー対

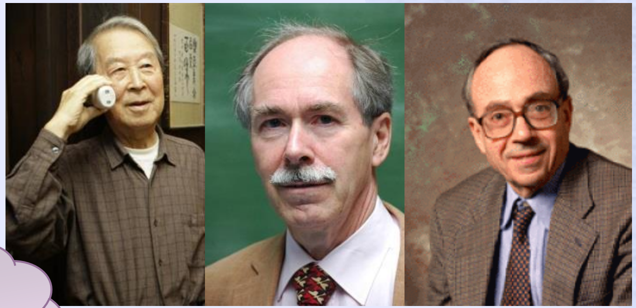
単磁荷不在

超伝導体





双対超伝導



[QCD]

クォーク

? 難しい

閉じ込め

flux tube

仮定

磁荷

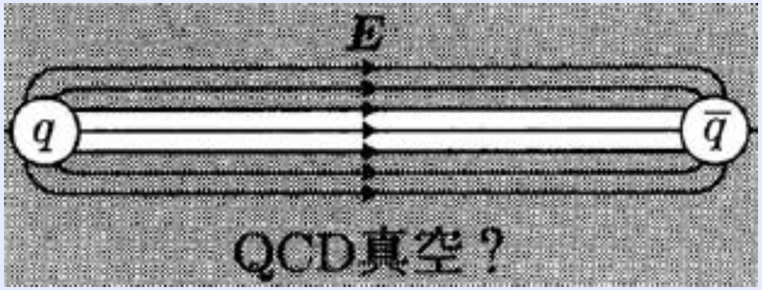
凝縮

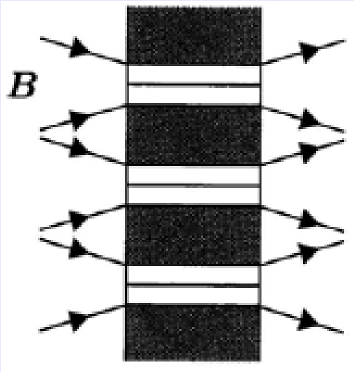
第2種超伝導体

強磁場

flux tube

双対性





疑問

- 単磁荷はあるのか？
  - t'Hooft：MA gauge (1981)
  - gaugeによらない処方が必要

    → **arXiv:1409.1599**
- 閉じ込めと超伝導は関係あるのか？
  - monopole current を有限温度で測定

    → **arXiv:1501.06271**

発表の流れ

- ✓ 背景
- monopoleの定義
  - SU(2) Y-M theory
- CDGFN分解
  - master Y-M
  - color direction
- SU(3)への拡張
- 格子上での定義
  - SU(3) Y-M theory
  - CDGFN分解
- 数値計算の結果
  - field strength
  - monopole current
- まとめ

磁荷をどう定義するか？

[電磁気]

$$g_m := \int d^3x \vec{\nabla} \cdot \vec{B} = \frac{1}{2}\int d^3x\, \epsilon^{0ijk}\partial_i F_{jk} = 0$$

[Y-M 理論]

$$g_m := \frac{1}{2}\int d^3x\, \epsilon^{0ijk}\partial_i F_{jk}$$

✕ gauge dependent

もしも…

$$n(x) \to \Omega(x) n(x) \Omega^\dagger(x)\, , 2tr\{n(x)n(x)\} = 1$$

$$F_{\mu\nu}(x) = f_{\mu\nu}(x) n(x)$$

○ gauge inv.

$$g_m := \frac{1}{2}\int d^3x\, \epsilon^{0ijk}\partial_i f_{jk} = \int d^3x \epsilon^{0ijk}\partial_i tr\{nF_{jk}\}$$

CDGFN分解

$$A_\mu^a(x) = V_\mu^a(x) + X_\mu^a(x)$$

Abelian part

cf. MA gauge
$V_\mu^a = \left(0,0,A_\mu^3\right)^a, X_\mu^a = \left(A_\mu^1, A_\mu^2, 0\right)^a$
$n^a(x) = (0,0,1)^a$

