Wilson Fermion Determinant in Lattice QCD

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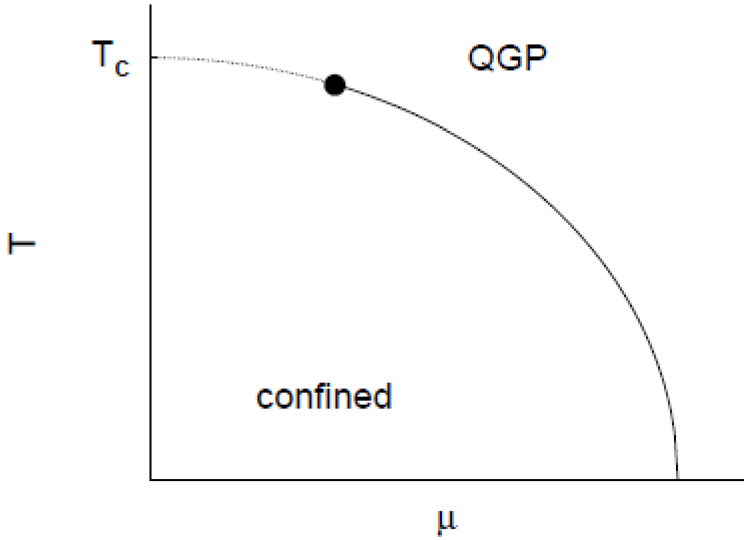
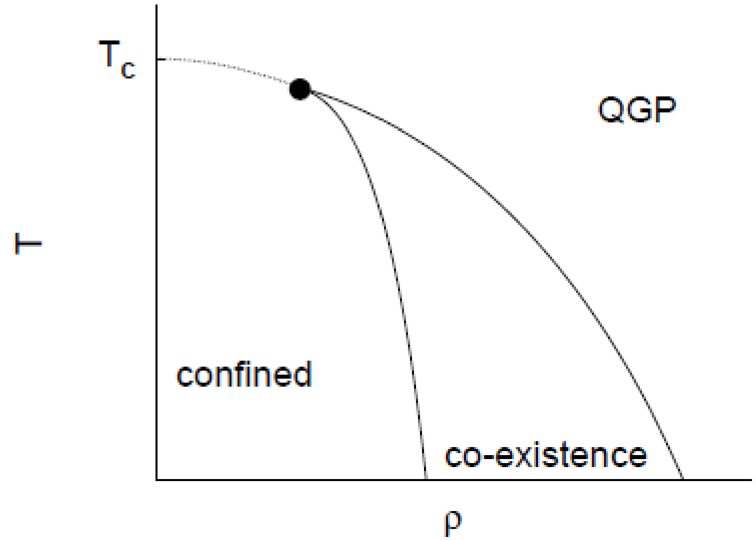
also, arXiv:1204.1412v2 [hep-lat] 7 Jun 2012

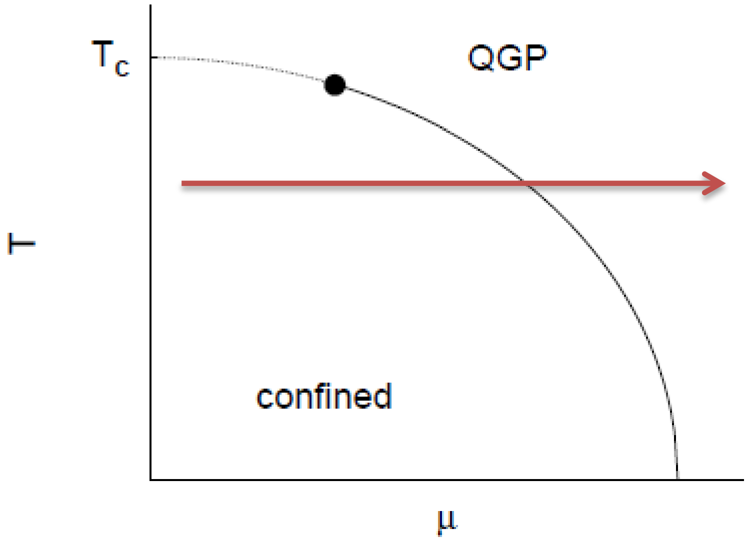
Motivation

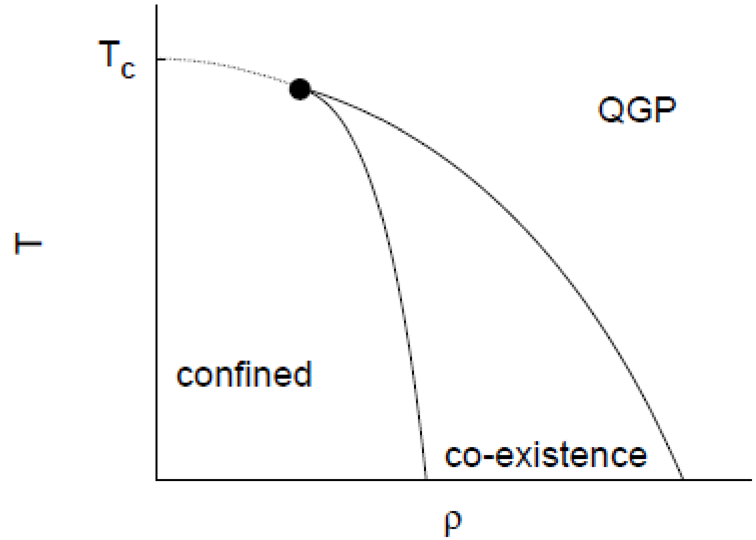




$T_c(\mu)$, $T_c(\rho)$が知りたい

Motivation





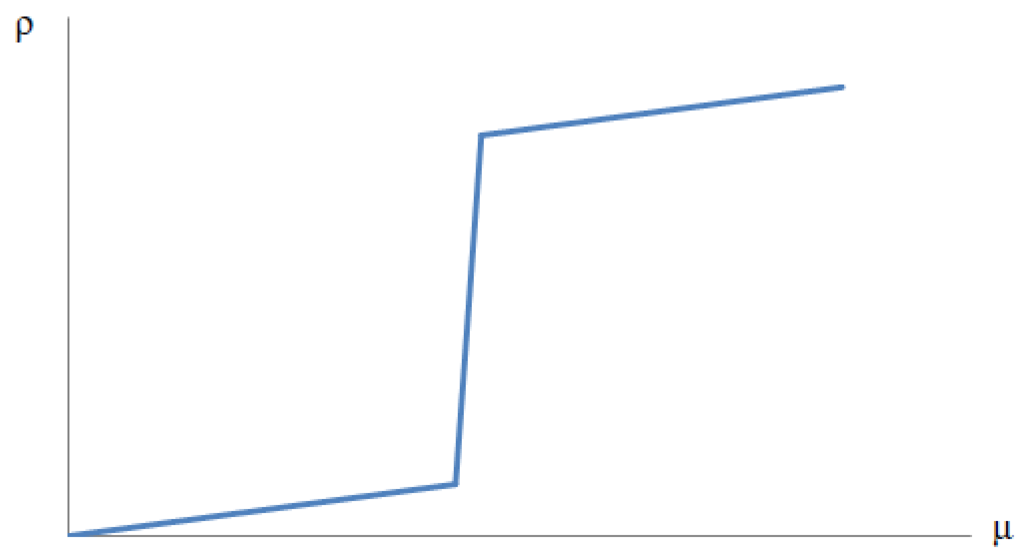
大分配関数

$Z_{G.C.}(T, \mu; V)$

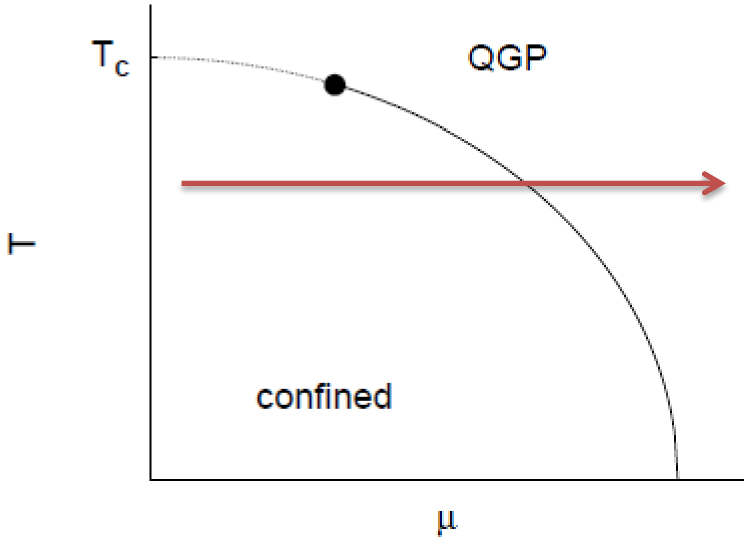
↓

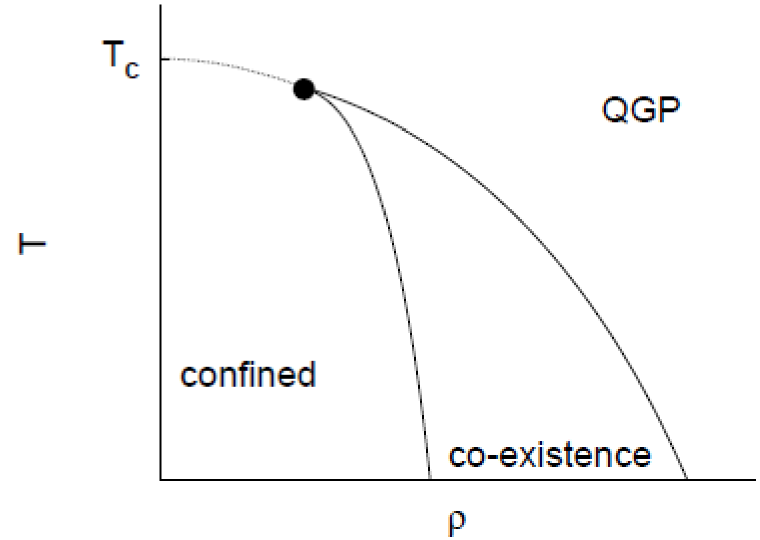
$J(T, \mu; V), \rho(T, \mu) \cdots$

熱力学関数



Motivation





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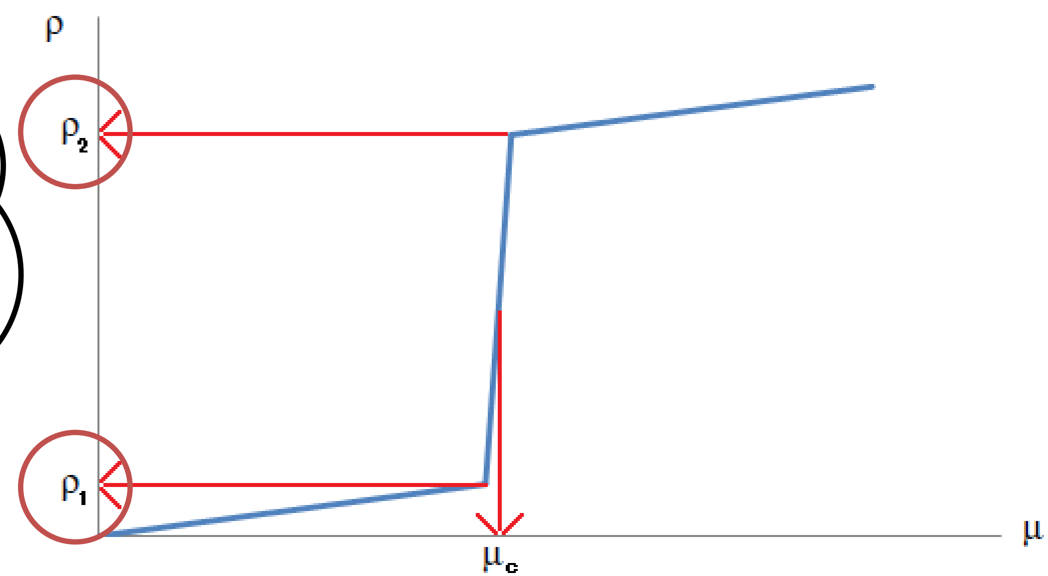

