Magnetic moments of light nuclei from lattice quantum chromodynamics

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Phys. Rev. Lett. 113, 252001 (2014)

(2015/3/6 Nemura)

Abstract

We present the results of lattice QCD calculations of the magnetic moments of the lightest nuclei, the deuteron, the triton, and ^{3}He, along with those of the neutron and proton. These calculations, performed at quark masses corresponding to $m_\pi \sim 800$ MeV, reveal that the structure of these nuclei at unphysically heavy quark masses closely resembles that at the physical quark masses. In particular, we find that the magnetic moment of ^{3}He differs only slightly from that of a free neutron, as is the case in nature, indicating that the shell-model configuration of two spin-paired protons and a valence neutron captures its dominant structure. Similarly a shell-model-like moment is found for the triton, $\mu_{^3\mathrm{H}} \sim \mu_p$. The deuteron magnetic moment is found to be equal to the nucleon isoscalar moment within the uncertainties of the calculations. Furthermore, deviations from the Schmidt limits are also found to be similar to those in nature for these nuclei. These findings suggest that at least some nuclei at these unphysical quark masses are describable by a phenomenological nuclear shell model.

The electromagnetic interactions of nuclei

- Structure and dynamics
  - The light nuclei behave like a collection of “weakly” interacting nucleons
- Nuclear shell model
  - Remarkable success
  - Nuclei are fundamentally bound states of quarks and gluons, the degrees of quantum chromodynamics (QCD)
- Nuclei are not simply collections of quarks and gluons,
  - defined by their global quantum numbers,
  - but have the structure of interacting protons and neutrons,
  - remains to be understood at a deep level

Lattice QCD calculations

- Our lattice QCD calculations were performed on one ensemble of gauge-field configurations generated with a $N_f = 3$ clover-improved fermion action [1] and a Lüscher-Weisz gauge action [2]. The configurations have $L = 32$ lattice sites in each spatial direction, $T = 48$ sites in the temporal direction, and a lattice spacing of $a \sim 0.12$ fm. All three light-quark masses were set equal to that of the physical strange quark, producing a pion of mass $m_\pi \sim 806$ MeV. A background electromagnetic $[U_Q(1)]$

- A background electromagnetic gauge field U
  - Uniform magnetic field along the z axis

techniques of Ref. [3]. Calculations were performed on ~ 750 gauge-field configurations, taken at uniform intervals from $\sim 10\,000$ trajectories. On each configuration, quark propagators were generated from 48 uniformly distributed Gaussian-smeared sources for each of four magnetic field strengths (for further details of the production, see Refs. [4,5]).

Background electromagnetic fields

- Continuum: minimal electromagnetic coupling

$$D_\mu = \partial_\mu + i g G_\mu + \boxed{i q A_\mu}$$

  - A_μ Electromagnetic four-potential
  - q Charge $q = Q e$ $\quad Q = \frac{2}{3}\ (\text{for up quark})$ $\quad Q = -\frac{1}{3}\ (\text{for down quark})$

- Lattice: $U_\mu(x) = \exp\{i a q A_\mu(x)\}$
  - Multiplying the gauge links by a phase factor
  - A uniform magnetic field B along the z axis

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$B_z = \partial_x A_y - \partial_y A_x$$

$$A_y = B x$$

$$A_x = \begin{cases} 0 & (\text{for } x < L-1) \\ -L B y & (\text{for } x = L-1) \end{cases}$$

Background electromagnetic fields

- Quantization condition
  - from periodic boundary condition:

$$\exp\{-i a^2 q B L^2\} = 1$$

$$a^2 q B L^2 = 2\pi n$$

- n Integer

$$e|B| = \frac{6\pi|n|}{a^2 L^2} \approx 0.046|n| GeV^2$$

$$n = 0, 1, -2, +4$$

The ground state energy of

- Nonrelativistic hadron of mass M and charge Qe in a uniform magnetic field

$$E(\boldsymbol{B}) = M + \frac{|Qe\boldsymbol{B}|}{2M} - \boldsymbol{\mu}\cdot\boldsymbol{B}$$
$$-2\pi\beta_{M0}|\boldsymbol{B}|^2 - 2\pi\beta_{M2}T_{ij}B_iB_j + \ldots$$