分解の仕方

$$\begin{cases} D_\mu^{[V]} n^a(x) = 0 \\ n_a(x) X_\mu^a(x) = 0 \end{cases}$$

$n^a(x)$ ~ Abelian な向きを表す場

$$\begin{cases} V_\mu^a(x) = A_\mu^b(x) n_b(x) n^a(x) + \dfrac{1}{2g} f^{abc} \partial_\mu n_b(x) n_c(x) \\ X_\mu^a(x) = \dfrac{1}{2g} f^{abc} n_b(x) D_\mu^{[A]} n_c(x) \end{cases}$$

※ $n^a(x)$ は後で決める

磁荷を求めてみる

field strength の Abelian part

$$F_{\mu\nu}^{[V]a} := \partial_\mu V_\nu^a - \partial_\nu V_\mu^a + \frac{g}{2} f^{abc} V_{b\mu} V_{c\nu}$$

$$= \left\{ \partial_\mu (A_\nu^b n_b) - \partial_\nu (A_\mu^b n_b) - \frac{n_b}{2g} f^{bcd} (\partial_\mu n_c)(\partial_\nu n_d) \right\} n^a$$

$f_{\mu\nu}$

$$g_m := \frac{1}{2} \int d^3x \, \epsilon^{0ijk} \partial_i f_{jk}$$

$$n^a(x) = (\sin\phi\cos\theta, \sin\phi\sin\theta, \cos\phi) \in S^2$$

$$g_m = g^{-1} \int_S dS^{jk} \sin\phi \frac{\partial(\phi,\theta)}{\partial(x_j,x_k)} = \boxed{\frac{4\pi}{g} N \ , N \in \mathbb{Z}}$$

Jacobian　　Dirac の量子化条件

master Y-M theory

$$A_\mu^a \,, n^a$$

$$\delta_\omega A_\mu^a = D_\mu^{[A]} \omega^a \qquad : SU(2)$$

$$\delta_\theta n^a = \frac{1}{2} g f^{abc} n^b \theta^c \quad : S^2 \simeq SU(2)/U(1)$$

$\theta = 0$

?

Y-M theory

$A_\mu^a : SU(2)$

CDG Y-M theory

$n^a, V_\mu^a, X_\mu^a : SU(2)$

$n(x)$ の決め方

$$\int d^4x\, X_\mu^a(x) X_\mu^a(x)$$

$$= \int d^4x\, D_\mu^{[A]} n^a(x) D_\mu^{[A]} n^a(x)$$ を最小化

$$\delta_{\omega,\theta} \int d^4x\, X_\mu^a(x) X_\mu^a(x) = 0 \quad \Rightarrow \quad \underline{\omega_\perp = \theta_\perp}$$

gauge 変換

$$n \to \Omega n \Omega^\dagger$$
$$V_\mu \to \Omega V_\mu \Omega^\dagger + i g^{-1} \Omega \partial_\mu \Omega^\dagger$$
$$X_\mu \to \Omega X_\mu \Omega^\dagger$$

SU(3)の場合

master Y-M theory $\underline{SU(3)} \times \underline{[SU(3)/H]}$

A_μ: *gauge* n: *color direction*

$$n = \cos\theta\, \Omega \frac{1}{2}\lambda_3 \Omega^\dagger + \sin\theta\, \Omega \frac{1}{2}\lambda_8 \Omega^\dagger = \begin{pmatrix} \Lambda_1 & & \\ & \Lambda_2 & \\ & & \Lambda_3 \end{pmatrix}$$