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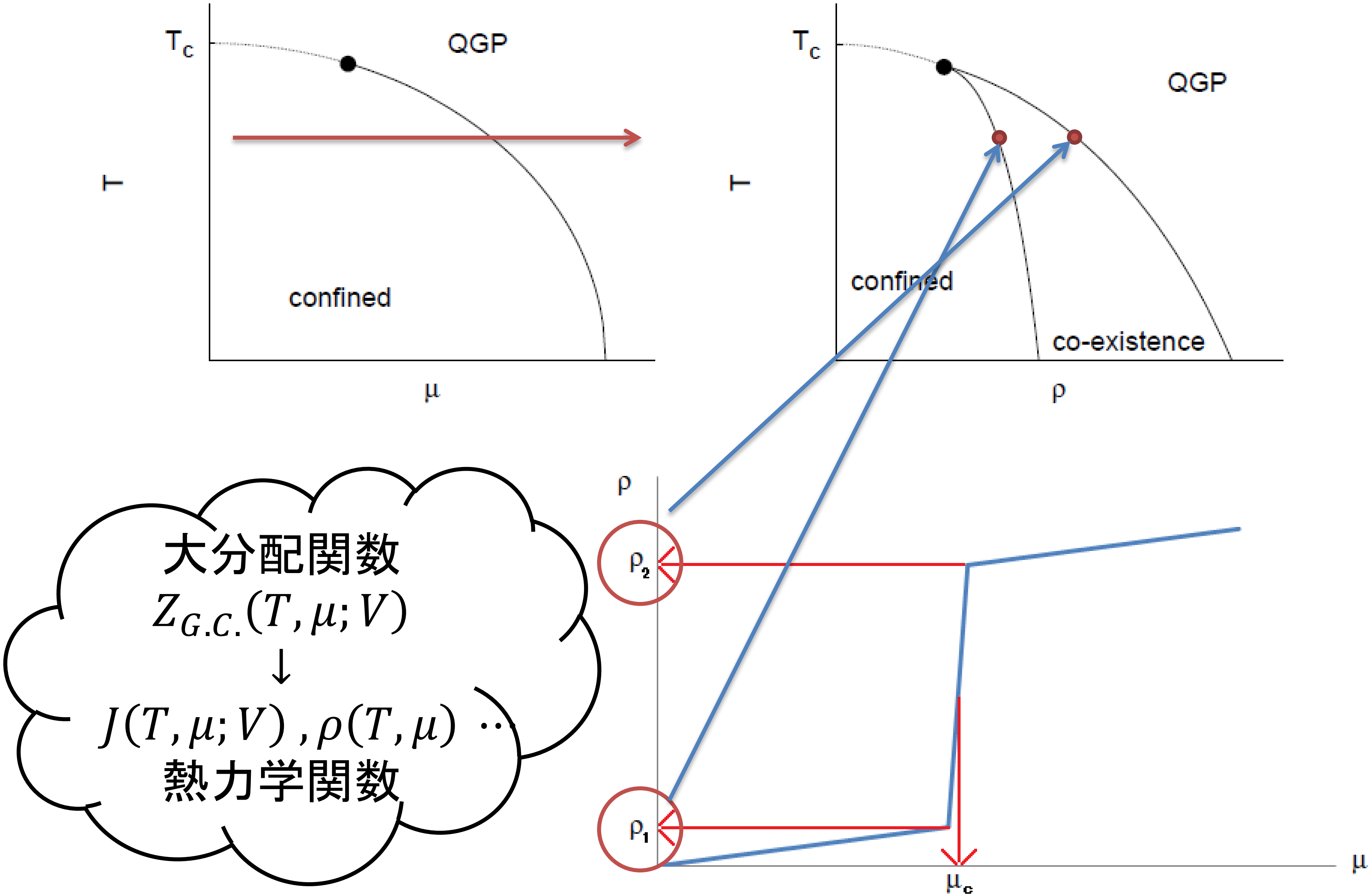

T_c
QGP
T
confined
μ
T_c
QGP
T
confined
co-existence
ρ
ρ
ρ_2
ρ_1
μ_c
μ
大分配関数
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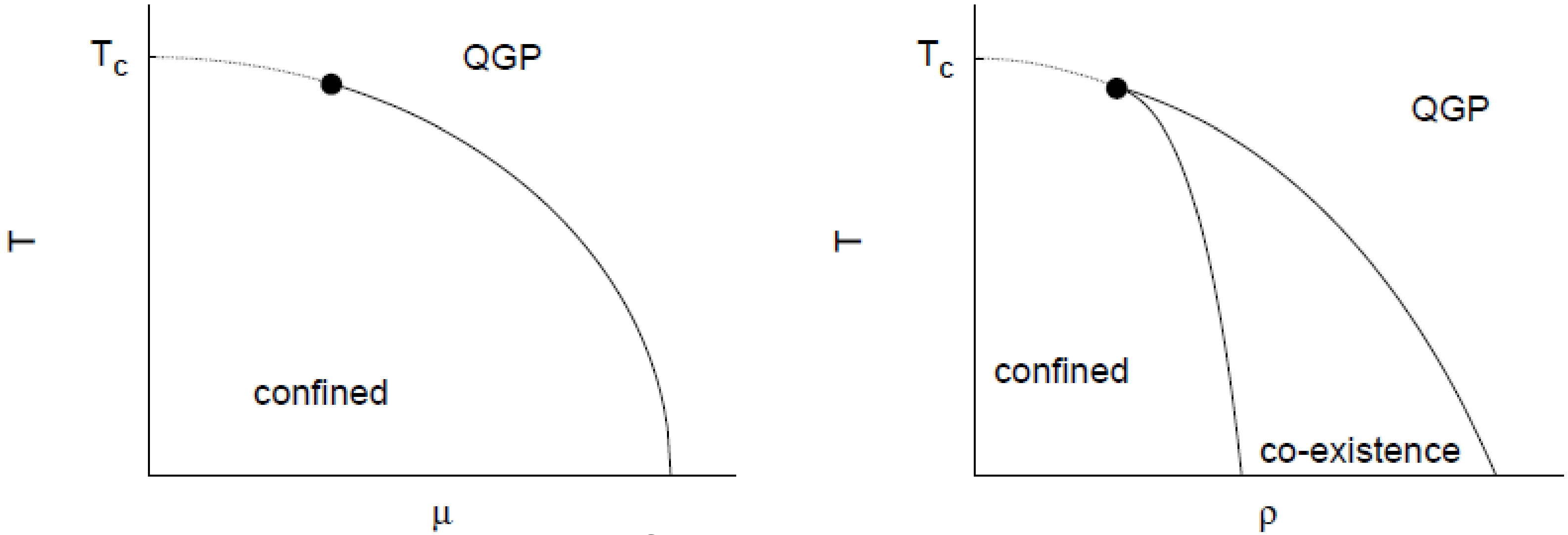
熱力学関数

論文の主張

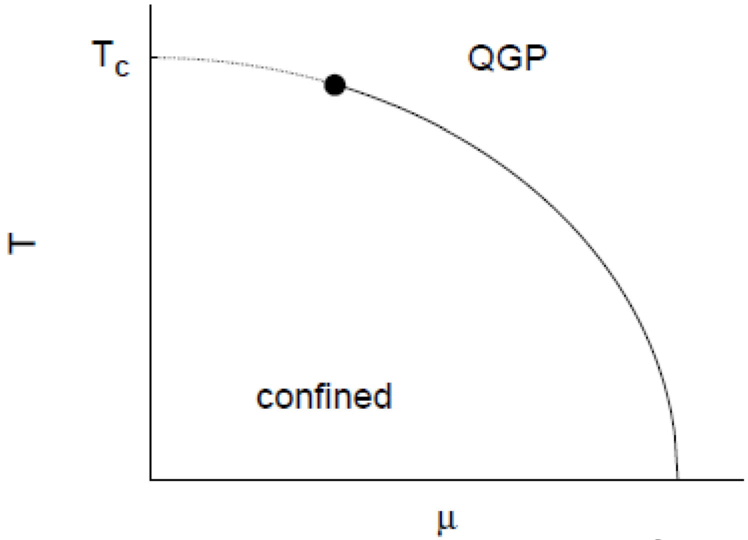


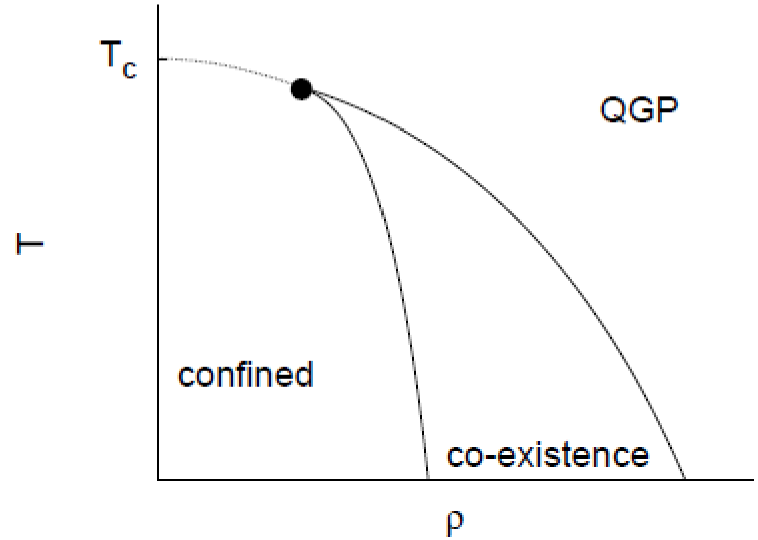
$$Z_{G.C.}(T,\mu,V) = \int \mathcal{D}U \;\; e^{-S_g} \boxed{\det D(\mu)} \quad \left(S_F = \sum \bar{\psi} D \psi\right)$$

Wilson fermion で $\det D(\mu)$ の reduction formula を作った

A. Borici,
Prog. Theor. Phys. Suppl. 153:335-339,2004

論文の主張





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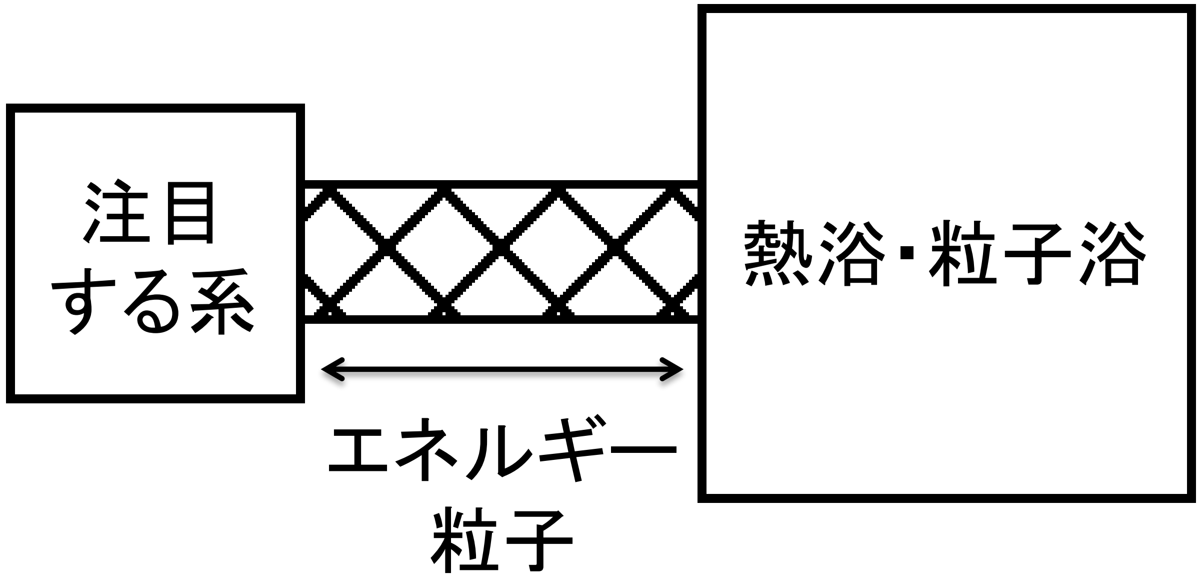
$$\det D(\mu) = C \sum_n C_n e^{\frac{\mu}{T} n}$$

