A proposal for B-physics on current lattices

B.Blossier, et al.[ETMC Collaboration], JHEP 1004(2010) 049

P.Dimopoulos et al.[ETMC Collaboration], JHEP01(2012) 046

PoS LATTICE2012 (2012) 104 (1211.0568)

PoS LATTICE2012 (2012) 105 (1211.0565)

N.Carrasco et al.[ETMC Collaboration], arXiv:1308.1851

PoS LATTICE2013(2013) (1311.2837)

journal club by Cho

B-physics on the lattice

- Precise SM predictions need good control of systematic errors on the lattice.
- discretization error am~1 in heavy quark sector $O(ma)$

A. lattice HQET and lattice NRQCD

B. Relativistic heavy quark action
(Fermilab,Tsukuba,RHQ,Oktay-Kronfeld)

C. light quark action

light quark action

- fine lattice ma < 1 - brute force
- improvement to suppress systematic errors

$O(ma)$

A. Clover improvement(Wilson)

B. HISQ(High Improved Staggered Quark)

- "Ratio method" with the improved action

[JHEP 1004(2010) 049]

Ratio method

[JHEP 1004(2010) 049]

- interpolation available data(around charm quark region) and static limit of Heavy quark effective theory

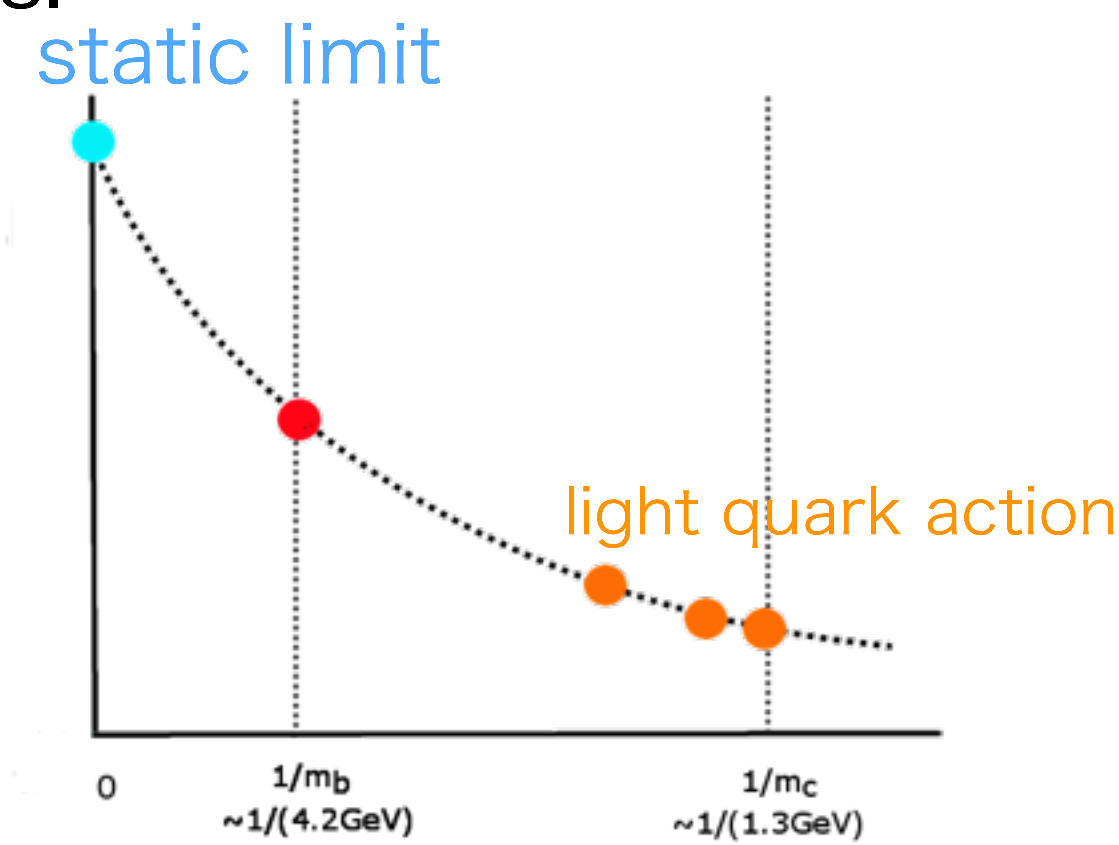
- Ratio for heavy-light observable - mild dependence of quark mass and cutoff effects.

leading order of HQET⇒

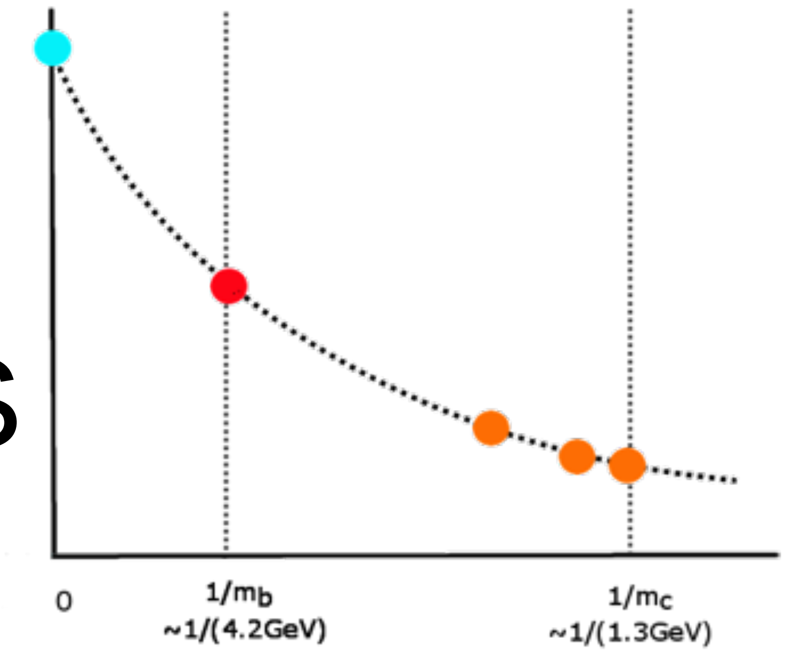
- Twisted mass Wilson fermion

O(a)-improved action

  - b-quark mass
  - decay constant



Ratio method for the b-quark mass



- HQET prediction in static limit

$$\lim_{\mu_h^{pole}\to\infty}\left(\frac{M_{hl}}{\mu_h^{pole}}\right) = const$$

M_{hl} : $heavy - light\ PS\ meson\ mass$
μ_h^{pole} : $heavy\ quark\ pole\ mass$

- Consider heavy quark mass set $\hat{\mu}_h^{(1)} < \hat{\mu}_h^{(2)} < \cdots < \hat{\mu}_h^{(n)}$

^(hat) : renormalized at 2.0GeV in MS bar scheme.