Cartan

$$\Lambda_1 \neq \Lambda_2 \neq \Lambda_3 \Rightarrow H = U(1) \times U(1)$$

$$\boxed{\Lambda_1 = \Lambda_2 \; etc. \quad \Rightarrow H = U(2)}$$

⇒ $W[A] \coloneqq tr\left[\mathcal{P}\left\{\exp\left(ig \oint A\right)\right\}\right] = W[k] + W[j]$

cf. non-Abelian Stokes theorem

格子上での分解

$$U_\mu(x) = X_\mu(x)\underline{V_\mu(x)}$$

Abelian part

$$D_\mu^{[V]} n(x) := \frac{1}{a}\{V_\mu(x) n(x+\mu) - n(x) V_\mu(x)\} = 0$$

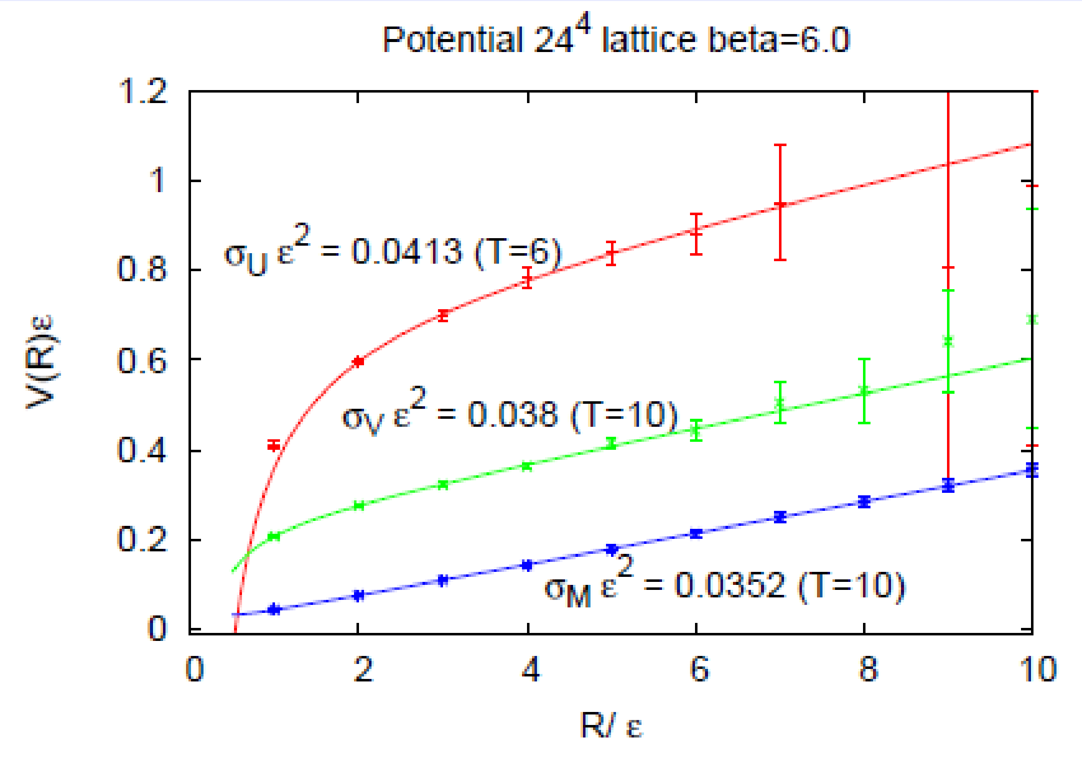
$\Longrightarrow$ $V_\mu(x)$ $\Longrightarrow$ $X_\mu(x) = U_\mu(x) {V_\mu}^\dagger(x)$

$$n(x) = \Omega \frac{1}{2} \lambda_8 \Omega^\dagger \in SU(3)/U(2)$$

$$\sum_{x,\mu} tr\left\{\left({D_\mu}^{[U]} n(x)\right)^\dagger \left({D_\mu}^{[U]} n(x)\right)\right\}$$ を最小化

Quark potential

- 24^4 Lattice $\beta = 6.0$ 500 conf.