（Canonical approach に適した形!）

発表の流れ

- 導入
- 有限温度QCD
- Reduction formula
- 数値計算の結果
- まとめ

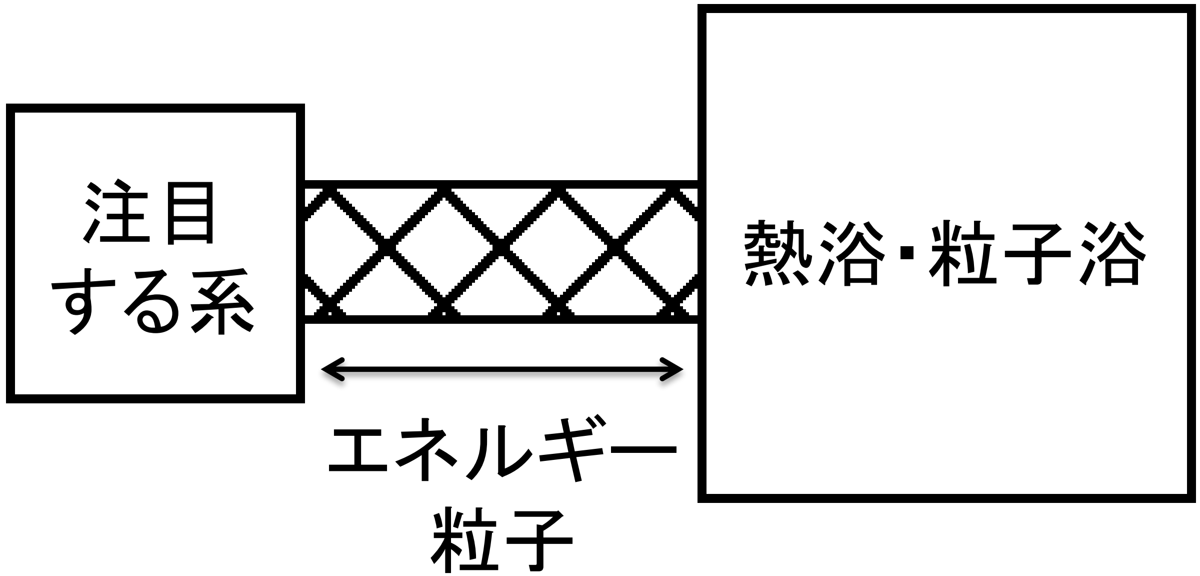
経路積分形式



$$Z_{G.C.}(T,\mu;V) = \mathrm{Tr}\left\{\exp\left(-\frac{1}{T}\left(\hat{H}-\mu\hat{N}\right)\right)\right\}$$

$$= \int \mathcal{D}\phi \ \left.\left\langle\phi\middle|\exp\left(-\frac{1}{T}\left(\hat{H}-\mu\hat{N}\right)\right)\middle|\phi\right\rangle\right|_{\phi(t=0)=\pm\phi(t=1/T)}$$

経路積分形式



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Coherent state

$$= \int \mathcal{D}\phi \left\langle \phi \middle| \exp\left(-\frac{1}{T}\left(\hat{H}-\mu\hat{N}\right)\right) \middle| \phi \right\rangle \Bigg|_{\phi(t=0)=\pm\phi(t=1/T)}$$

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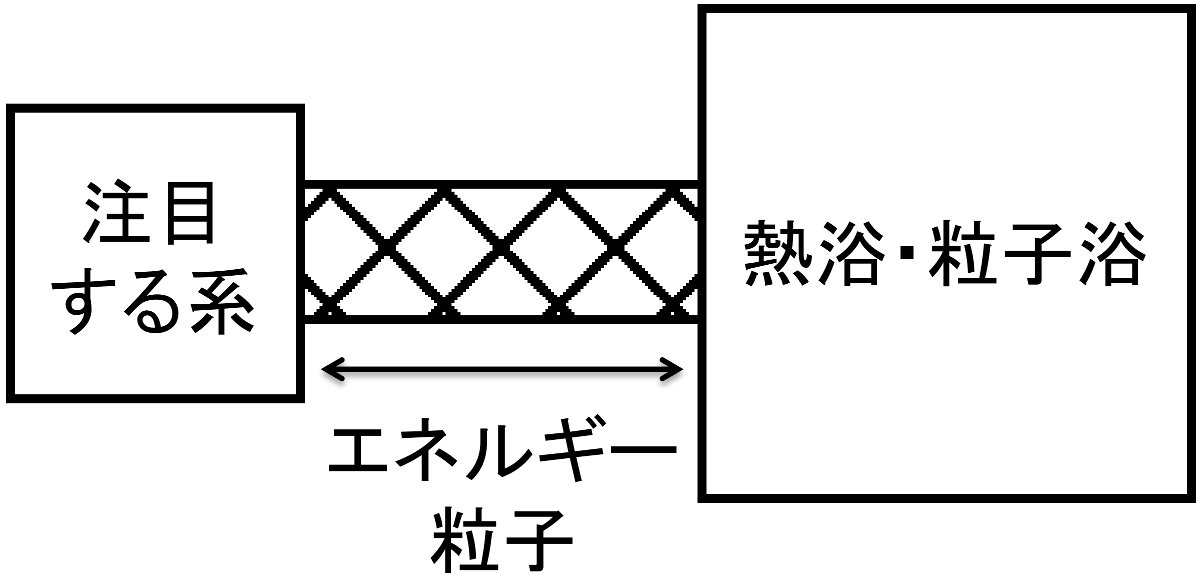

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Coherent state

$$= \int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi \exp\left(-\underline{\int_0^{1/T} dt} \int d^3x \left(\mathcal{L}_{QCD} - \mu\psi^\dagger\psi\right)\right)$$

場の理論との比較

- 有限温度

$$Z_{G.C.}(T, \mu = 0; V)$$
$$= \boxed{\mathrm{Tr}}\left\{e^{-\frac{1}{T}\hat{H}}\right\}$$
$$= \int \mathcal{D}\phi \langle \phi | e^{-\hat{H}/T} | \phi \rangle$$

- 場の理論

$$Z = \lim_{T\to\infty} \langle \phi_f | e^{-\hat{H}T} | \phi_i \rangle$$

$$= \lim_{T\to\infty} \int \mathcal{D}\phi \langle \phi | e^{-\hat{H}T} | \phi \rangle$$

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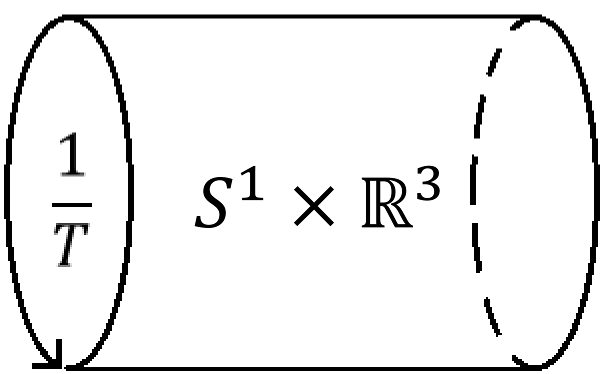
$$\phi(t=0) \ = \ \pm\, \phi(t = 1/T)$$

周期境界条件



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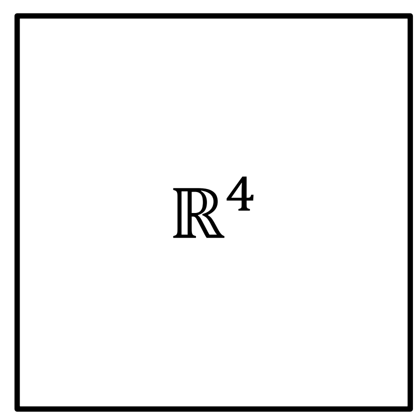
$$\phi(t=0) \ = \phi_i$$
$$\phi(t=T) \ = \phi_f$$



格子作用の決定

$$S_{cont.} = \int d^4x\, \bar{\psi}(\gamma_i D_i + m + \mu\gamma_4)\psi$$

$\boxed{\lim_{a\to 0} S_{latt.} = S_{cont.}\text{ で作るべし！}}$

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$$S_{latt.} = (M+4)\sum_{n,m} \bar{\psi}(n)\Delta(\mathrm{n},\mathrm{m})\psi(m)$$

$$\begin{aligned}
&\Delta(n,m)\\
&= \delta_{n,m} - \kappa \sum_{i=1}^{3}\{(1-\gamma_i)U_i(n)\delta_{m,n+\hat{\imath}} + (1+\gamma_i){U_i}^{\dagger}(m)\delta_{m,n-\hat{\imath}}\}\\
&\quad -\kappa\{(1-\gamma_4)e^{+\mu}U_4(n)\delta_{m,n+\hat{4}} + (1+\gamma_4)e^{-\mu}{U_4}^{\dagger}(m)\delta_{m,n-\hat{4}}\}\\
&\quad -\delta_{n,m}C_{SW}\kappa\sum_{i\le j}\frac{[\gamma_i, \gamma_j]}{2i}F_{ij}(n)
\end{aligned}$$

$\kappa = \frac{1}{2(M+4)}$と C_{SW} はパラメータ

格子作用の決定

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Wilson fermion

$$\Delta(n,m)$$
$$= \delta_{n,m} - \kappa\sum_{i=1}^{3}\{(1-\gamma_i)U_i(n)\delta_{m,n+\hat{\imath}} + (1+\gamma_i)U_i{}^{\dagger}(m)\delta_{m,n-\hat{\imath}}\}$$
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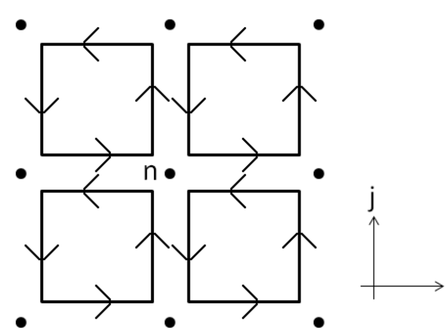
Clover term



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問題点

$$\det(D(\mu = 0))^* = \det\left(D^\dagger(\mu = 0)\right)$$
$$= \det(D(\mu = 0))$$

$$D^\dagger = \gamma_5 D \gamma_5$$

$$\boldsymbol{det(D(\mu = 0))}$$ **実数**

$$\boldsymbol{det(D(\mu \neq 0))}$$ **複素数！**

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$$Z_{G.C.}(T, \mu \neq 0; V) = \int \mathcal{D}U \ \boxed{e^{-S_g} \det\left(D(\mu)\right)}$$