- lattice ratios

$$y^L\left(\hat{\mu}^{(n)};\hat{\mu}_l,a\right) = \frac{M_{hl}^L\left(\hat{\mu}_h^{(n)};\hat{\mu}_l,a\right)}{M_{hl}^L\left(\hat{\mu}_h^{(n-1)};\hat{\mu}_l,a\right)}\cdot\frac{\rho(\hat{\mu}_h^{(n-1)})\hat{\mu}_h^{(n-1)}}{\rho(\hat{\mu}_h^{(n)})\hat{\mu}_h^{(n)}} \underset{\substack{\hat{\mu}_l\to\hat{\mu}_{u/d}\\ a\to 0}}{\rightarrow} \underset{n\to\infty}{\rightarrow} 1$$

$\rho(\hat{\mu}_h)\hat{\mu}_h = \mu_h^{pole}$ $\rho(\hat{\mu}_h)$: transform factor

This factor is known up to NNNL order in continuum perturbation theory.[hep-ph/9912391]

Ratio method for the b-quark mass

- quark mass sequence with fixed ratio

$$\lambda = \frac{\hat{\mu}_h^{(n)}}{\hat{\mu}_h^{(n-1)}} = \frac{x^{(n-1)}}{x^{(n)}} \qquad x^{(n)} = \frac{1}{\hat{\mu}_h^{(n)}}$$

- y ratio with λ

$$y\left(x^{(n)}, \lambda; \hat{\mu}_{u/d}\right) \equiv \lim_{\hat{\mu}_l \to \hat{\mu}_{u/d}} \lim_{a \to 0} y^L\left(x^{(n)}, \lambda; \hat{\mu}_l\right)$$
$$= \lambda^{-1} \frac{M_{hu/d}\left(1/x^{(n)}\right)}{M_{hu/d}\left(1/\lambda x^{(n)}\right)} \cdot \frac{\rho(\lambda x^{(n)})}{\rho(x^{(n)})}$$

- ansatz for y

$$y\left(x, \lambda; \hat{\mu}_{u/d}\right)\Big|_p = 1 + \eta_1 x + \eta_2 x^2 \qquad \lim_{x \to 0} y\left(x^{(n)}, \lambda; \hat{\mu}_{u/d}\right) = 1$$

ρ up to $N^{p-1}L$ order

Simulation details

- Tree-level Symanzik gauge action
- Nf=2 dynamical configurations
- Twisted mass fermion action at maximal twist
- Renormalization constant are determined by RI-MOM scheme

λ=1.278

$$\hat{\mu}_h^{(1)} = 1.230GeV,\ \hat{\mu}_h^{(2)} = \lambda\hat{\mu}_h^{(1)} = 1.572GeV,\ \hat{\mu}_h^{(3)} = \lambda^2\hat{\mu}_h^{(2)} = 2.009GeV,\ \hat{\mu}_h^{(4)} = \lambda^3\hat{\mu}_h^{(3)} = 2.568GeV$$

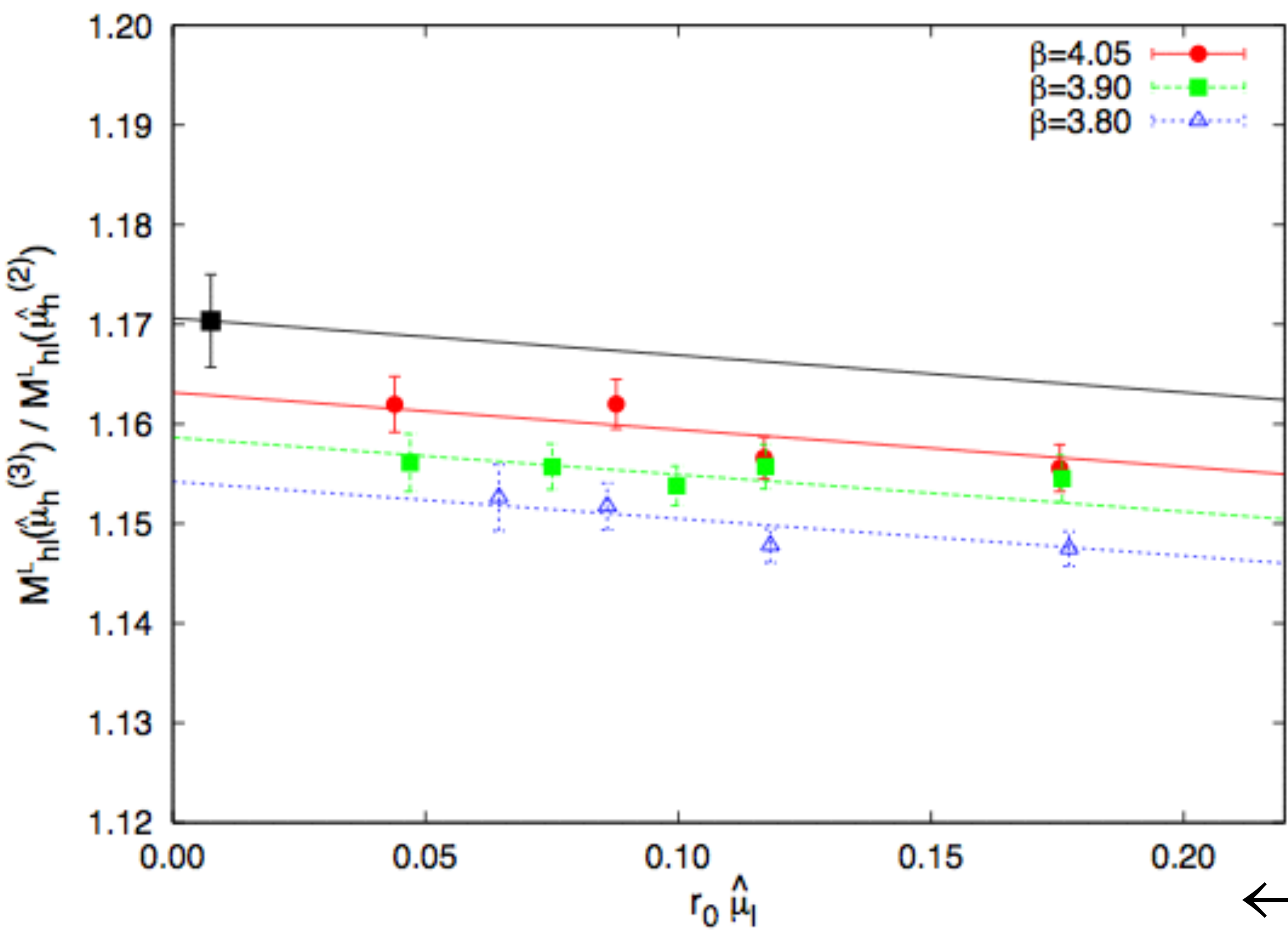
β	$a^{-4}(L^3 \times T)$	$a\mu_\ell = a\mu_{sea}$	$a\mu_s$	$a\mu_h$
3.80	$24^3 \times 48$	0.0060, 0.0080	0.0200, 0.0250	0.2700, 0.3100
		0.0110, 0.0165	0.0300, 0.0360	0.3550, 0.4350
				0.5200
3.90	$24^3 \times 48$	0.0040, 0.0064	0.0220, 0.0270	0.2500, 0.3200
		0.0085, 0.0100	0.0320	0.3900, 0.4600
		0.0150		
3.90	$32^3 \times 64$	0.0030, 0.0040	0.0220, 0.0270	0.2500, 0.3200
4.05	$32^3 \times 64$	0.0030, 0.0060	0.0150, 0.0180	0.2000, 0.2300
		0.0080, 0.0120	0.0220, 0.0260	0.2600, 0.3150

