Reanalysis of lattice QCD spectra leading to the $D^*_{s0}(2317)$ and $D_{s1}(2460)$

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Key topics:

- Analysis of lattice data
- Compositeness

References:

- Phys. Rev. Lett 111 (2013) 222001
- Phys. Rev. D 90 (2014) 034510
- Phys. Rev. 137 (1965) B672
- Int. J. Mod. Phys. A28 (2013) 1330045

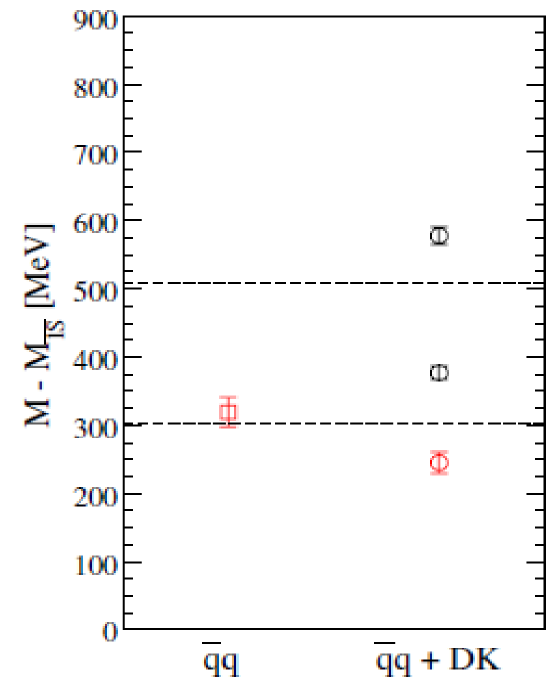
Introduction

- The scalar $D^*_{s0}(2317)$ and axial $D_{s1}(2460)$ mesons were experimentally found slightly below KD and $KD*$ thresholds.
  - Observed in B-decays by BaBar, CLEO Coll. and confirmed by Belle Coll.
  - Focus on $D^*_{s0}(2317)$ =DK and $D_{s1}(2460)$ =D^*K mesons

- Lattice simulation
  - 2+1 flavor with mp=156MeV.
  - KD, KD* and sc interpolators are used.
  - The scattering length and effective range are determined from lowest two energy levels and effective range formula.
  - Bound states were found about 40MeV below the KD and KD* thresholds.

- Reasons for reanalysis
  - Two-energy levels → 130MeV difference → effective range expansion??
  - The theird energy level was not used in analysis

- Using the auxiliary potential method and generalized Weinbereg compositeness condition
  We can determine the amount of KD and KD* component in the respective wave function

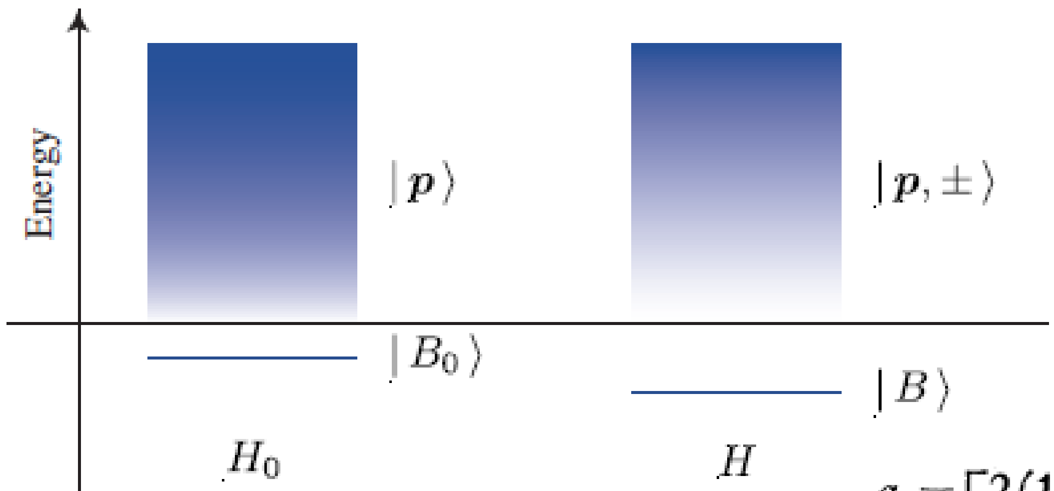


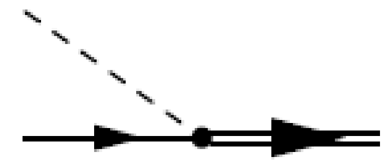
Compositeness of states

- Introduction of the bare state

$H = H_0 + V$

Suppose that H_0 has an elementary particle state





Diagramatic rep of V

Overlap of the physical bound state and bare bound state

$$Z \equiv |\langle B_0 | B \rangle|^2$$

$B \to 0$

$$a_s = [2(1-Z)/(2-Z)]R + O(m_\pi^{-1}),$$
$$r_e = [-Z/(1-Z)]R + O(m_\pi^{-1}),$$
$$R \equiv (2\mu B)^{-1/2}$$