確率

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~~確率~~

$$= \int \mathcal{D}U \; \boxed{e^{i\theta}} \; \boxed{e^{-S_g}|\det(D(\mu))|}$$

→sign problem

Monte Carlo 計算が難しい

いくつかのアプローチ

- Reweighting (Z. Fodor, S.D. Katz, Phys. Lett. B534(2002)87)
  - $Z_{G.C.}(T,\mu;V) = \int \mathcal{D}U \ \frac{\det(D(\mu))}{\det(D(0))} \mathrm{e}^{-S_{\mathrm{g}}(\beta_0)} \boxed{\det(D(0)) e^{-S_g(\beta)+S_g(\beta_0)}}$
- Taylor展開
  - $Z_{G.C.}(\mu) = Z_{G.C.}(0) + \mu \left. \frac{\partial Z_{G.C.}}{\partial \mu} \right|_{\mu=0} + \cdots$
- 進撃の複素ランジュバン
  Complex Langevin

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- Canonical approach
  - $Z_{G.C.}(T,\mu;V)$ に替えて $Z_{can.}(T;V,N)$ を使う

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- Canonical approach
  - $Z_{G.C.}(T,\mu;V)$ 大分配関数 に替えて $Z_{can.}(T;V,N)$ 分配関数 を使う

熱力学的に等価

Reduction formula

$$\Delta(n,m) = \boxed{\delta_{n,m} - \kappa \sum_{i=1}^{3} \{(1-\gamma_i)U_i(n)\delta_{m,n+\hat{i}} + (1+\gamma_i){U_i}^{\dagger}(m)\delta_{m,n-\hat{i}}\} - \delta_{n,m}\Delta_{Clover}}$$

$$-\kappa\{ (1-\gamma_4)e^{\mu}U_4(n)\delta_{m,n+\hat{4}} + (1+\gamma_4)e^{-\mu}{U_4}^{\dagger}(m)\delta_{m,n-\hat{4}} \}$$

$$\left(\begin{array}{c|c|c|c|c}
B_1(n_t=1) & 0 & \cdots & 0 & 0 \\ \hline
0 & B_2(n_t=2) & \cdots & 0 & 0 \\ \hline
\cdots & \cdots & \ddots & \cdots & \cdots \\ \hline
0 & 0 & \cdots & B_{N_t-1}(n_t=N_t-1) & 0 \\ \hline
0 & 0 & \cdots & 0 & B_{N_t}(n_t=N_t)
\end{array}\right) \quad 4N_cN_tN_s^3$$

$$4N_cN_s^3$$

Reduction formula

$$\Delta(n,m)$$

$$= \boxed{\delta_{n,m} - \kappa \sum_{i=1}^{3} \{(1-\gamma_i)U_i(n)\delta_{m,n+\hat{\imath}} + (1+\gamma_i){U_i}^{\dagger}(m)\delta_{m,n-\hat{\imath}}\} - \delta_{n,m}\Delta_{Clover}}$$

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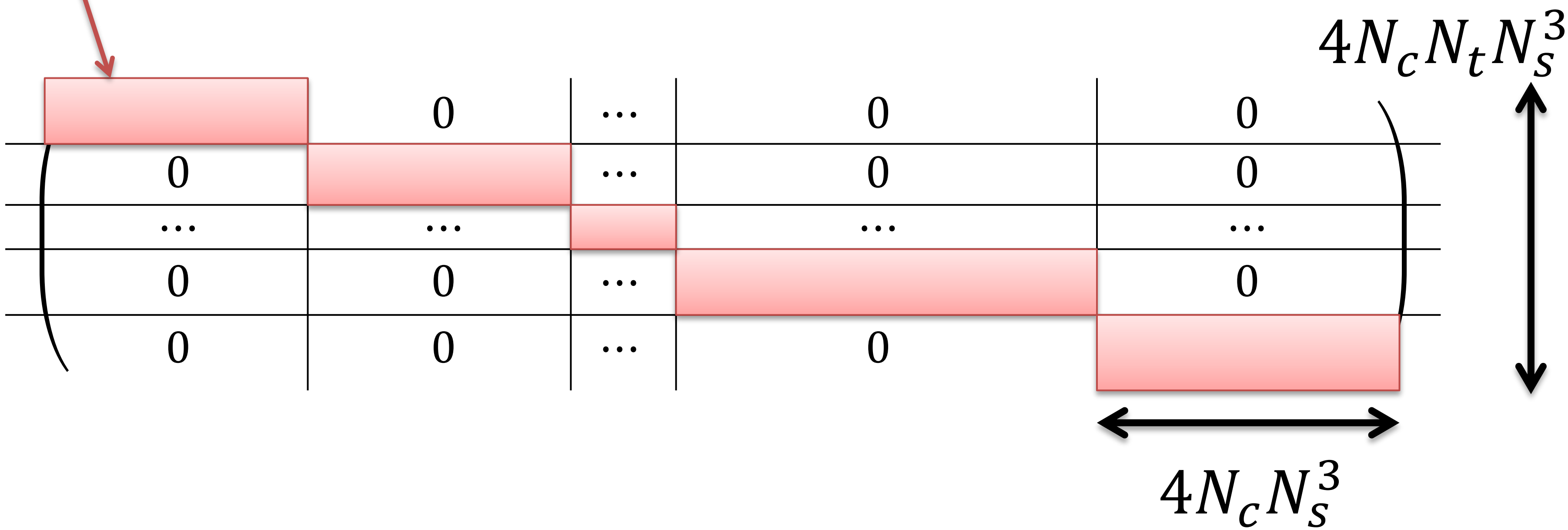

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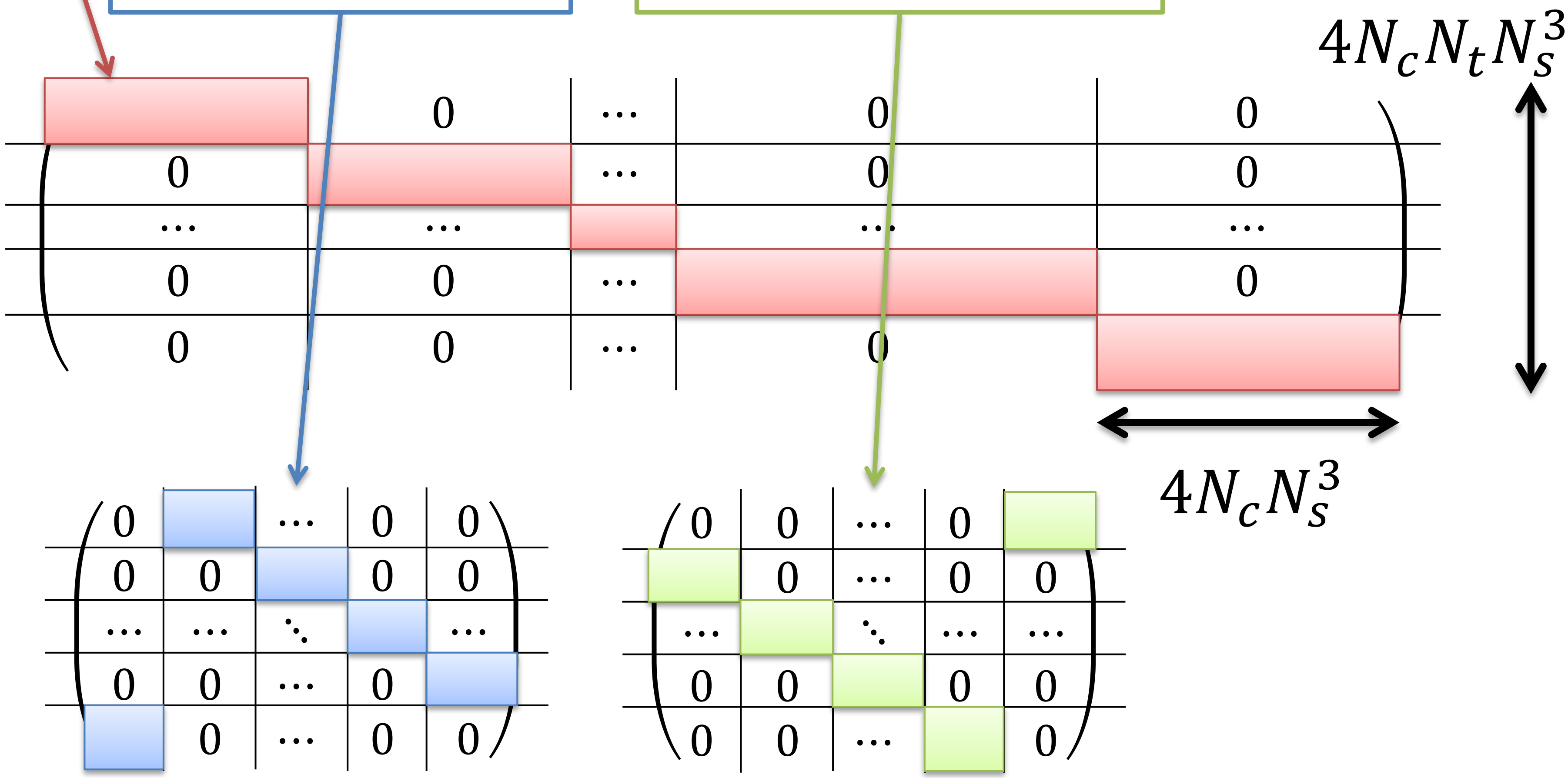
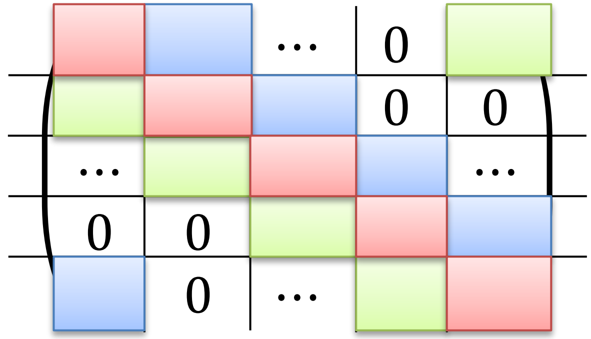


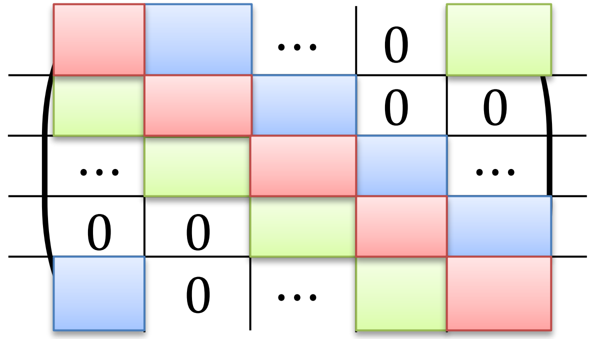
Reduction formula

$\Delta(\mu) =$



$$P_{n,m} := \frac{1 - \gamma_4}{2}\delta_{n,m} + \frac{1+\gamma_4}{2}U_4{}^{\dagger}(m)\delta_{m,n+\hat{4}}e^{-\mu}$$

Reduction formula

$\Delta(\mu) =$ 

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射影行列

$$\left(\frac{1\pm\gamma_4}{2}\right)^2 = \frac{1\pm\gamma_4}{2}$$

$$\frac{1-\gamma_4}{2}\frac{1+\gamma_4}{2} = 0$$

$(1\pm\gamma_4)/2$ を含まない

$(1-\gamma_4)/2$ を含む

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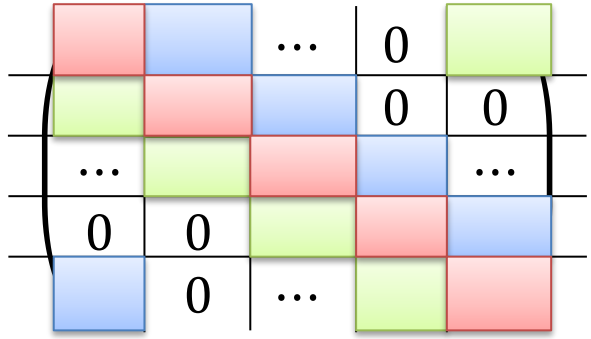
Reduction formula