cutoff scale

1/a=1.7-2.6GeV

measured correlator = 240

cutoff-scale and light quark mass dependence



extrapolation with HMChPT

←light quark

Figure 1: *Lattice spacing and $\hat{\mu}_\ell$ dependence of the ratio $M^L_{h\ell}(\hat{\mu}_h^{(3)};\hat{\mu}_\ell,a)/M^L_{h\ell}(\hat{\mu}_h^{(2)};\hat{\mu}_\ell,a)$. The black square with its error is the combined continuum and chirally ($\hat{\mu}_\ell \to \hat{\mu}_{u/d}$) extrapolated value. Here and in all the following figures uncertainties possibly affecting the value of the variable in the horizontal axis are propagated to the quantity plotted on the vertical axis.*

y ratios from current data

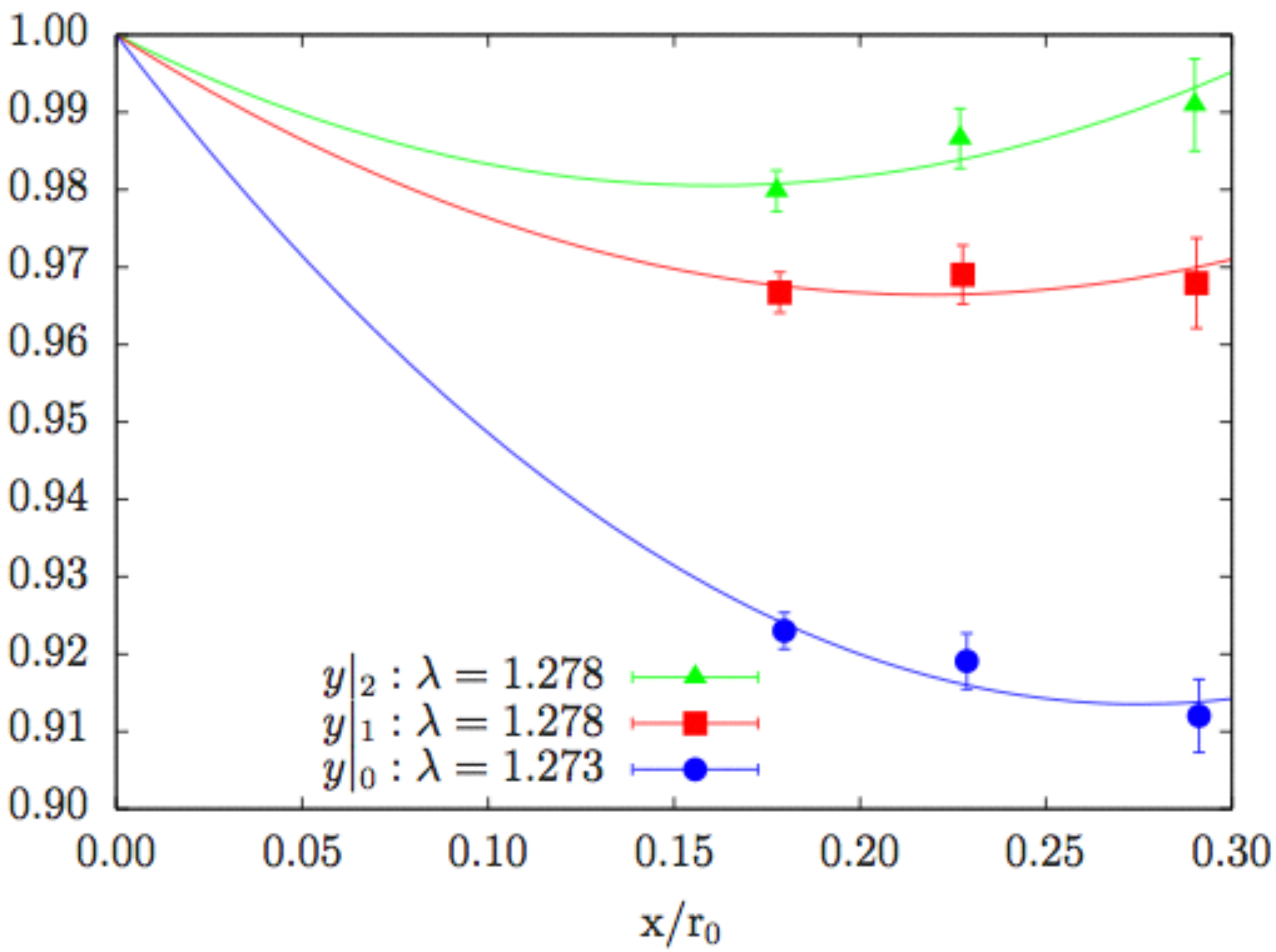


Figure 2: *Continuum data for* $y|_0$ *(blue dots),* $y|_1$ *(red squares),* $y|_2$ *(green triangles). The corresponding best fit curves are drawn with* $\lambda = 1.273$ *(lower curve, in blue) and* $\lambda = 1.278$ *(middle curve, in red and upper curve, in green). In all cases* $\mu_\ell \to \mu_{u/d}$.

$$\frac{\rho(\hat{\mu}_h^{(n-1)})\hat{\mu}_h^{(n-1)}}{\rho(\hat{\mu}_h^{(n)})\hat{\mu}_h^{(n)}}$$