  - (1): Hadron’s rest mass
  - (2): Lowest-lying Landau level
  - (3): Interaction of its magnetic moment
  - (4,5): scalar and quadrupole magnetic polarizabilities

Determine μ using lattice QCD

- Two-point correlation function:

$$C_{j_z}^{(B)}(t)$$

$$j_z = \pm j$$
$$B = \hat{\mathbf{z}} \cdot \mathbf{B}$$

- The energy difference:

$$\delta E^{(B)} = E_{+j}^{(B)} - E_{-j}^{(B)}$$

  - Determined from the large time behaveior

$$R(B) = \frac{C_j^{(B)}(t) C_{-j}^{(0)}(t)}{C_{-j}^{(B)}(t) C_j^{(0)}(t)} \xrightarrow{t \to \infty} Z e^{-\delta E^{(B)} t}. \qquad (4)$$

$$p, n, d, \ {}^3He, \ {}^3H, \qquad n = +1, -2, +4$$

$$\delta E^{(B)} = -2\mu |B| + \gamma |B|^3$$

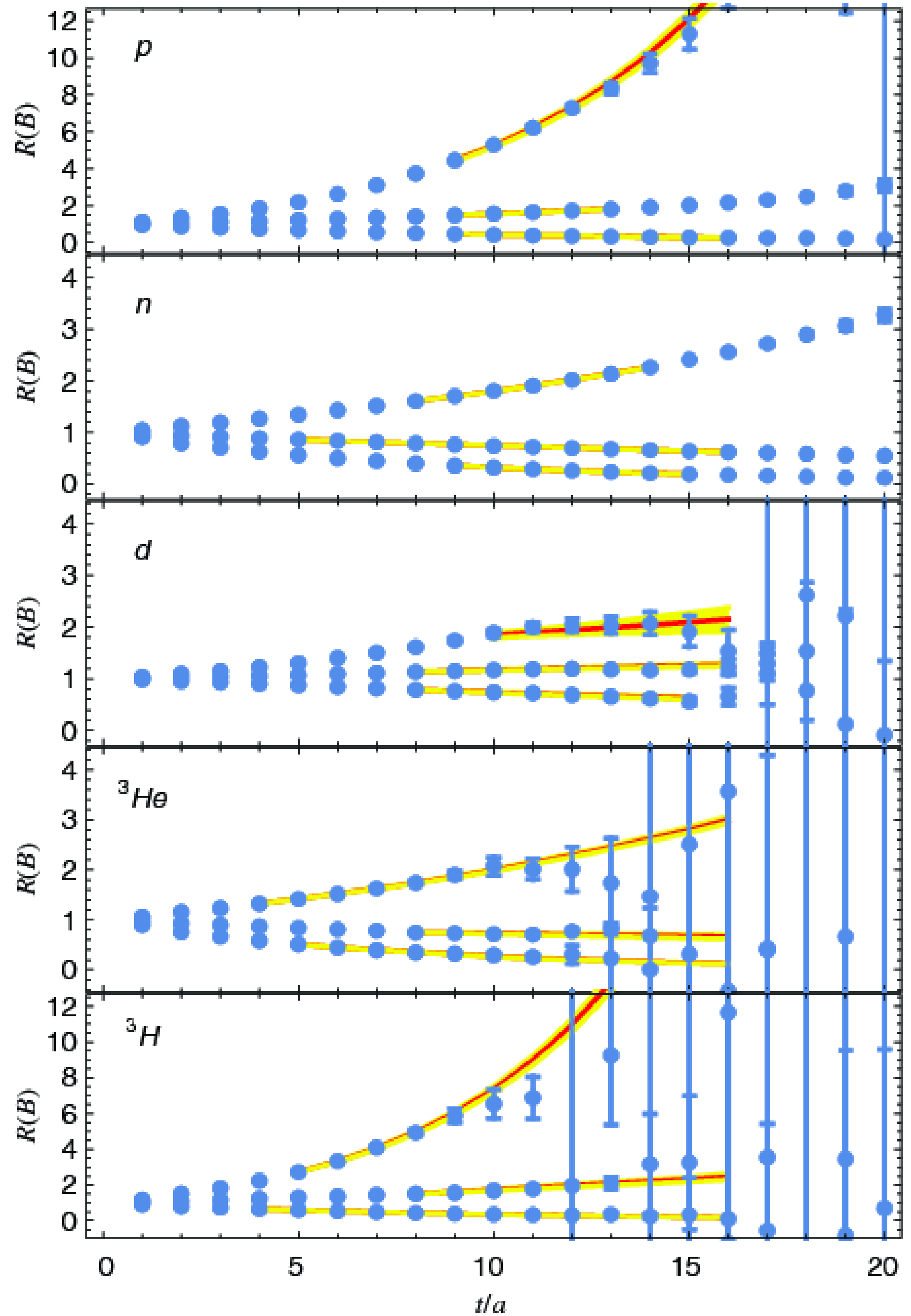

FIG. 1 (color online). The correlator ratios $R(B)$ as a function of time slice for the various states (p, n, d, ^{3}He, and ^{3}H) for $\tilde{n} = +1, -2, +4$. Fits to the ratios are also shown.