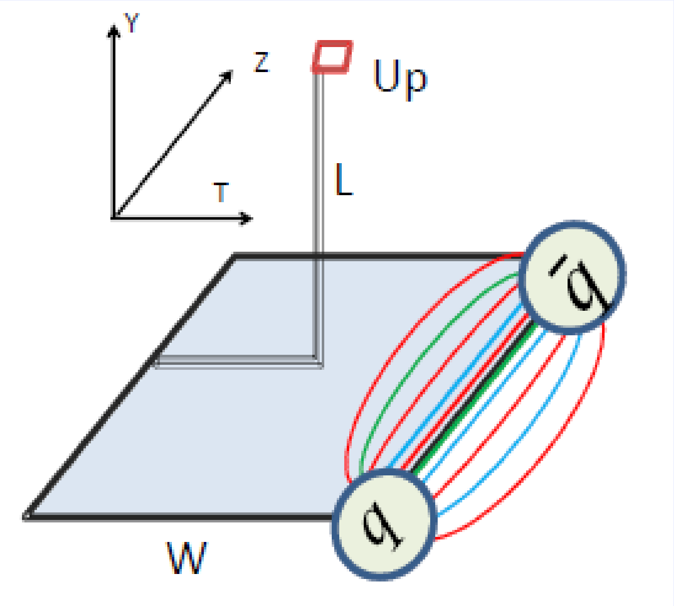
$$V(R) = -\lim_{T\to\infty}\frac{1}{T}\log\langle W_{R\times T}\rangle$$