Overlap of the bound state and scattering state (compositeness)

$$X \equiv \int d\boldsymbol{p} |\langle \boldsymbol{p} | B \rangle|^2 \qquad Z = 1 - X$$

This can be expressed by scattering t-matrix as

$$X = 4\pi\sqrt{2\mu^3} \int_0^\infty dE \frac{\sqrt{E}}{E+B} \left[\mathrm{Re}\, t(E) - v - 4\pi\sqrt{2\mu^3} \mathcal{P} \int_0^\infty dE' \frac{\sqrt{E'}|t(E')|^2}{E-E'} \right]$$

Compositeness of states

- Start with Bethe-Salpeter equation (on-shell factorization)

$$T = V + VGT \quad \rightarrow \quad T = \frac{1}{V^{-1} - G}$$

- Around the pole position ($s = s_0$), in the case of energy independent potential

$$g_i g_j = \lim (s - s_0) T_{ij} \qquad \sum_i \left(-g_i^2 \frac{\partial G_i}{\partial s} \right) = 1$$

The generalized Weinberg compositeness condition

- Probability to have channel *i* in the wave function of the bound state is given by

$$P_i = -g_i^2 \frac{\partial G_i}{\partial s}$$

Sum of P_i saturate the wave function

- For the case of energy dependent potential, we have

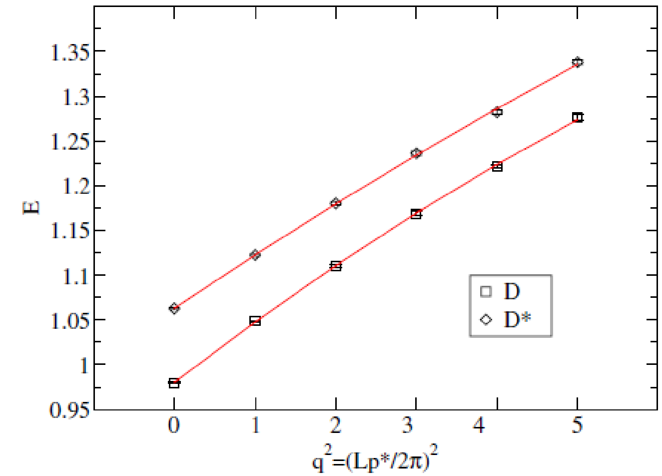
$$-\sum_i g_i^2 \frac{\partial G}{\partial s} - \sum_{ij} g_i g_j G_i \frac{\partial V_{ij}}{\partial s} G_j = 1$$

Analysis of the lattice spectra

- 2+1flavor lattice, close to physical pion mass mπ=156MeV
- Lattice spacing a=0.0907fm and box size L=2.9fm.
- Charm quark is treated by Fermilab method.

Dispersion E(p) for D and D* mesons

W4 and the deviation M4 from M2 capture a lattice artifact



- Neglect the term with coefficient W4 and fit M1, M2 and M4

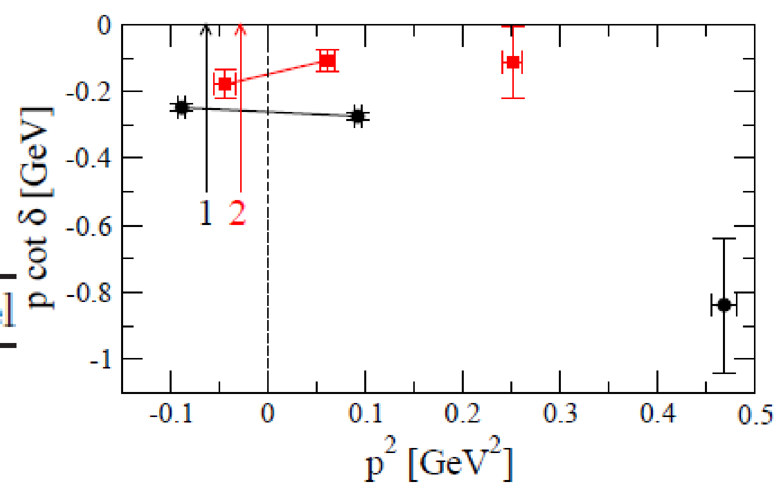
$$E(\boldsymbol{p}) = M_1 + \frac{\boldsymbol{p}^2}{2M_2} - \frac{a^3 W_4}{6}\sum_i p_i^4 - \frac{(\boldsymbol{p}^2)^2}{8M_4^3} + \cdots$$