Results

- Natural nuclear magneton (nNM) $\frac{e}{2M_N^{latt}}$

$$\mu_p = 3.119(33)(64)\, nNM$$
$$\mu_n = -1.981(05)(18)\, nNM$$

$$\mu_p^{expt} = 2.792847356(23)\, NM$$
$$\mu_n^{expt} = -1.9130427(05)\, NM$$

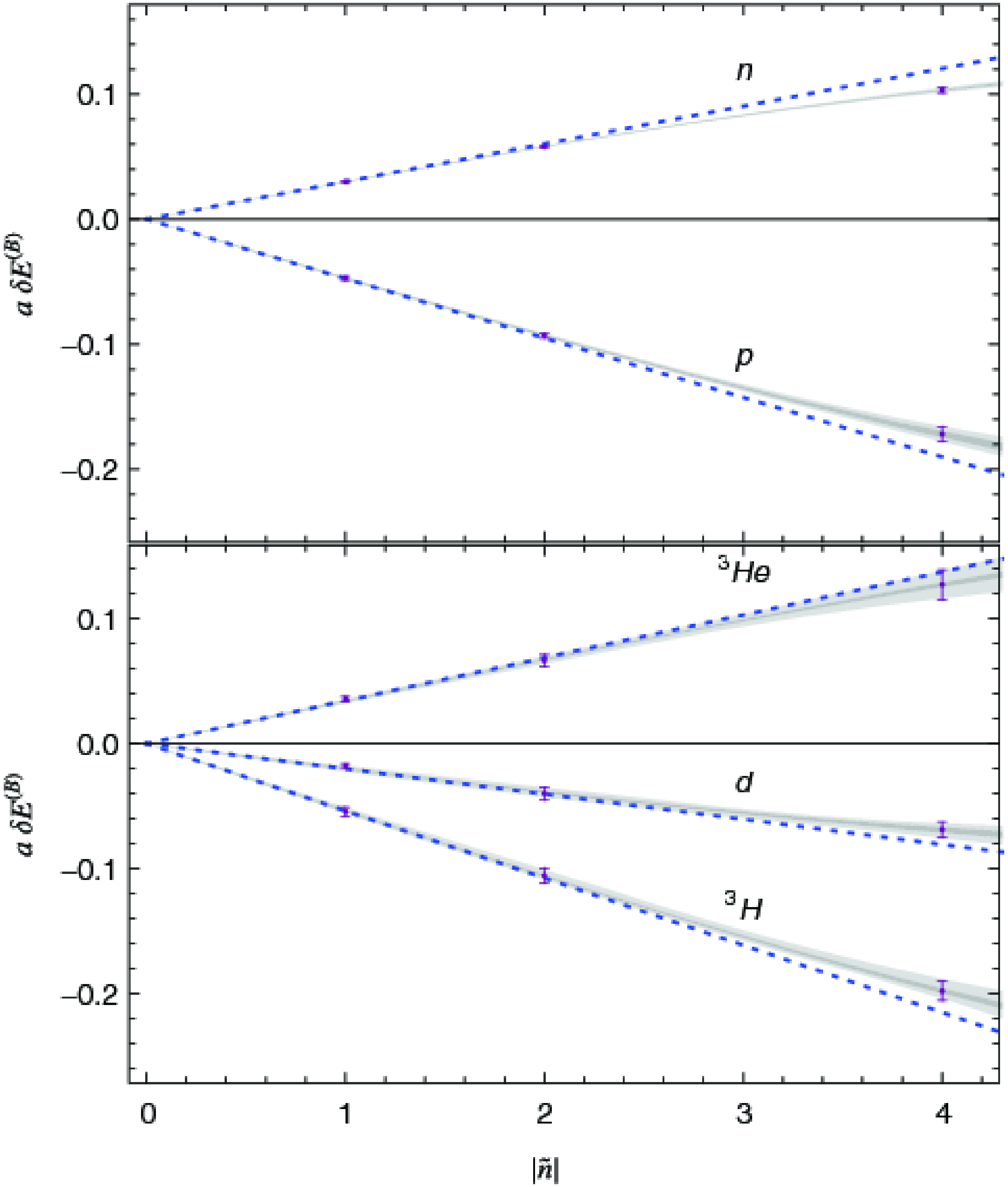

FIG. 2 (color online). The calculated $\delta E^{(B)}$ of the proton and neutron (upper panel) and light nuclei (lower panel) in lattice units as a function of $|\tilde{n}|$. The shaded regions corresponds to fits of the form $\delta E^{(B)} = -2\mu|\mathbf{B}| + \gamma|\mathbf{B}|^3$ and their uncertainties. The dashed lines correspond to the linear contribution alone.