$$y\left(x, \lambda; \hat{\mu}_{u/d}\right)\Big|_p = 1 + \eta_1 x + \eta_2 x^2$$

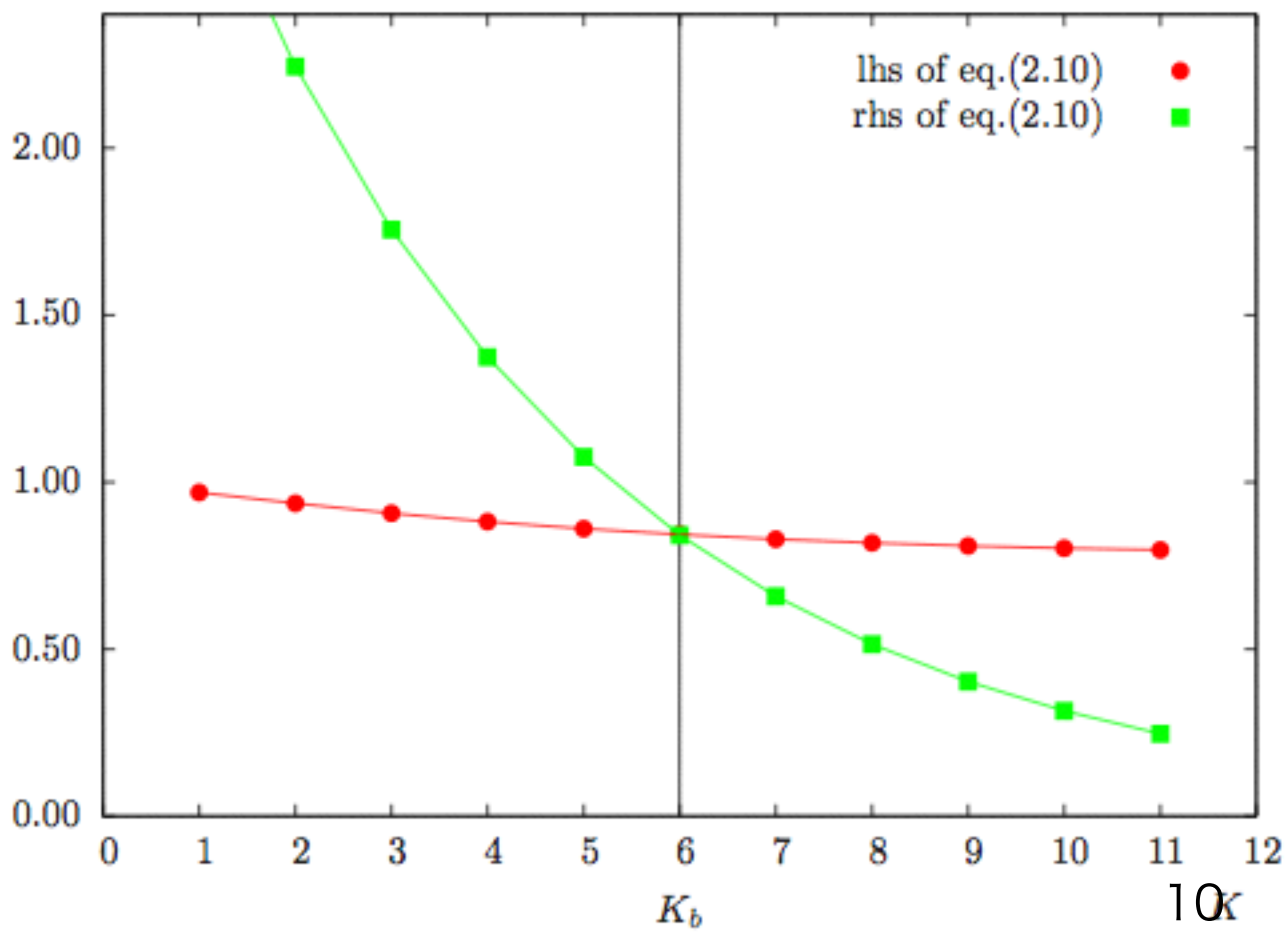
$$\Rightarrow \quad \eta_1, \ \eta_2$$

determination of B-quark mass

- A following chain equation beyond current data

M_B^{exp} $\qquad \hat{\mu}_b = \lambda^{K_b} \hat{\mu}^{(1)}$

$$y_p^{(2)} y_p^{(3)} \cdots y_p^{(K+1)} = \lambda^{-K} \frac{M_{hu/d}\left(\hat{\mu}_h^{(K+1)}\right)}{M_{hu/d}\left(\hat{\mu}_h^{(1)}\right)} \cdot \left[\frac{\rho(\hat{\mu}_h^{(1)})}{\rho(\hat{\mu}_h^{(K+1)})}\right]_p$$



$$\hat{\mu}_b^{(\overline{MS}, N_f=2)} = 4.63(27) GeV$$

$$\hat{\mu}_b^{(\overline{MS}, PDG2014)} = 4.18(3)\ GeV$$

sea s,c quark effect

$$\hat{\mu}_b^{(\overline{MS}, N_f=4)} = 4.29(13) GeV$$

[N.Carrasco et al, arXiv:1311.2837]

Discussion and error budget

$$y_p^{(2)} y_p^{(3)} \cdots y_p^{(K+1)} = \lambda^{-K} \frac{M_{hu/d}\left(\hat{\mu}_h^{(K+1)}\right)}{M_{hu/d}\left(\hat{\mu}_h^{(1)}\right)} \cdot \left[\frac{\rho(\hat{\mu}_h^{(1)})}{\rho(\hat{\mu}_h^{(K+1)})}\right]_p$$

- reliability of truncation of ρ (NL order)

L order ~ 2%

NNL order ~ 1%

- Pseudo-scalar meson mass in charm region

5%(scale setting,renormalization constant,⋯)

decay constant : f_B, f_{Bs}

- Precise determination of PS decay constant f_B, f_{Bs} is significant for “New Physics” sensitive processes e.g. B->$\tau\ \nu$, Bs->$\mu^+\mu^-$.

- ratio method for f_B, f_{Bs}

$$\lim_{\hat{\mu}_h^{pole}\to\infty} f_{hl}\sqrt{\mu_h^{pole}} = \mathrm{constant}$$

- A very similar strategy with determination of b quark mass

ratio method for decay constant

- lattice ratio

$$z\left(x,\lambda;\hat{\mu}_l\right)=\lambda^{1/2}\frac{f_{hl}(1/x)}{f_{hl}(1/\lambda x)}\cdot\frac{C_A^{stat}(\lambda x)}{C_A^{stat}(x)}\frac{[\rho(x)]^{1/2}}{[\rho(\lambda x)]^{1/2}}$$

$$\Phi_{hl}(\mu^*)=[C_A^{stat}(\mu^*,\hat{\mu}_h)]^{-1}\cdot\Phi_{hl}^{QCD}(\hat{\mu}_h)$$ up to 3 loops [hep-ph/0303113]