$$m_{D(D^*)} = M_1$$

	D meson	D^* meson
M_1 (MeV)	1639	1788
M_2 (MeV)	1801	1969
M_4 (MeV)	1936	2132

- $$p\cot\delta(p) = \frac{2Z_{00}}{L\sqrt{\pi}} \approx \frac{1}{a_0} + \frac{1}{2}r_0 p^2 + \mathcal{O}(p^4)$$

	KD channel	KD^* channel
E_1 (MeV)	2086 (34)	2232 (33)
E_2 (MeV)	2218 (33)	2349 (34)
E_3 (MeV)	2419 (36)	2528 (53)



Analysis of the lattice spectra

- Effective range formula

The effective range approximation reads

$$p\,\mathrm{ctg}\delta = \frac{1}{a_0} + \frac{1}{2} r_0 p^2, \quad T = -\frac{8\pi E}{p\,\mathrm{ctg}\delta - ip}.$$

Below threshold $p = i\tilde{p}$, ⇒ $\frac{1}{2} r_0 \tilde{p}^2 - \tilde{p} - \frac{1}{a_0} = 0.$ $B = -\frac{\tilde{p}^2}{2\mu}$, $\mu = \frac{m_K m_{D/D^*}}{m_K + m_{D/D^*}}$

Channel	a_0 [fm]	r_0 [fm]	B [MeV]	$\lvert g\rvert$ [GeV]	$-g^2\partial G/\partial s$
KD	$-1.33(20)$	$0.27(17)$	$38(9)$	$12.6(1.5)$	$1.14(0.15)$
KD^*	$-1.11(0.11)$	$0.10(0.10)$	$44(6)$	$12.6(0.7)$	$0.96(0.06)$

At pole, g^2 is $g^2 = \frac{16\pi s\tilde{p}}{\mu(1 - r_0\tilde{p})}$,

- It is interesting to test the sum rule → all compatible with unity

- There are two reasons for reanalysis
  - the first two levels are separated by 132MeV (Is the effective range formula applicable?)
  - Furthermore the information of the third level is not used.