Results

- Natural nuclear magneton (nNM) $\frac{e}{2M_N^{latt}}$

$$\mu_d = 1.218(38)(87)\,nNM$$

$$\mu_{^3He} = -2.29(03)(12)\,nNM$$

$$\mu_{^3H} = 3.56(05)(18)\,nNM$$

$$\mu_p = 3.119(33)(64)\,nNM$$

$$\mu_n = -1.981(05)(18)\,nNM$$

$$\mu_d^{expt} = 0.8574382308(72)\,NM$$

$$\mu_{^3He}^{expt} = -2.127625306(25)\,NM$$

$$\mu_{^3H}^{expt} = 2.978962448(38)\,NM$$

$$\mu_p^{expt} = 2.792847356(23)\,NM$$

$$\mu_n^{expt} = -1.9130427(05)\,NM$$

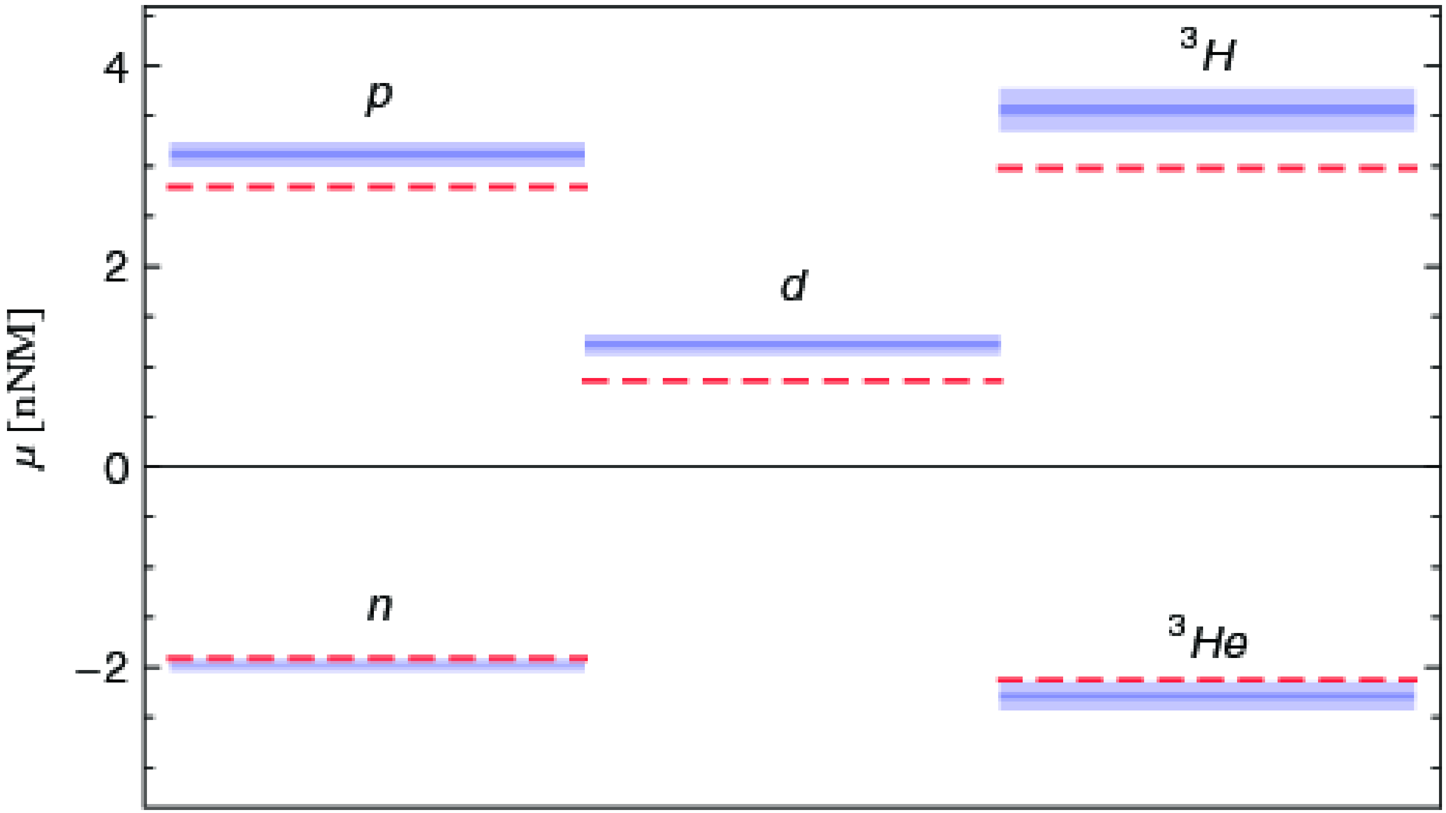

FIG. 3 (color online). The magnetic moments of the proton, neutron, deuteron, ^{3}He, and triton. The results of the lattice QCD calculation at a pion mass of $m_\pi \sim 806$ MeV, in units of natural nuclear magnetons ($e/2M_N^{\rm latt}$), are shown as the solid bands. The inner bands corresponds to the statistical uncertainties, while the outer bands correspond to the statistical and systematic uncertainties combined in quadrature, and include our estimates of the uncertainties from lattice spacing and volume. The red dashed lines show the experimentally measured values at the physical quark masses.

Results

- The naive shell-model predictions

$$\mu_d^{SM} = \mu_p + \mu_n \qquad \mu_{^3He}^{SM} = \mu_n \qquad \mu_{^3H}^{SM} = \mu_p$$