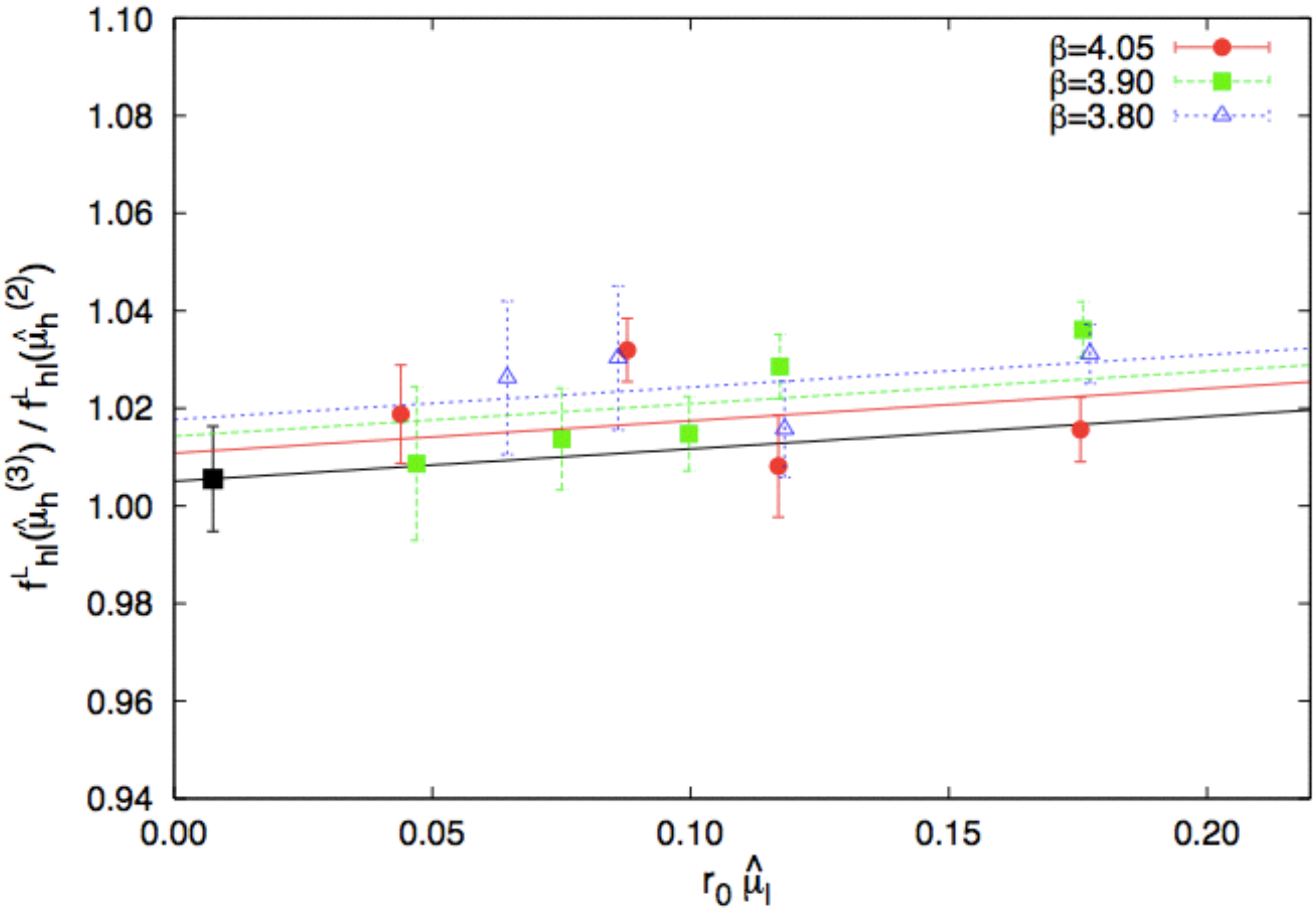
- f_B : light -> u/d, f_{Bs} : light-> s

- asymptotic behavior z $\quad \lim_{x\to 0} z\left(x,\lambda;\hat{\mu}_{u/d}\right)=1$

- ansatz

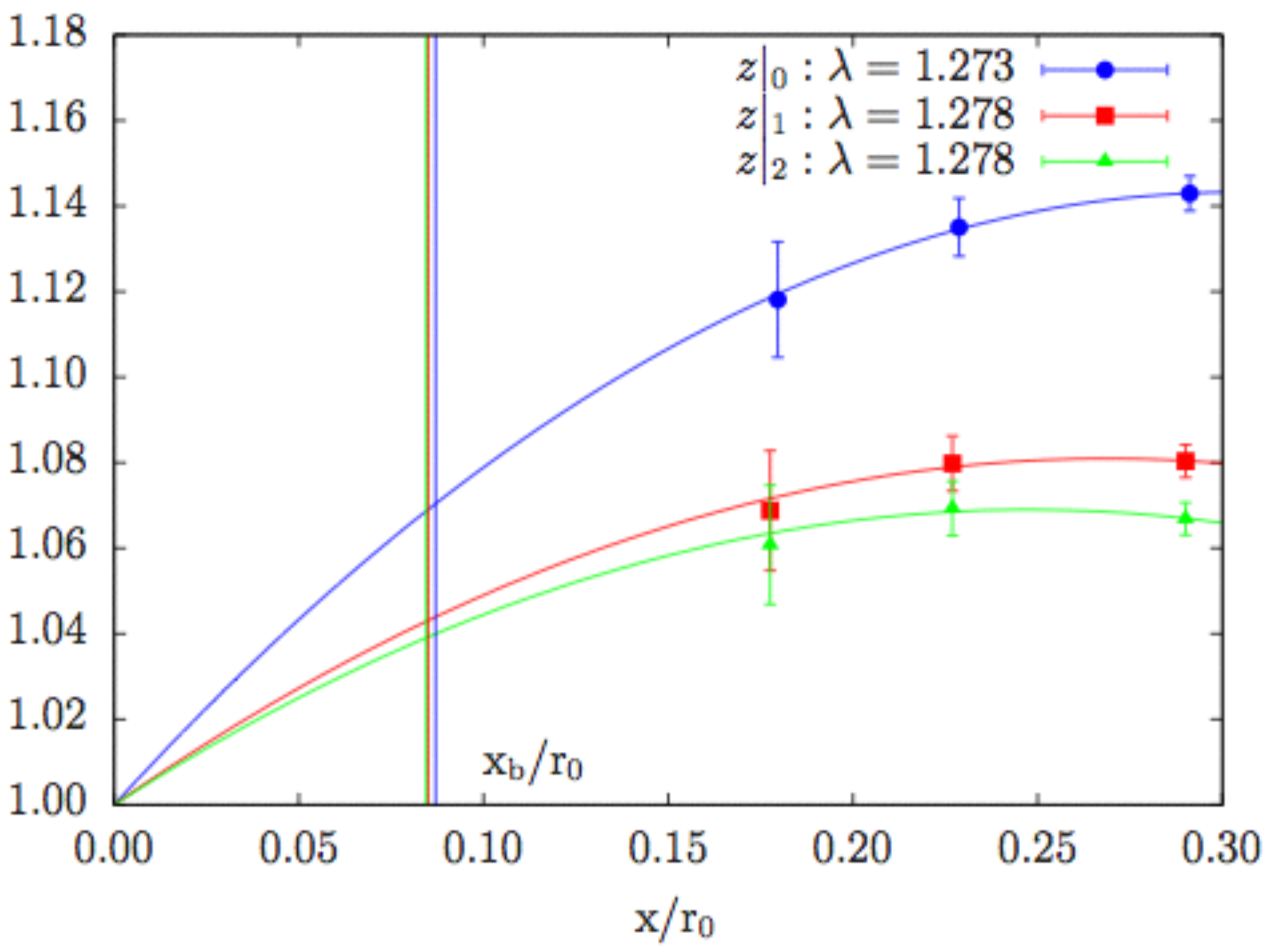
$$z\left(x,\lambda;\hat{\mu}_{u/d}\right)\Big|_p=1+\zeta_1 x+\zeta_2 x^2 \rightarrow \zeta_1,\zeta_2$$