赤 : $V^{[U]}(R)$　緑 : $V^{[V]}(R)$

青 : W_{mono}　から作った

$$\sigma_V/\sigma_U \sim 0.92\,, \sigma_M/\sigma_U \sim 0.85$$

Field strength

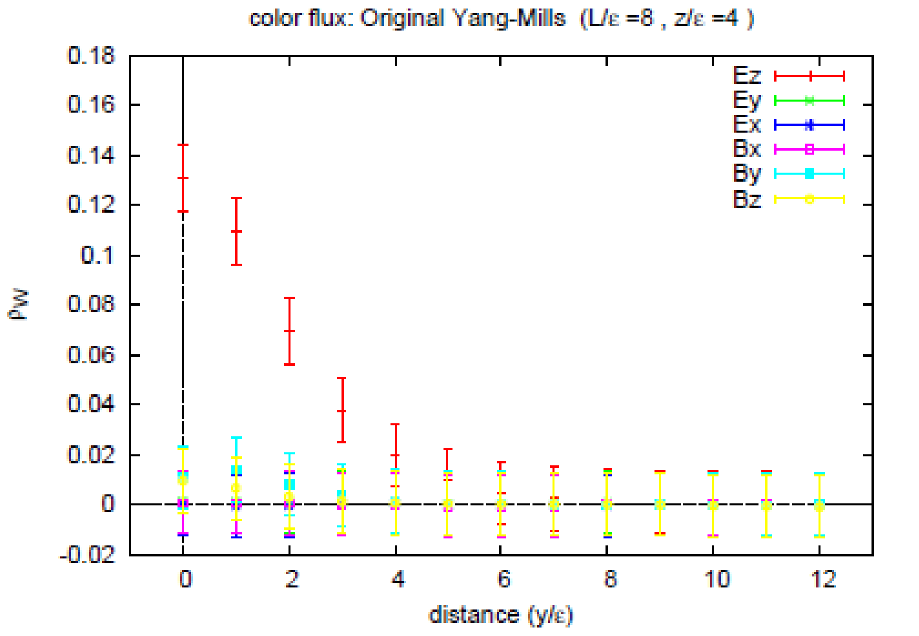


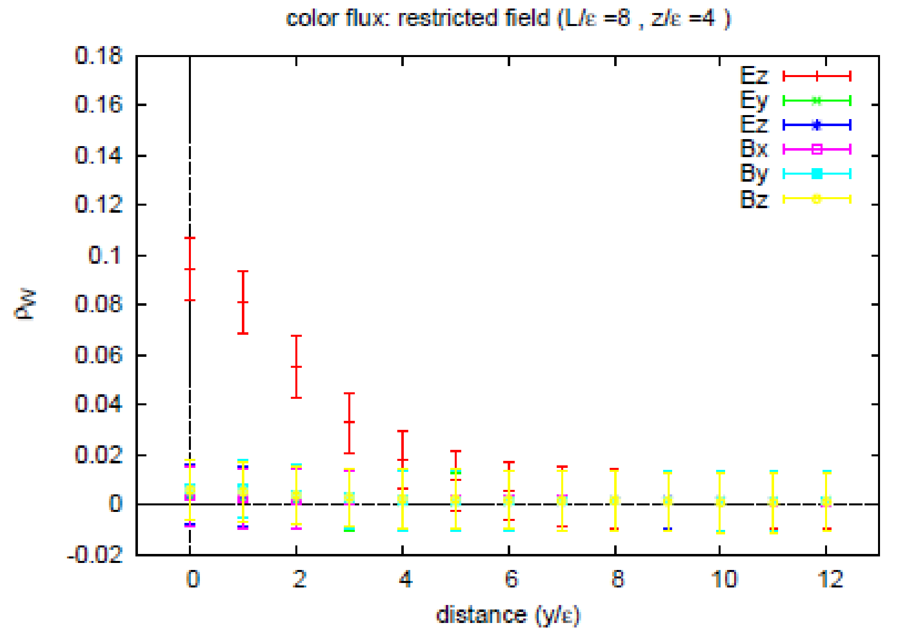
$$\rho_{\mu\nu}(U) := \frac{\langle tr\{U_p(\mu,\nu)L^\dagger W L\}\rangle}{\langle tr\{W\}\rangle} - \frac{1}{3}\frac{\langle tr\{U_p\} tr\{W\}\rangle}{\langle tr\{W\}\rangle}$$

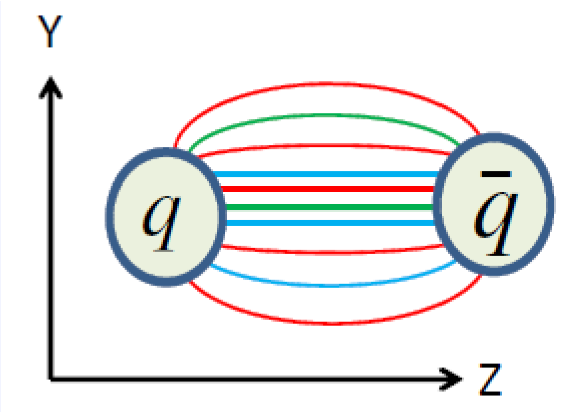
$$= -\frac{\langle tr\{iga^2 F_{\mu\nu}(U)L^\dagger W L\}\rangle}{\langle tr\{W\}\rangle} + O(a^4)$$

Field strength

- 24^4 Lattice $\beta = 6.2$ 500 conf.