$\Delta(\mu) =$



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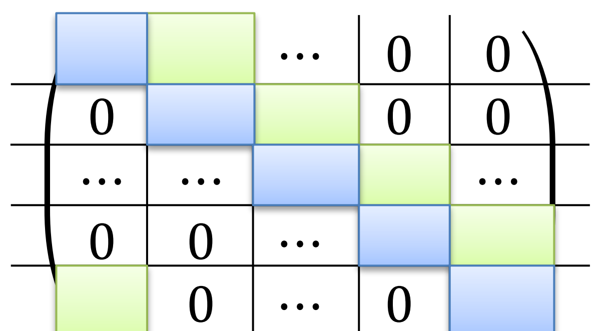
$(1 - \gamma_4)/2$ を含む

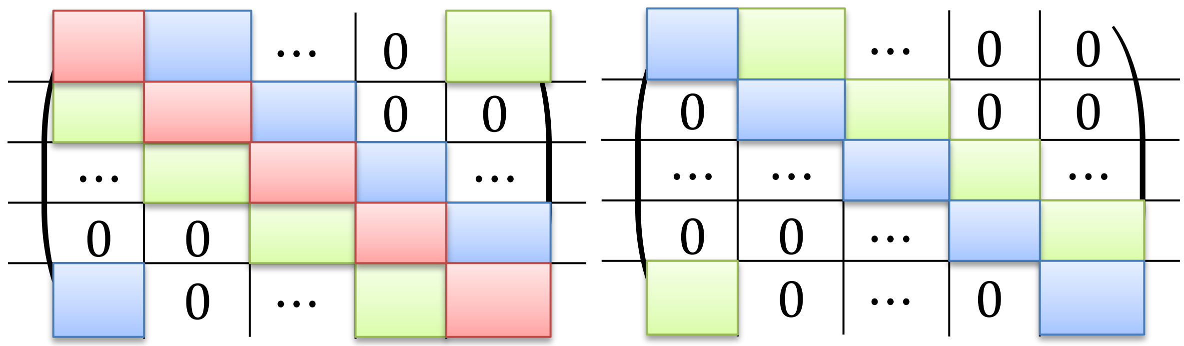
$(1 + \gamma_4)/2$ を含む

$P(\mu) =$



Reduction formula

$\Delta P =$



$$
= \begin{pmatrix}
\alpha_1 & \beta_1 e^{\mu} & \cdots & 0 & 0 \\
0 & \alpha_2 & \cdots & 0 & 0 \\
\cdots & \cdots & \ddots & \cdots & \cdots \\
0 & 0 & \cdots & \alpha_{N_t-1} & \beta_{N_t-1} e^{\mu} \\
-\beta_{N_t} e^{\mu} & 0 & \cdots & 0 & \alpha_{N_t}
\end{pmatrix}
$$

$$
\begin{cases}
\alpha_i^{ab,\mu\nu}(\vec{n},\vec{m}) = \dfrac{1}{2} B^{ab,\mu\sigma}(\vec{n},\vec{m},t_i)(1-\gamma_4)^{\sigma\nu} - \kappa(1+\gamma_4)^{\mu\nu}\delta^{ab}\delta^3{}_{\vec{n},\vec{m}} \\
\beta_i^{ab,\mu\nu}(\vec{n},\vec{m}) = \dfrac{1}{2} B^{ac,\mu\sigma}(\vec{n},\vec{m},t_i)(1+\gamma_4)^{\sigma\nu} U_4^{cb}(\vec{m},t_i) - \kappa(1-\gamma_4)^{\mu\nu}\delta^3{}_{\vec{n},\vec{m}} U_4^{ab}(\vec{m},t_i)
\end{cases}
$$

$$
\det(\Delta P) = \prod \det(\alpha_i) \det\left\{1 + e^{\mu/T} \prod \det(\alpha_j^{-1}\beta_j)\right\}
$$

Reduction formula

$$\det(P) = \det\left(\begin{array}{c|c|c|c} \frac{1-\gamma_4}{2} & \frac{1+\gamma_4}{2}U_4(n_t=1)e^{\mu} & \cdots & 0 \\ \hline 0 & \frac{1-\gamma_4}{2} & \cdots & 0 \\ \hline \cdots & \cdots & \ddots & \cdots \\ \hline \frac{1+\gamma_4}{2}U_4(n_t=N_t)e^{\mu} & 0 & \cdots & \frac{1-\gamma_4}{2} \end{array}\right) = e^{N\mu/2}$$

$$\det\big(\Delta(\mu)\big) = \frac{\det(\Delta P)}{\det(P)}$$

$\{\lambda_i\}$ は $\prod \alpha_i^{-1}\beta_i$ の固有値

$$= e^{N\mu/2}\left(\prod \det(\alpha_i)\right)\prod\left(e^{-\mu/T} + \lambda_i\right)$$

$$= \sum_{n=-2N_cN_s^3}^{2N_cN_s^3} C_n\, e^{\frac{\mu}{T}n}$$

Reduction formula

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$$\boxed{C_n = \left(\prod \det(\alpha_i)\right) \frac{1}{n!\,(N-n)!} \sum_{\text{permutation } \sigma} \lambda_{\sigma(1)} \lambda_{\sigma(2)} \cdots \lambda_{\sigma(N-n)}}$$

Reduction formula

$$\det(P) = \det\begin{pmatrix} \frac{1-\gamma_4}{2} & \frac{1+\gamma_4}{2}U_4(n_t=1)e^{\mu} & \cdots & 0 \\ 0 & \frac{1-\gamma_4}{2} & \cdots & 0 \\ \cdots & \cdots & \ddots & \cdots \\ \frac{1+\gamma_4}{2} \cdots (n_t = N_t)e^{\mu} & 0 & \cdots & \frac{1-\gamma_4}{2} \end{pmatrix} = e^{N\mu/2}$$

$4N_tN_cN_s^3$

$$\det(\Delta(\mu)) = \frac{\det(\Delta P)}{\det(P)}$$

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Canonical approach

$$Z_{G.C.}(T,\mu;V) = \int \mathcal{D}U \, \frac{\det(D(\mu))}{\det(D(0))} \, \mathrm{e}^{-S_{\mathrm{g}}} \det(D(0))$$

$$\boxed{\det \mathrm{D}(\mu) = C \sum C_N e^{\frac{\mu}{T}N}}$$

$$= \sum_N \int \mathcal{D}U \, \frac{C * C_N}{\det(D(0))} e^{-S_g} \det(D(0)) \, e^{\frac{\mu}{T}N}$$

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統計力学で
よく知られた関係式

$$Z_{G.C.}(T,\mu;V) = \sum_N Z_{can.}(T;V,N)\, e^{\frac{\mu}{T}N}$$

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統計力学で
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$$Z_{G.C.}(T,\mu;V) = \sum_N \boxed{Z_{can.}(T;V,N)} e^{\frac{\mu}{T}N}$$

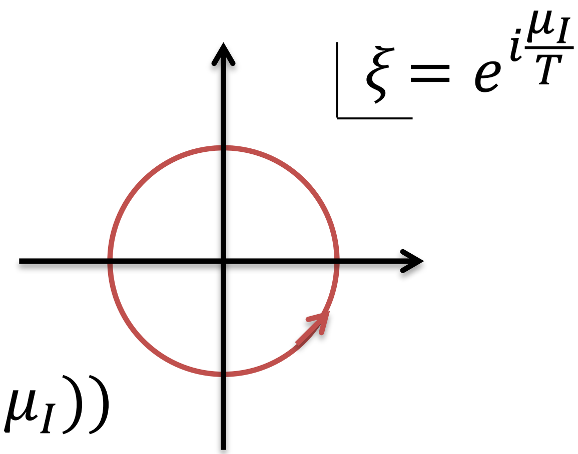
$$\therefore Z_{can.}(T;V,N) = \int \mathcal{D}U \, \frac{C * C_N}{\det(D(0))} e^{-S_g} \det(D(0))$$

今までのアプローチとの比較

$$\det(\Delta(\mu)) = \sum_{n=-\infty}^{\infty} C_n\ \xi^n\ , \xi = e^{\frac{\mu}{T}}$$

$$C_n = \frac{1}{2\pi i}\oint_C d\xi\ \xi^{-n-1}\det(\Delta(\xi))$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} d\frac{\mu_I}{T}\ e^{-i\frac{\mu_I}{T}n}\ \det(\Delta(i\mu_I))$$



Fourier変換

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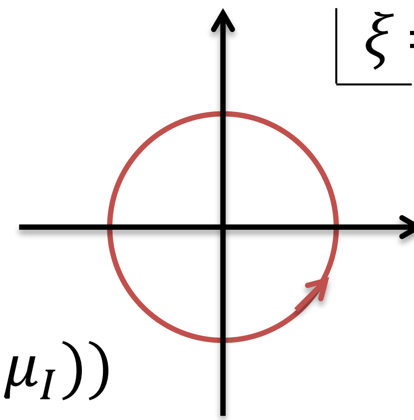
$\xi = e^{i\frac{\mu_I}{T}}$



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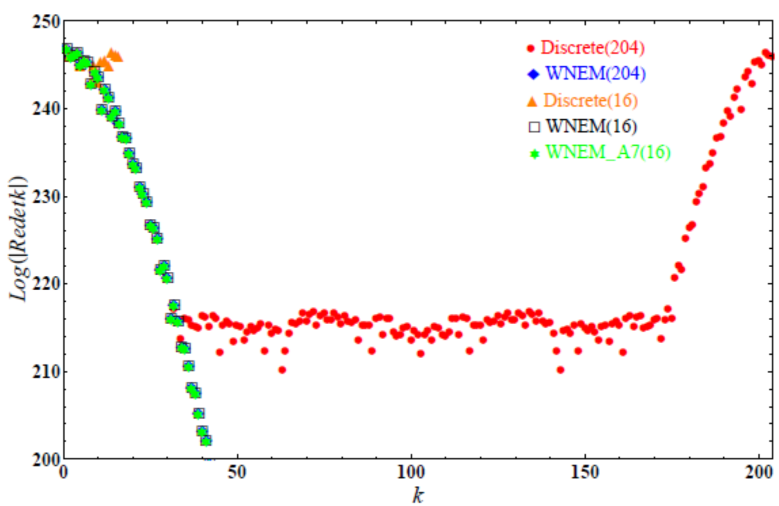

X. Meng, A. Li, A .Alexandru and K-F. Liu
arXiv:0811.2112[hep-lat](2008)
$6^3 \times 4\ , \beta = 5.2\ , \kappa = 0.158$

粒子数 n が大きいと
Fourier変換がうまくいかない!