Analysis of the lattice spectra

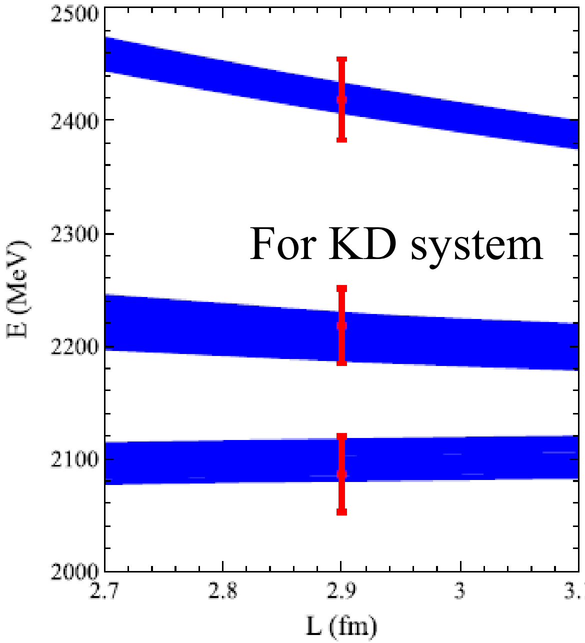
- Auxiliary potential approach

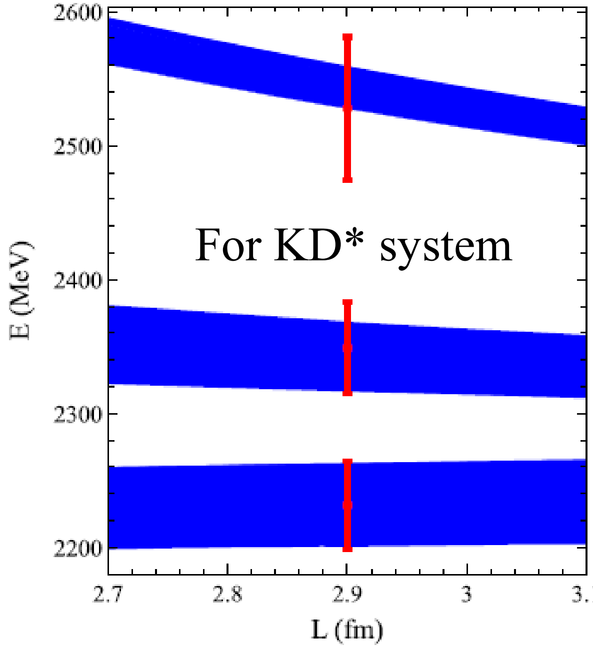
A potential linear in s

$$V = \alpha + \beta(s - s_{th}), \quad s_{th} = (M_{D^{(*)}} + M_K)^2,$$

At the energy of lattice spectra, $V^{-1} = \tilde{G}$, the continuum T is

$$T = \frac{1}{V^{-1} - G} = \frac{1}{\tilde{G} - G} = \frac{1}{\lim\limits_{q_{\max}\to\infty}\left[\frac{1}{L^3}\sum\limits_{q_i}^{q_{\max}} I(\vec{q}_i) - \int\limits_{q<q_{\max}} \frac{d^3q}{(2\pi)^3} I(\vec{q})\right]}$$





$$I(\vec{q}) = \frac{\omega_1(\vec{q}) + \omega_2(\vec{q})}{2\omega_1(\vec{q})\omega_2(\vec{q})\left[P^2 - (\omega_1(\vec{q}) + \omega_2(\vec{q}))^2 + i\epsilon\right]}$$

$$P(KD) = 72 \pm 12\ \%, \text{ for the } D^*_{s0}(2317),$$
$$P(KD^*) = 63 \pm 16\ \%, \text{ for the } D_{s1}(2460).$$

$B(KD) = m_D + m_K - E_B(KD) = 31 \pm 17$ MeV, $B(KD^*) = m_{D^*} + m_K - E_B(KD^*) = 32 \pm 20$ MeV.

Analysis of the lattice spectra

- Fit with CDD pole

A synthetic analysis by potential with CDD pole

$$V = \alpha + \beta(s - s_{\text{th}}) + \frac{\gamma^2}{s - M_{\text{CDD}}^2}$$

Refit the lattice levels

The statistics of the obtained fits shows a clear preference for solutions with a M_{CDD} value that lies far away from the KD, KD^* thresholds, such that it effectively provides a linear dependence in $(s - s_{\text{th}})$.

$$P(KD) = 67 \pm 14\ \%,\ \text{for the } D_{s0}^*(2317) \qquad B(KD) = 29 \pm 15\ \text{MeV}\ ,$$

$$P(KD^*) = 61 \pm 26\ \%,\ \text{for the } D_{s1}(2460) \qquad B(KD^*) = 37 \pm 23\ \text{MeV}\ ,$$

Both results are compatible with the liner potential results

- Two channel calculation

Since we only have three energy levels, we can only use an energy independent potential Which has three paraemeters V11, V12, V22.

→ We do not find any suitable fit to the data.

The energy levels do not contain information on the ηDs and ηDs*.

Scattering length and effective range

- We also obtain the scattering length and the effective range in each of the cases

$$p\,\mathrm{ctg}\delta = \mathrm{Re}\left\{-\frac{8\pi E}{T}\right\} \simeq \frac{1}{a_0} + \frac{1}{2} r_0 p^2 \qquad E = \sqrt{m_K^2 + p^2} + E_{D(D^*)}(p)\ ,$$

For single channel linear potential

$$a_0 = -1.2 \pm 0.6\ \mathrm{fm}, \quad r_0 = 0.04 \pm 0.16\ \mathrm{fm\ for}\ KD,$$
$$a_0 = -0.9 \pm 0.3\ \mathrm{fm}, \quad r_0 = -0.3 \pm 0.4\ \mathrm{fm\ for}\ KD^*$$

For CDD pole potential

$$a_0 = -1.4 \pm 0.4\ \mathrm{fm}, \quad r_0 = -0.2 \pm 0.4\ \mathrm{fm\ for}\ KD,$$
$$a_0 = -1.3 \pm 0.6\ \mathrm{fm}, \quad r_0 = -0.1 \pm 0.2\ \mathrm{fm\ for}\ KD^*$$

The obtained values with the different methods are similar.

The values also agree with those from PRL paper but should be considered more accurate.

Conclusions

- Reanalysis of the lattice data with the third level is performed with an auxiliary potential
- We found a bound state for KD and KD* scattering associated to D*s0(2317) and Ds1(2460)
- Two methods are considered,
  - 1.single channel energy dep.
  - 2.CDD pole potential.
  - The results with both methods were compatible within errors.
- The existence of the bound state with a binding of the order of 40MeV.
- The states are mostly of meson-meson nature of the order of 70%.