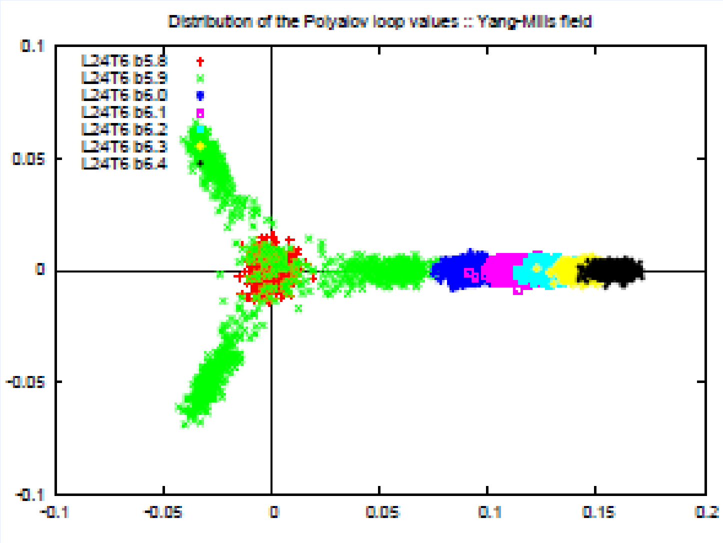




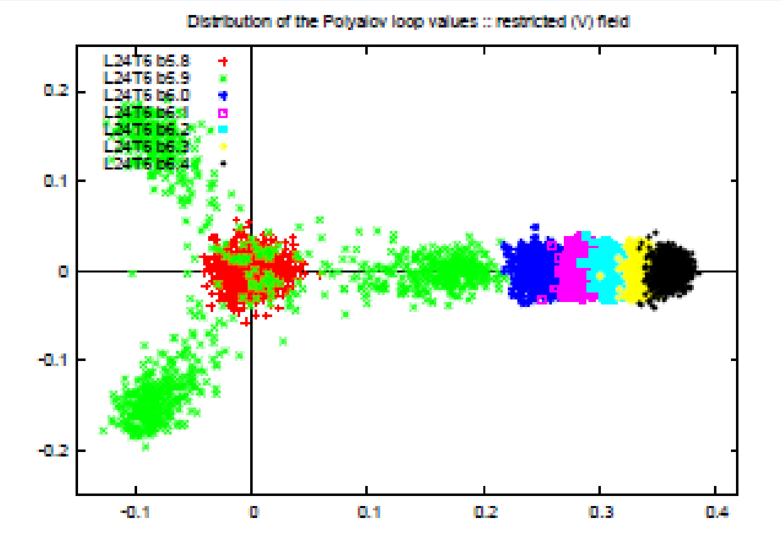


Polyakov loop

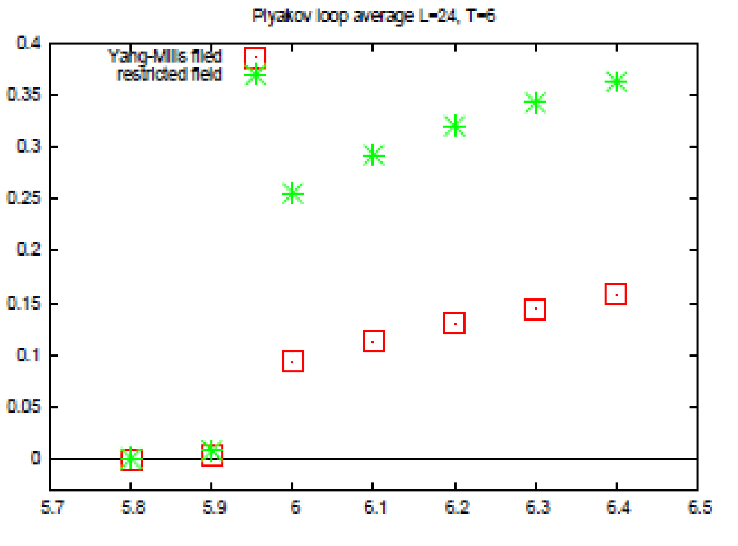
- $24^3 \times 6$ Lattice $\beta = 5.8\sim6.4$ 500 conf.