- Deviations from the Schmidt values

$$\delta\mu_{^3He} = \mu_{^3He} - \mu_n = -0.340(24)(93)\, nNM$$

$$\delta\mu_{^3He}^{expt} = -0.215\, NM$$

$$\delta\mu_{^3H} = \mu_{^3H} - \mu_p = +0.45(04)(16)\, nNM$$

$$\delta\mu_{^3H}^{expt} = +0.186\, NM$$

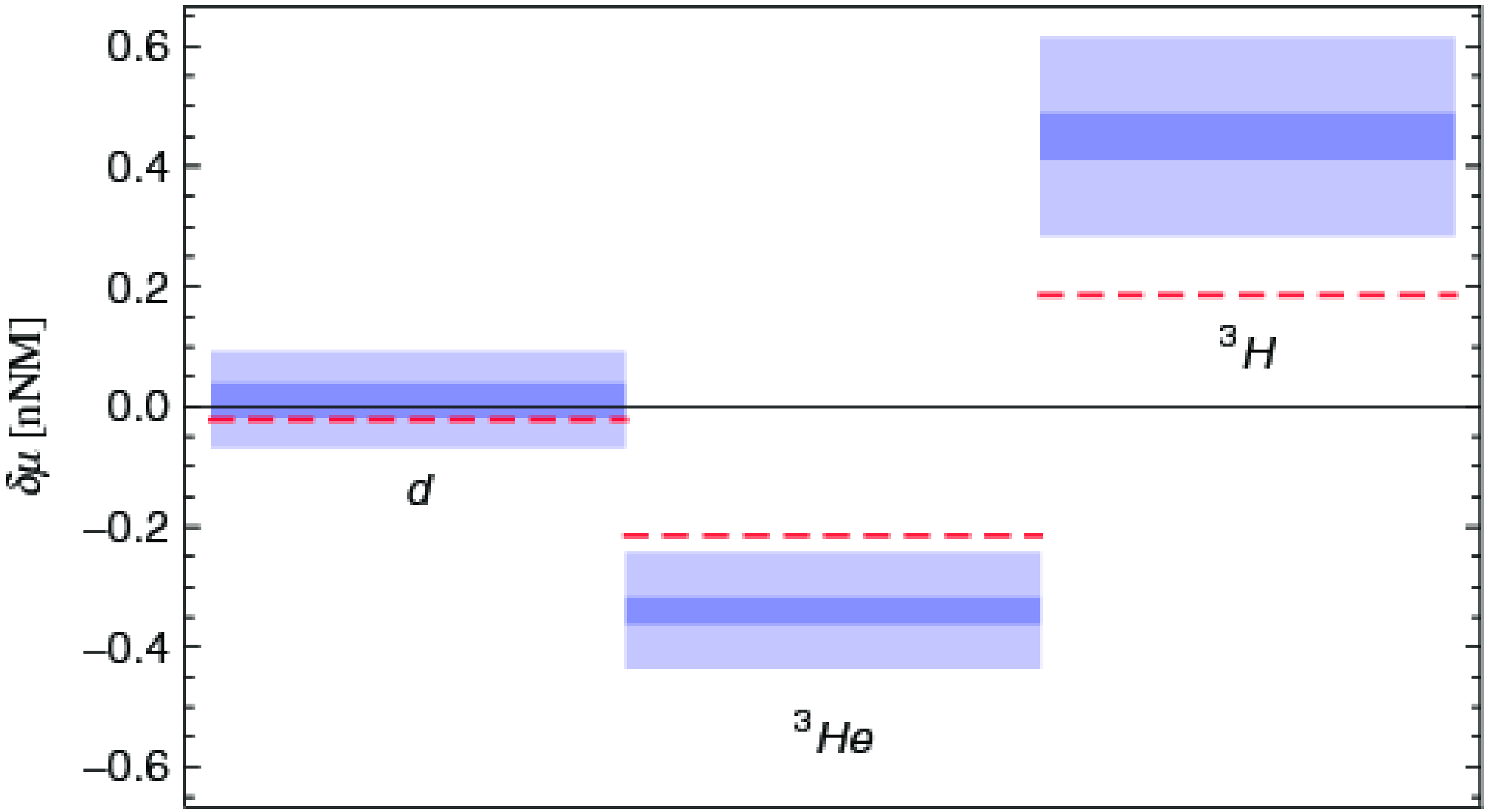

FIG. 4 (color online). The differences between the nuclear magnetic moments and the predictions of the naive shell model. The results of the lattice QCD calculation at a pion mass of $m_\pi \sim 806$ MeV, in units of natural nuclear magnetons ($e/2M_N^{\rm latt}$), are shown as the solid bands. The inner band corresponds to the statistical uncertainties, while the outer bands correspond to the statistical and systematic uncertainties combined in quadrature, including estimates of the uncertainties from lattice spacing and volume. The red dashed lines show the experimentally measured differences.

Summary

- Lattice QCD calculations of the magnetic moments of the lightest nuclei at the flavor SU(3) symmetric point
  - Rescaled by the mass of the nucleon
  - Remarkably close to their experimental values
    - The present calculations performed at a single lattice spacing in one lattice volume
- Future work
  - Continuum extrapolation
    - The systematic uncertainty remains to be quantified
  - Lighter quark mass
  - Larger nuclei