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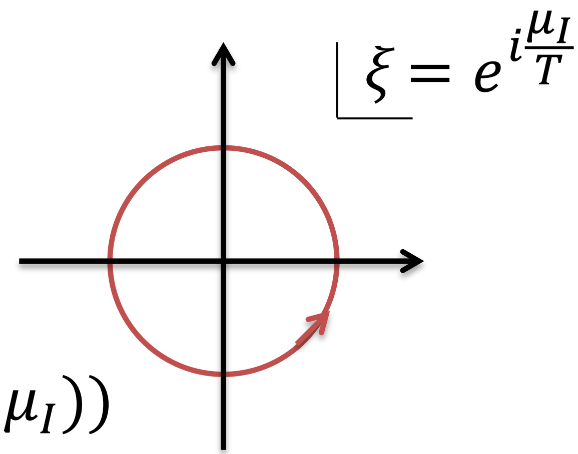

Fourier変換

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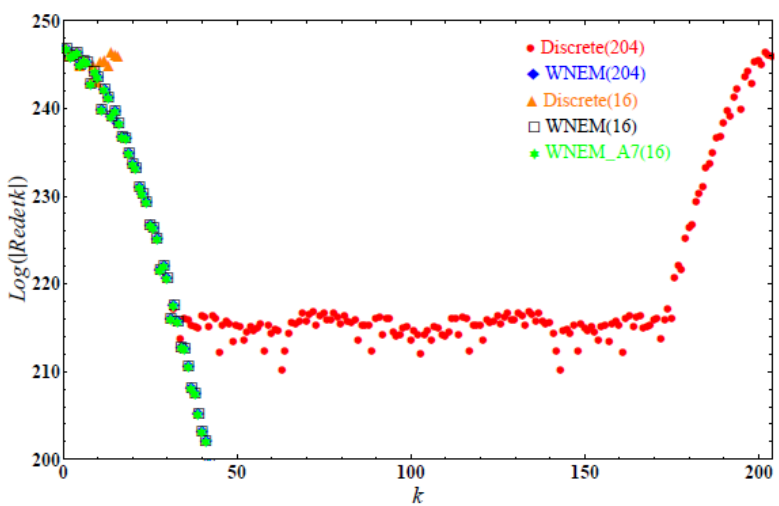
今回の方法
固有値の計算から C_n を求めた



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$6^3 \times 4\ , \beta = 5.2\ , \kappa = 0.158$

Numerical results

- Lattice size 4^4
- $\beta = 1.85$
  - $(\kappa, C_{SW}) = (0.14007, 1.5759)$
- $\beta = 2.0$
  - $(\kappa, C_{SW}) = (0.1369, 1.5058)$

※固有値 $\{\lambda_i\}$ の性質を調べる(groundwork として)

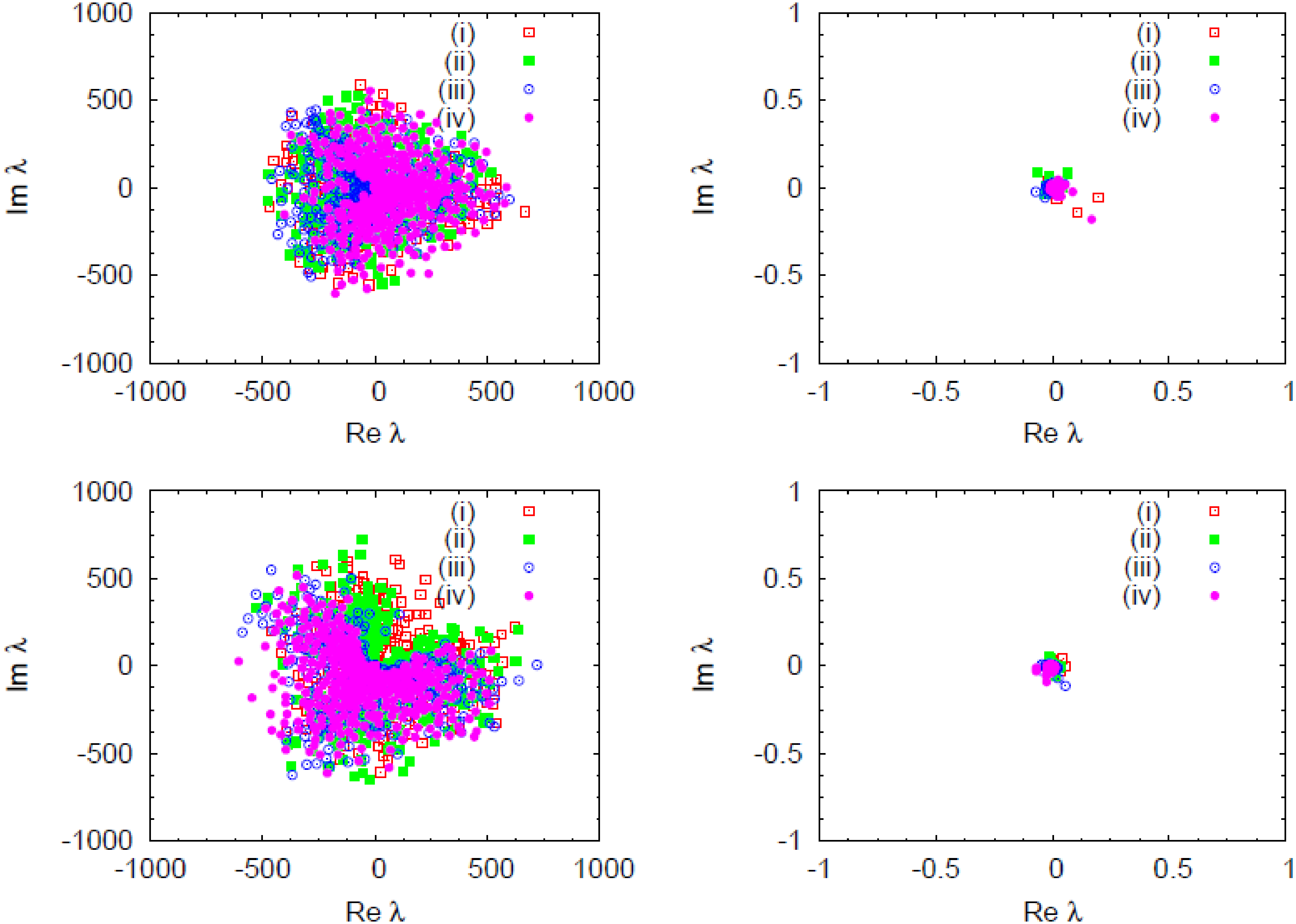
	$\det \Delta(0)$	Polyakov loop	Plaquette
(i)	3.0957×10^{-19}	$0.04377 - 0.25418i$	0.53338
(ii)	2.0921×10^{-21}	$-0.03234 + 0.08711i$	0.50668
(iii)	2.2560×10^{-21}	$-0.16365 - 0.10135i$	0.52471
(iv)	5.1115×10^{-18}	$0.49234 - 0.12163i$	0.53313

$\beta = 1.85.$

	$\det \Delta(0)$	Polyakov loop	Plaquette
(i)	8.7586×10^{-12}	$0.37590 + 0.0041i$	0.57810
(ii)	3.0329×10^{-12}	$0.13827 - 0.1978i$	0.57107
(iii)	1.1159×10^{-12}	$-0.22324 - 0.4285i$	0.57491
(iv)	1.2578×10^{-12}	$-0.35711 - 0.6028i$	0.57954

$\beta = 2.0.$

固有値の分布 $\begin{pmatrix} 上: \beta = 1.85 \\ 下: \beta = 2.0 \end{pmatrix}$



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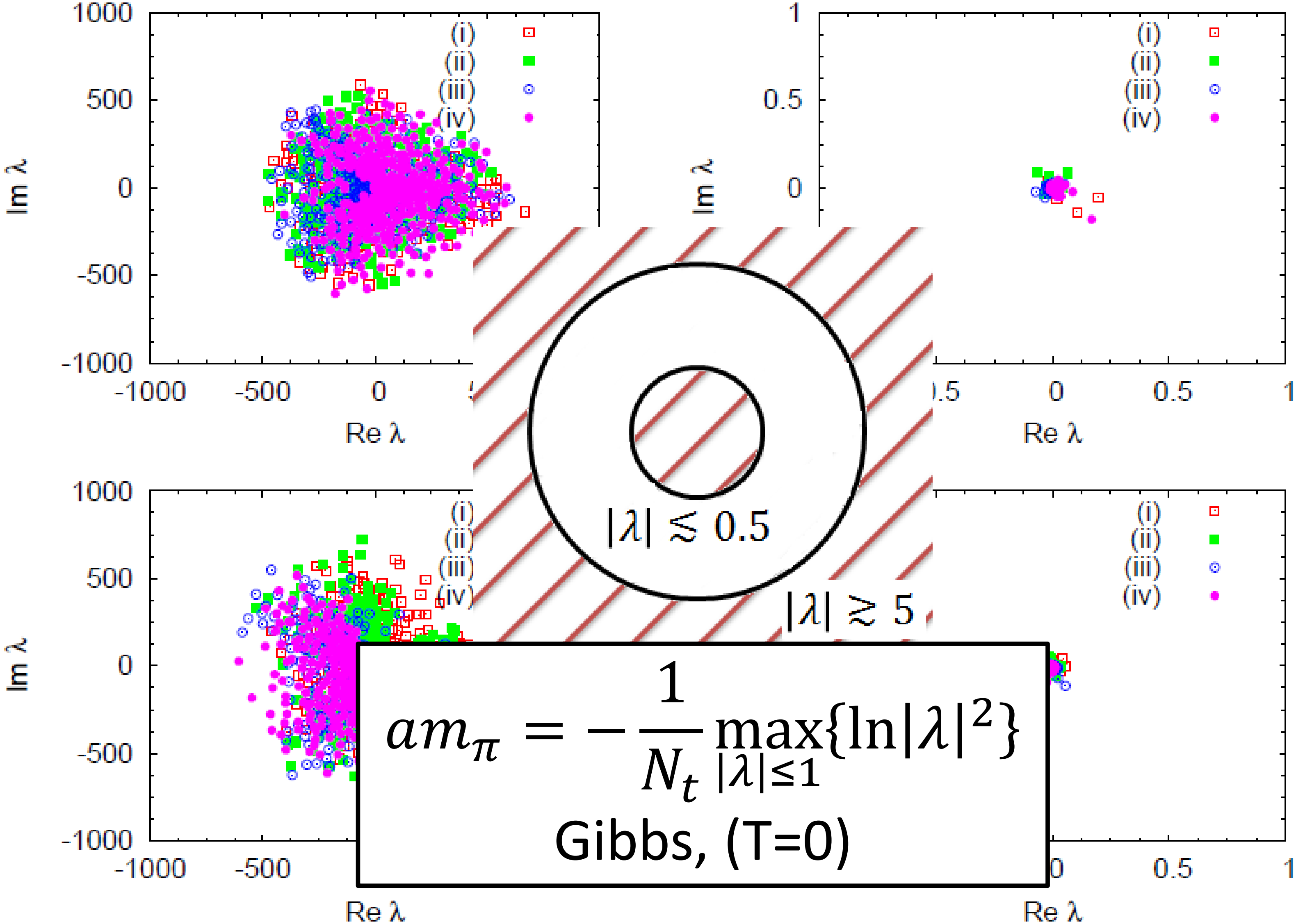
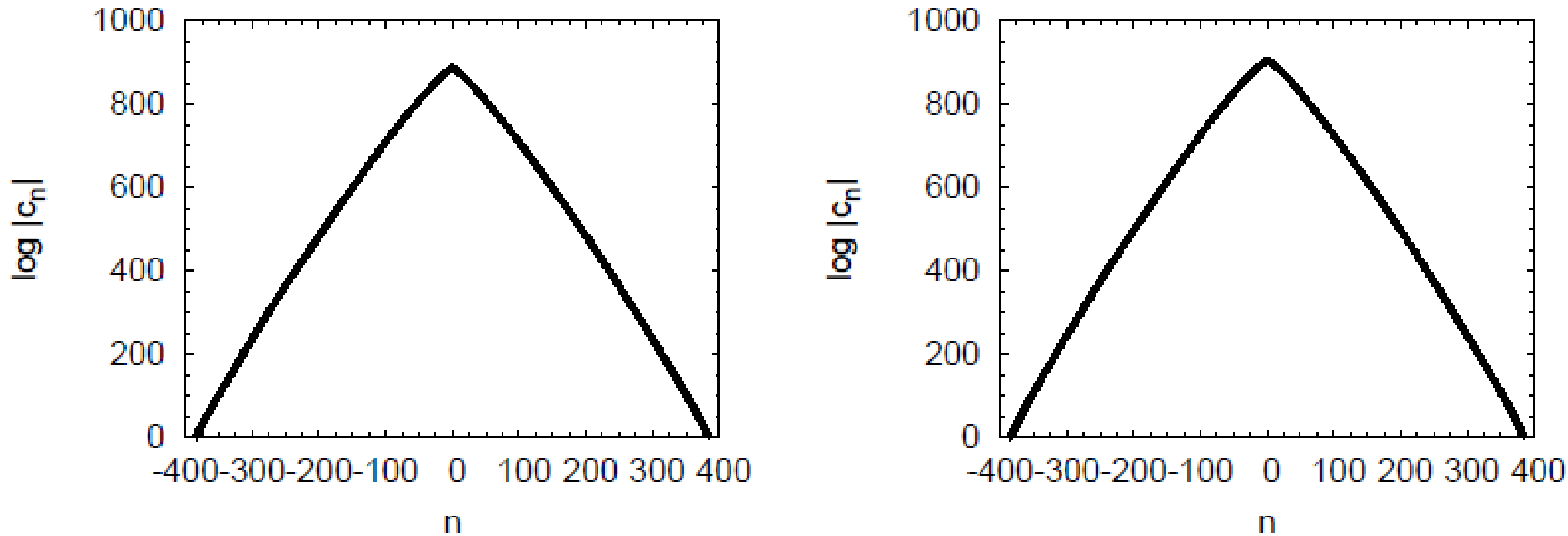