Chiral and continuum extrapolation

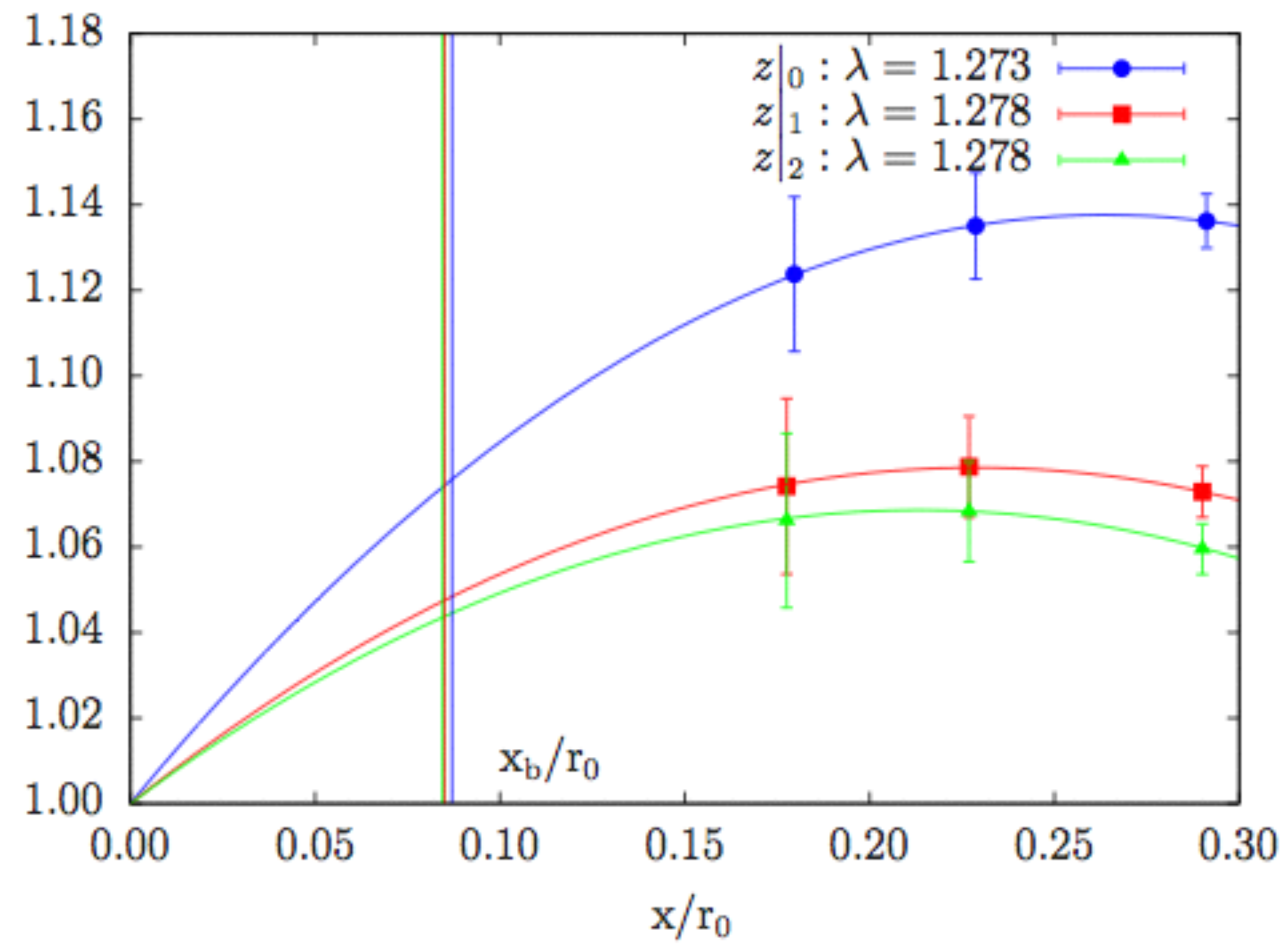


interpolation to b quark region

$$z\left(x,\lambda;\hat{\mu}_{u/d}\right)\Big|_{p} = 1 + \zeta_1 x + \zeta_2 x^2$$



light - u/d



light - s

determination of f_B and f_{Bs}

- the iterative formula

$$z_p^{(2)} z_p^{(3)} \cdots z_p^{(K+1)} = \lambda^{K/2} \frac{f_{hl}\left(\hat{\mu}_h^{(K+1)}\right)}{f_{hl}\left(\hat{\mu}_h^{(1)}\right)} \cdot \left[\frac{C_A^{stat}(\hat{\mu}_h^{(1)})}{C_A^{stat}(\hat{\mu}_h^{(K+1)})} \sqrt{\frac{\rho(\hat{\mu}_h^{(K+1)})}{\rho(\hat{\mu}_h^{(1)})}} \right]_p$$