$Pol(U)$



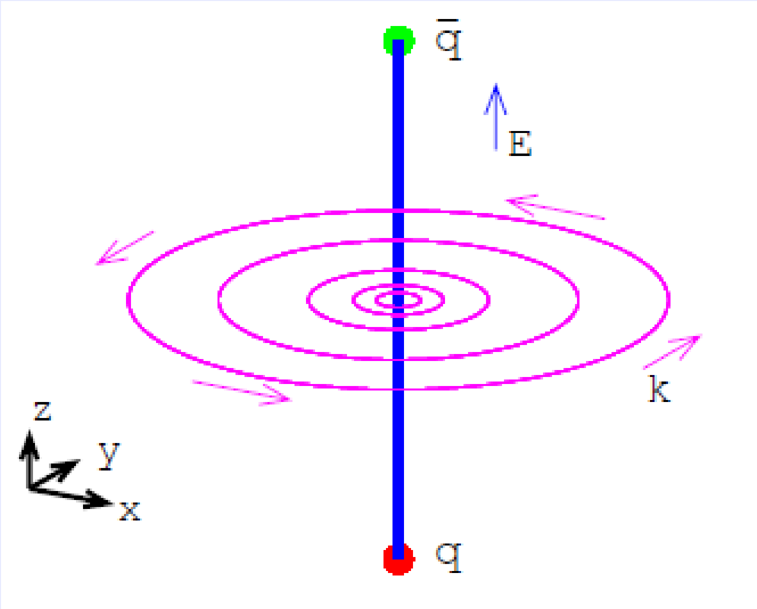
$Pol(V)$

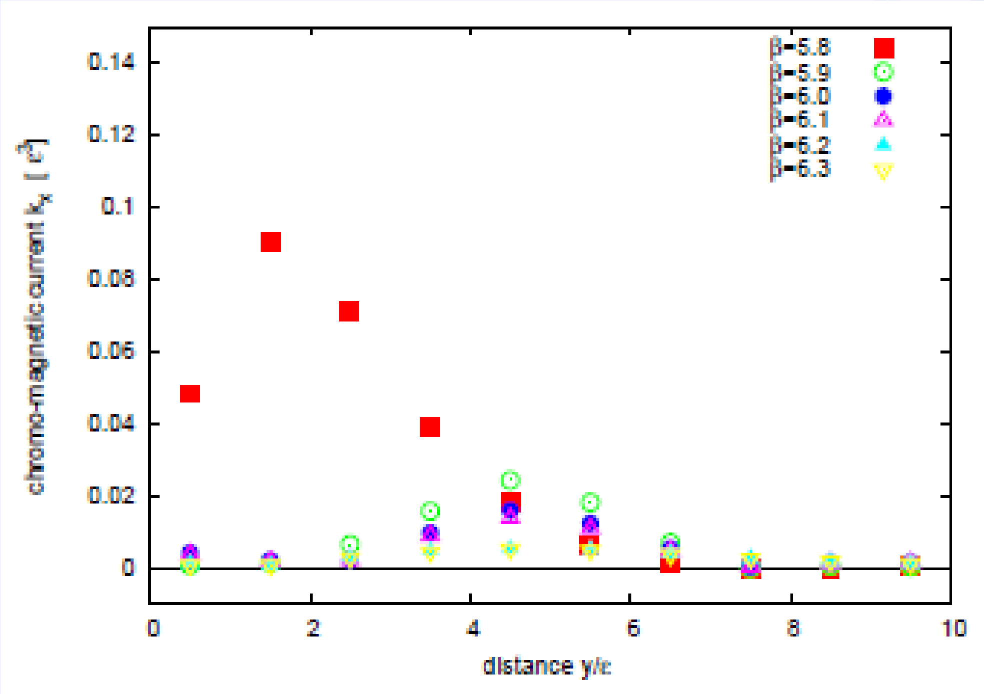


$\langle |Pol(V)| \rangle$

Monopole current

- $24^3 \times 6$ Lattice $\beta = 5.8 \sim 6.4$ 500 conf.





$$k_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial^\nu F^{\rho\sigma}[V]$$

$$|k|$$

まとめ

- CDGFN分解を用いてmonopoleを定義
  - color direction field
  - Master Y-M theory
- string tension
  - V dominance , monopole dominance
- field strength
  - Z方向の電場だけが存在
- finite temperature
  - monopole current の温度依存性