$$am_\pi = -\frac{1}{N_t}\max_{|\lambda|\leq 1}\{\ln|\lambda|^2\}$$

Gibbs, (T=0)

$$n \text{ vs. } \log|C_n| \quad \begin{pmatrix} 左: \beta = 1.85 \\ 右: \beta = 2.0 \end{pmatrix}$$



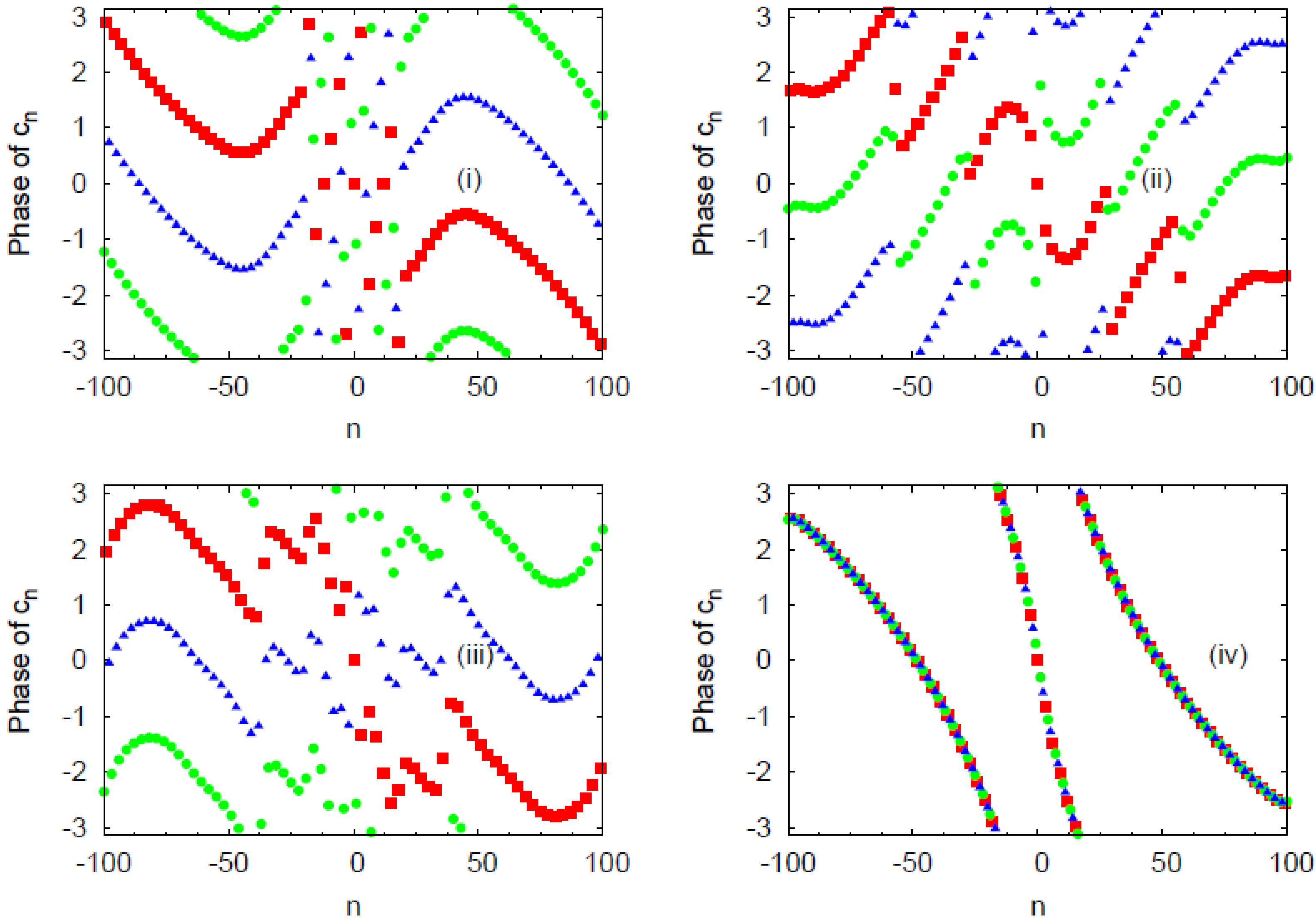
$n = 0$ でmax

指数関数的に減少

$$|C_n| = |C_{-n}|$$

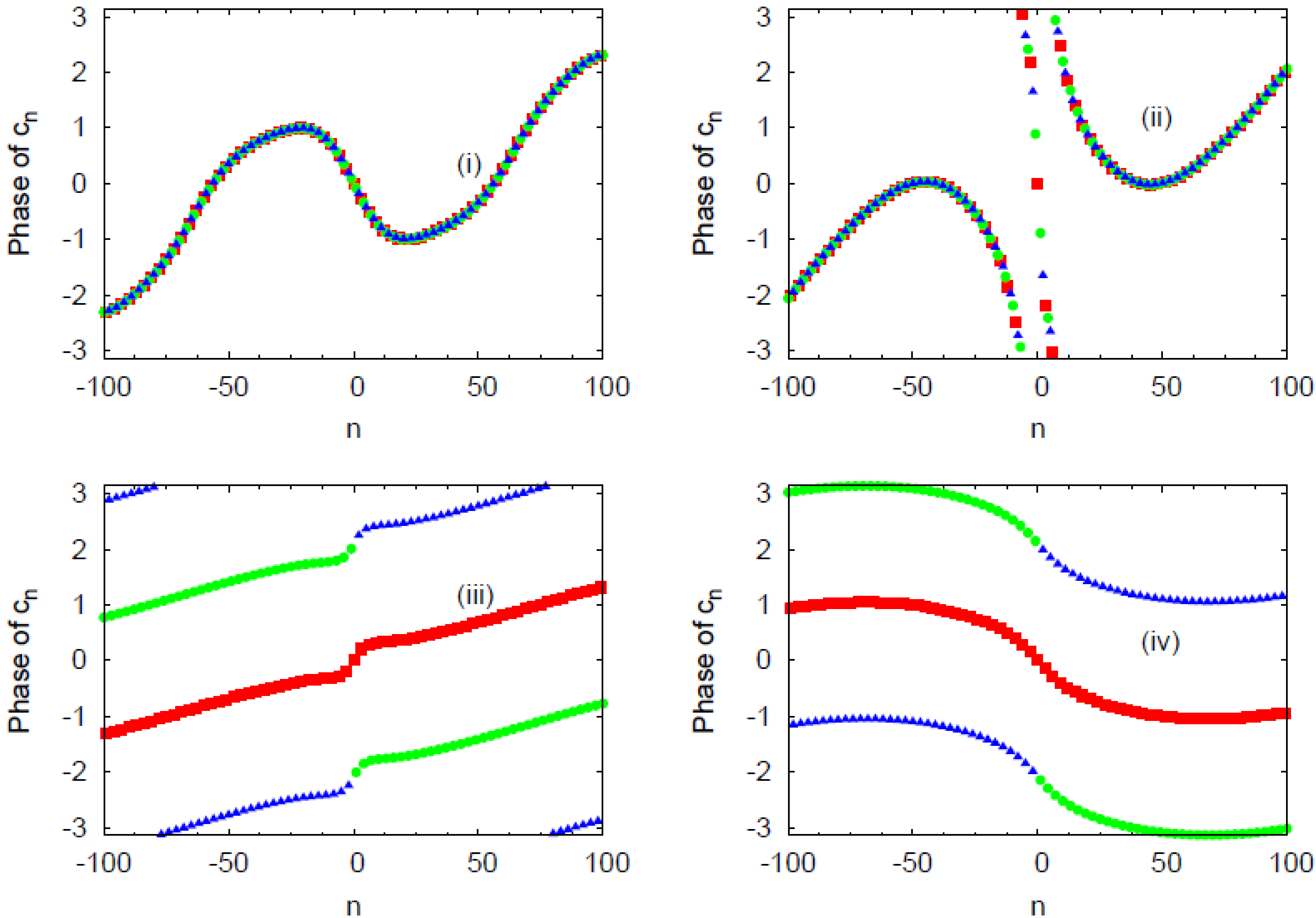
n vs. $\arg(C_n)$ ($\beta = 1.85$)

赤 $n = 3m$
緑 $n = 3m + 1$
青 $n = 3m + 2$



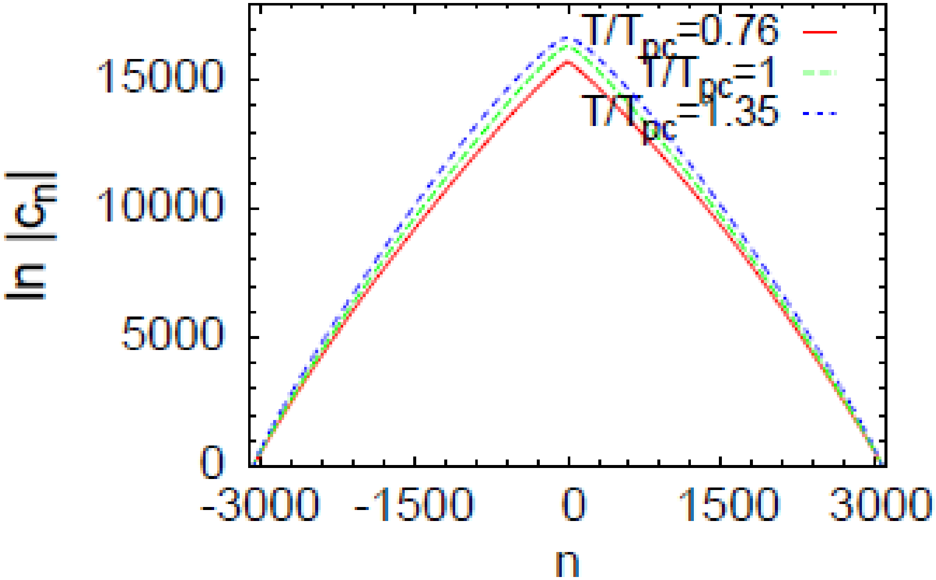
n vs. $\arg(C_n)$ ($\beta = 2.0$)