- final results

$$f_B = f_{hu/d}(\hat{\mu}_b) = 194(16)\mathrm{MeV}$$

$$f_{B_s} = f_{hs}(\hat{\mu}_b) = 235(12)\mathrm{MeV}$$

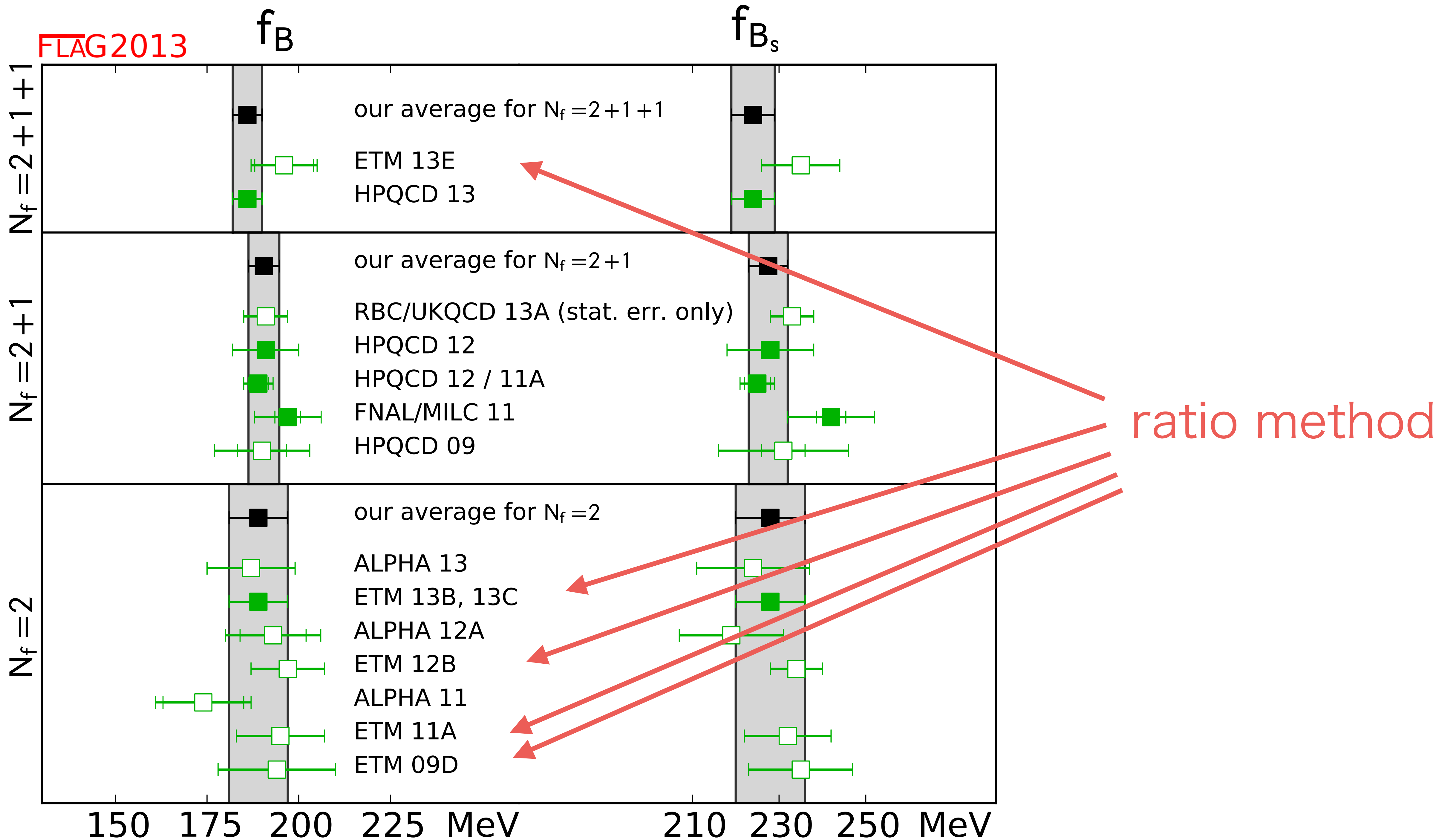
- error budget

L order ~ 1%, NNL order ~ 0.1%

ζ depends logx ~ 1, 2 %

uncertainty of m_b

FLAG: f_B, f_{Bs}



FLAG2013
f_B
f_{B_s}
$N_f=2+1+1$
our average for $N_f=2+1+1$
ETM 13E
HPQCD 13
$N_f=2+1$
our average for $N_f=2+1$
RBC/UKQCD 13A (stat. err. only)
HPQCD 12
HPQCD 12 / 11A
FNAL/MILC 11
HPQCD 09
$N_f=2$
our average for $N_f=2$
ALPHA 13
ETM 13B, 13C
ALPHA 12A
ETM 12B
ALPHA 11
ETM 11A
ETM 09D
150 175 200 225 MeV
210 230 250 MeV
ratio method

Summary

- Ratio method can deal with B-quark on current lattices.
- The procedure seems complicated, but good control of systematic errors.
- key factor - ratio, interpolation with static limit
- Is this method promising for B-physics ?

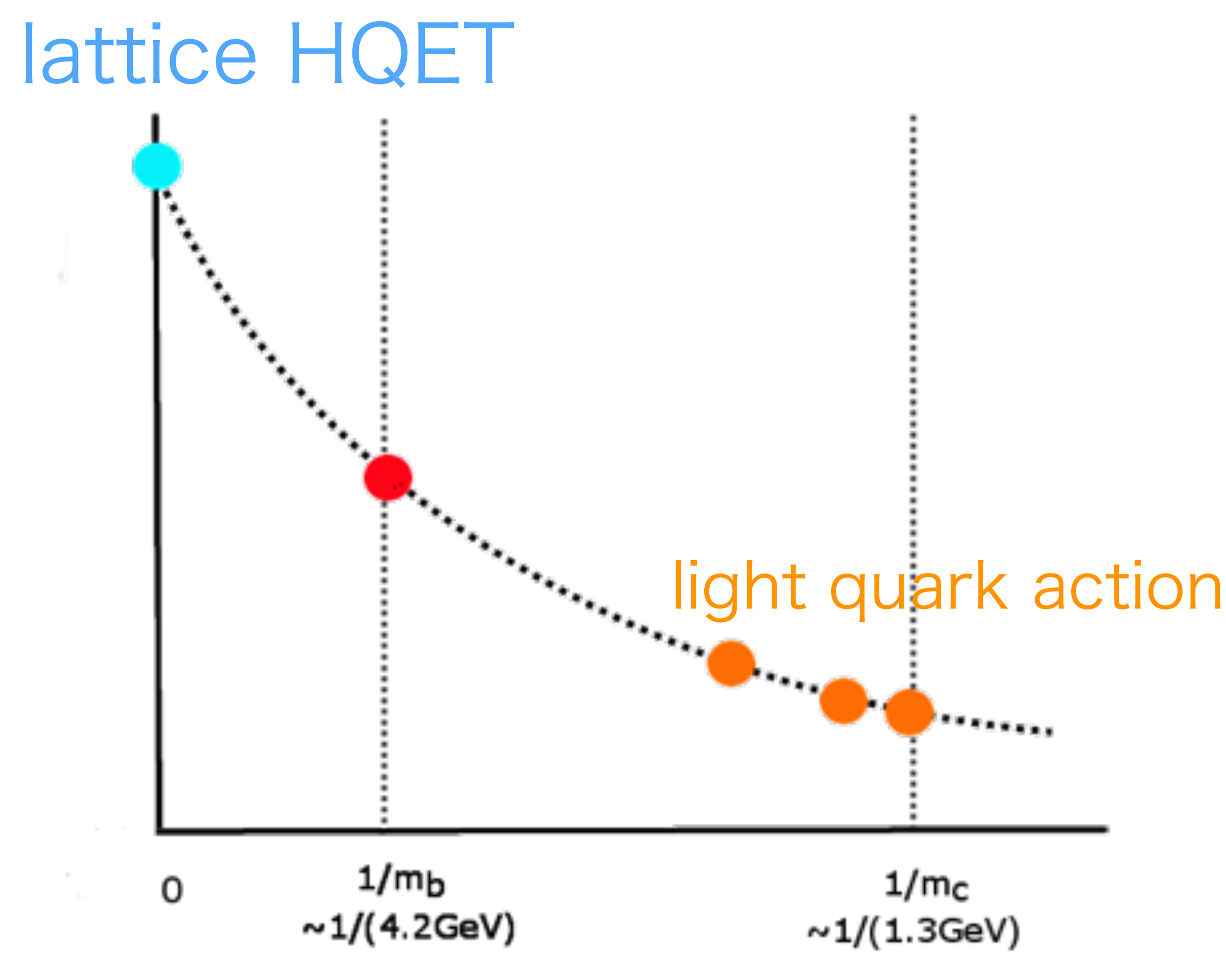
interpolation method

[JHEP01(2012) 046]

- So far results of static limit is used.
- Lattice action of static limit is employed.
- average for

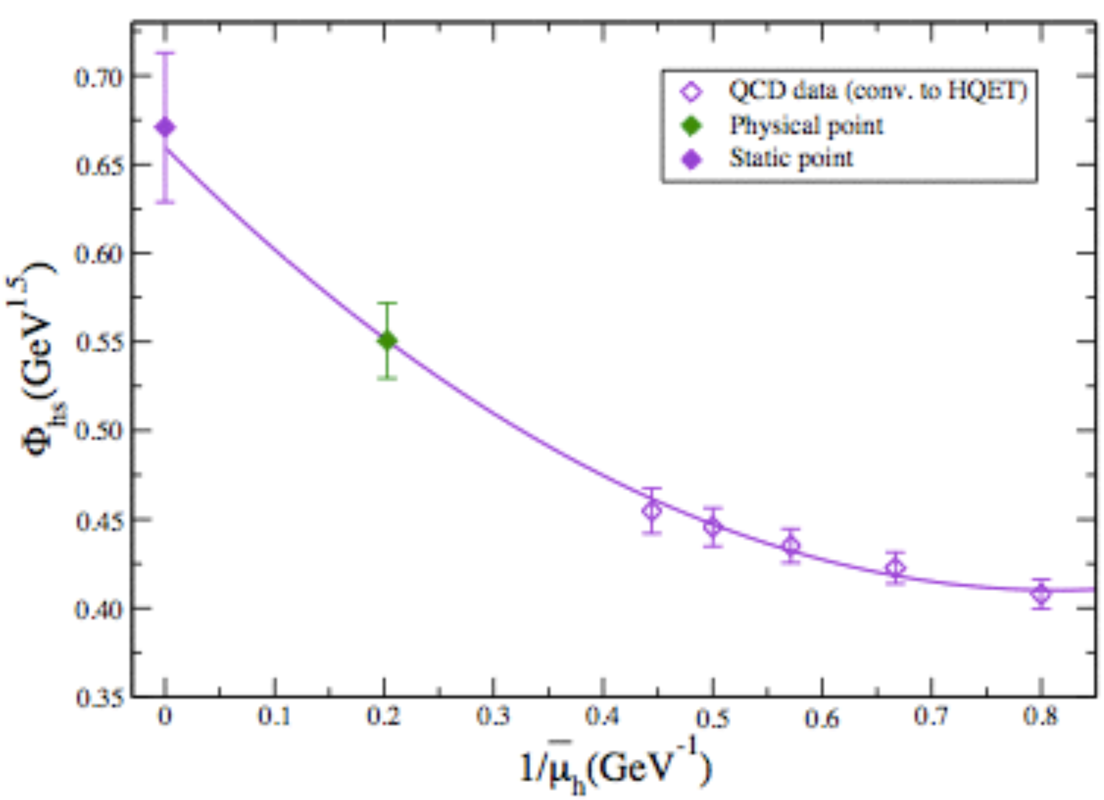
  ratio method

  interpolation method



interpolation between lattice HQET and QCD data

$$S_{stat} = a^4 \sum_x \bar{\psi}_h(x) D_0 \psi_h(x) = a^3 \sum_x \bar{\psi}_h(x)[\psi_h(x) - V^{\dagger}_{HYP}(x - a\hat{0}, 0)\psi(x - a\hat{0})]$$



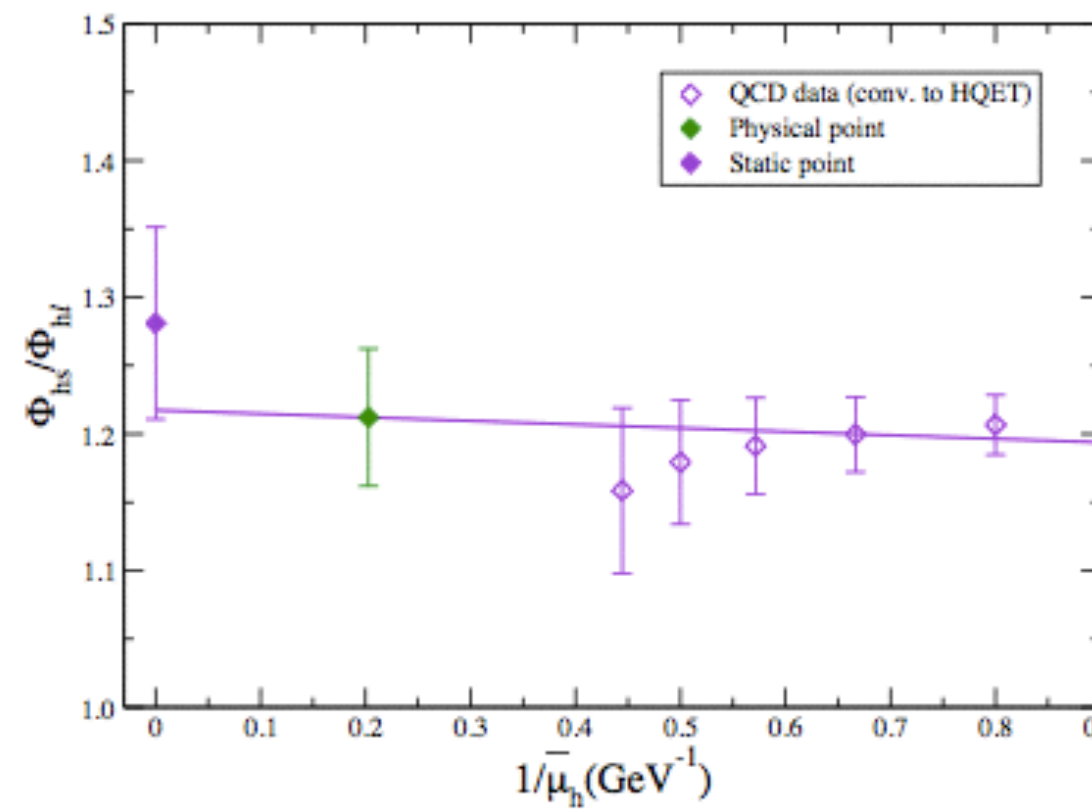


Figure 8: *Dependence of Φ_{hs} (left) and $\Phi_{hs}/\Phi_{h\ell}$ (right), in the chiral and continuum limit, on the inverse of the heavy quark mass.*

final results

f_{Bs}[MeV]				f_{Bs}/f_B			
Ratio Method		Interpol. Method		Ratio Method		Interpol. Method	
Lin.	Quad.	Lin.	Quad.	HMChPT	Polyn.	HMChPT	Polyn.
225(7)(4)	225(7)(4)	237(9)(4)	238(9)(4)	1.22(2)(0)	1.14(2)(0)	1.22(5)(2)	1.16(6)(2)
225(7)(4)		238(9)(4)		1.18(2)(4)		1.19(5)(3)	
232(10)				1.19(5)			

Table 2: *Collection of the results obtained for f_{Bs} and f_{Bs}/f_B from the ratio and interpolation methods. The statistical and systematic uncertainties are summed in quadrature. The third and fourth lines provide info on the results obtained by extrapolating to the physical pion mass point by using different chiral fit ansatz (see text). The final values, given in the last row, are an average of the results of the two methods.*

HeavyMesonChPT

- SU(2) Chiral effective theories for heavy-light
- fixed heavy quark mass
- For pseudo scalar meson mass

$$M_{hl} r_0 = C_0 + C_1 \hat{\mu}_l r_0 + \frac{a^2}{r_0^2} C_L$$

- For decay constant of PS meson

$$f_{hl} r_0 = D_0 + D_1 \hat{\mu}_l r_0 + d_1 \frac{2B_0 \hat{\mu}_l}{(4\pi f_0)^2} + \frac{a^2}{r_0^2} D_L$$

An improved ratio method for decay constant [JHEP03,016]

- Old method depends the measurement of the b-quark mass.

- propose a following asymptotic behavior.

$$\lim_{m \to \infty} \sqrt{M_{hl}} f_{hl} = const$$

- lattice ratio

$$z(x, \lambda; \mu) = \frac{f_{hl}(1/x)}{f_{hl}(1/\lambda x)} \cdot \frac{C_A^{stat}(\lambda x)}{C_A^{stat}(x)} \cdot \frac{M_{hl}(1/x)}{M_{hl}(1/\lambda x)}$$