赤 $n = 3m$
緑 $n = 3m + 1$
青 $n = 3m + 2$

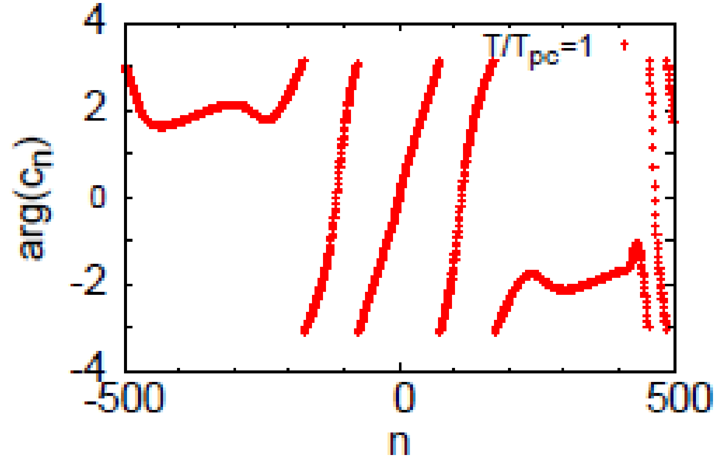


分配関数 $Z_{can.}(T;V,N)$

- arXiv:1204.1412v2 [hep-lat]   7 Jun 2012
  - K. Nagata, S. Motoki, Y. Nakagawa, A. Nakamura, and T. Saito
- *clover-Wilson + RG-gauge(Nf=2)*
- *Volume : 8^3x4*
- *quark mass : mps/mV ~ 0.8*
- *Configurations : HMC at mu=0*
- *Eigen values : 400 configs.*

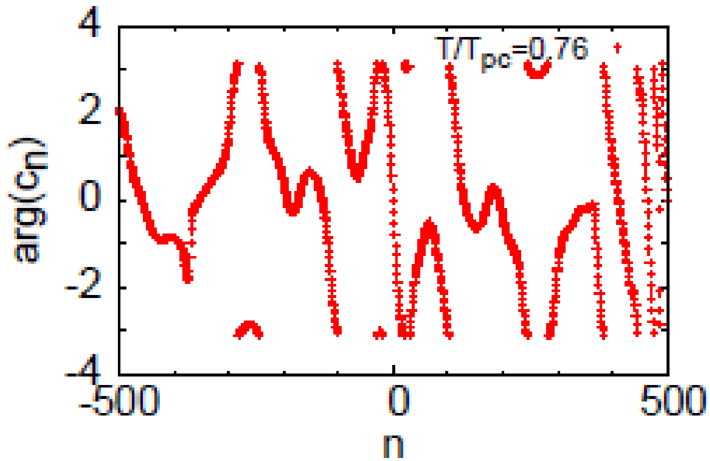


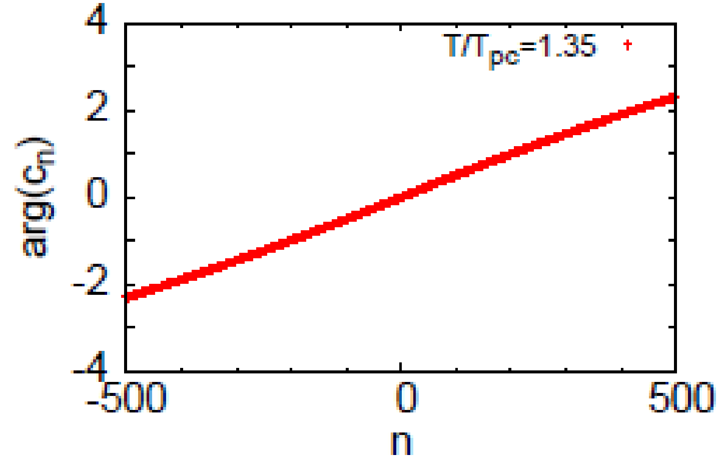
$$n\ \mathrm{vs.}\ \log|C_n| \quad \begin{pmatrix} \text{赤}: T = 0.76\,T_c \\ \text{緑}: T = T_c \\ \text{青}: T = 1.35\,T_c \end{pmatrix}$$



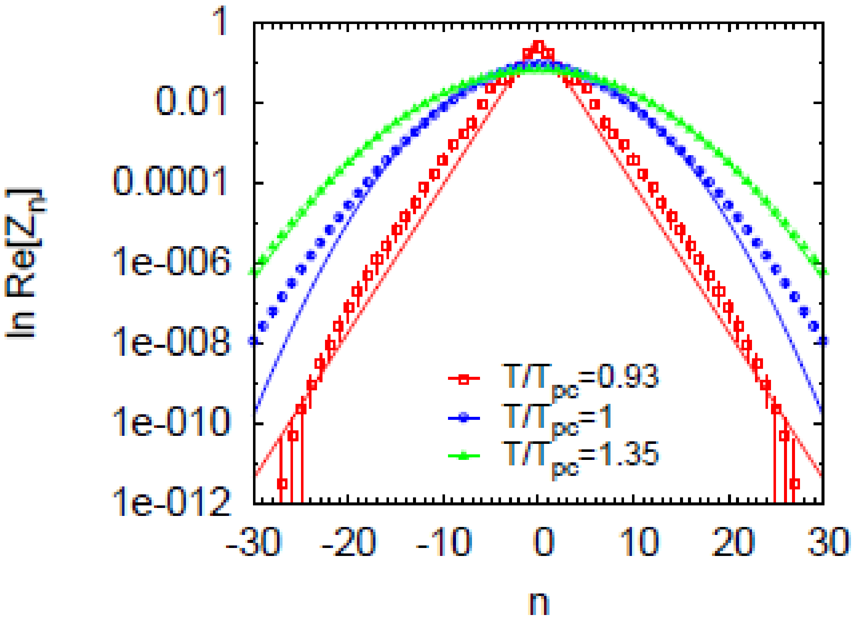
$$n\ \mathrm{vs.}\ \arg(C_n)$$

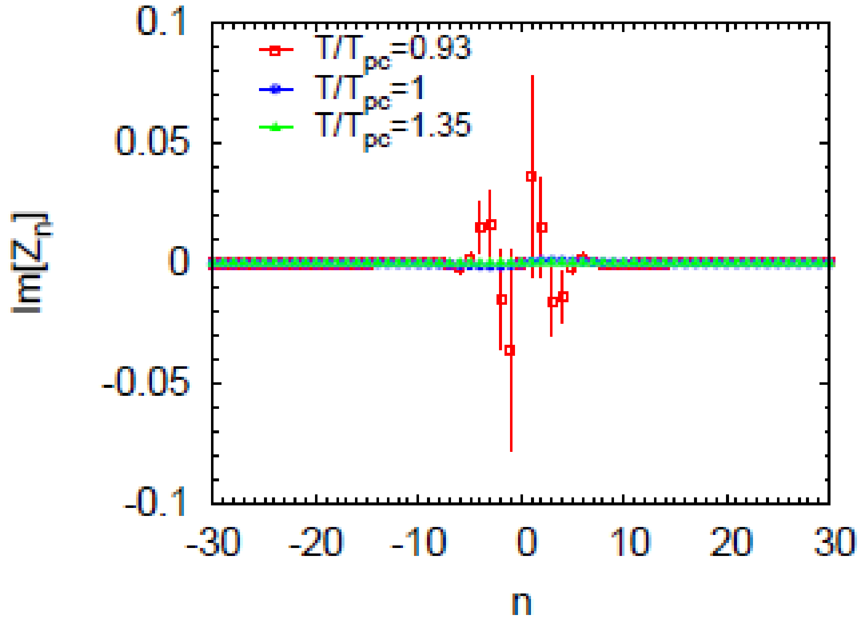
$$\begin{pmatrix} \text{左}: T = 0.76\,T_c \\ \text{上}: T = T_c \\ \text{下}: T = 1.35\,T_c \end{pmatrix}$$



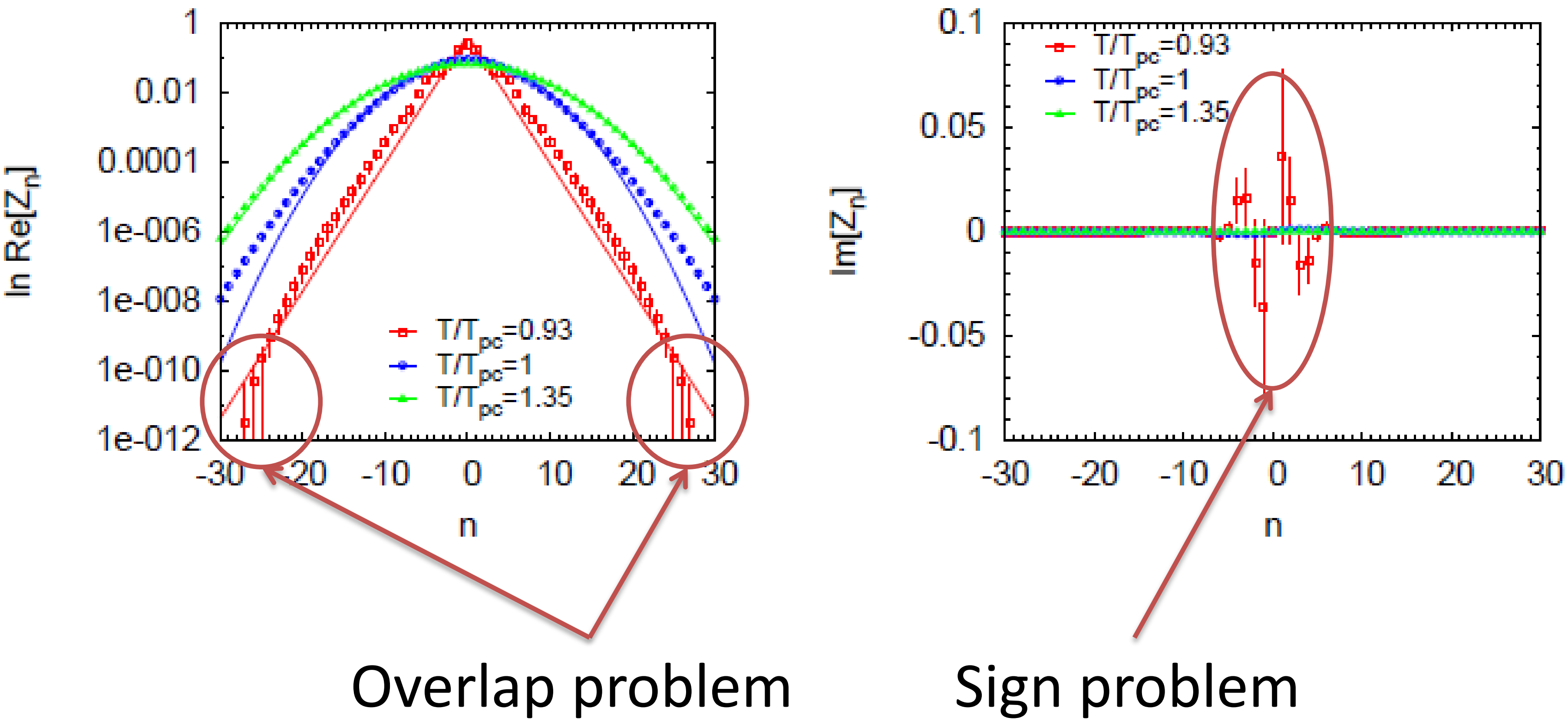


分配関数 $Z_{can.}(T;V,N)$





分配関数 $Z_{can.}(T; V, N)$



低温は難しい？

まとめ

- Wilson fermion に対する reduction formula をうまい射影 P を用いることで、
  $\det D(\mu) = C \sum_n C_n e^{\frac{\mu}{T}n}$ という形で作った
  - Canonical approach に適した形
- C_nの振る舞いを、大きなnまで調べることができた
- canonical分配関数を、温度を変えて求めた
  - 低温側では難しい？