進撃のcomplex Langevin

Taniguchi for journal club

Parisi (1983)

Aarts, James, Seiler, Stamatescu(0912.0617, 1110.5749)

Seiler, Sexty, Stamatescu (1211.3709)

Sexty (1307.7748)

Lattice 2014 NEWS

Parallel session, Fri 24/6 by Jaeger

Exploring the phase diagram of QCD with complex Langevin simulations

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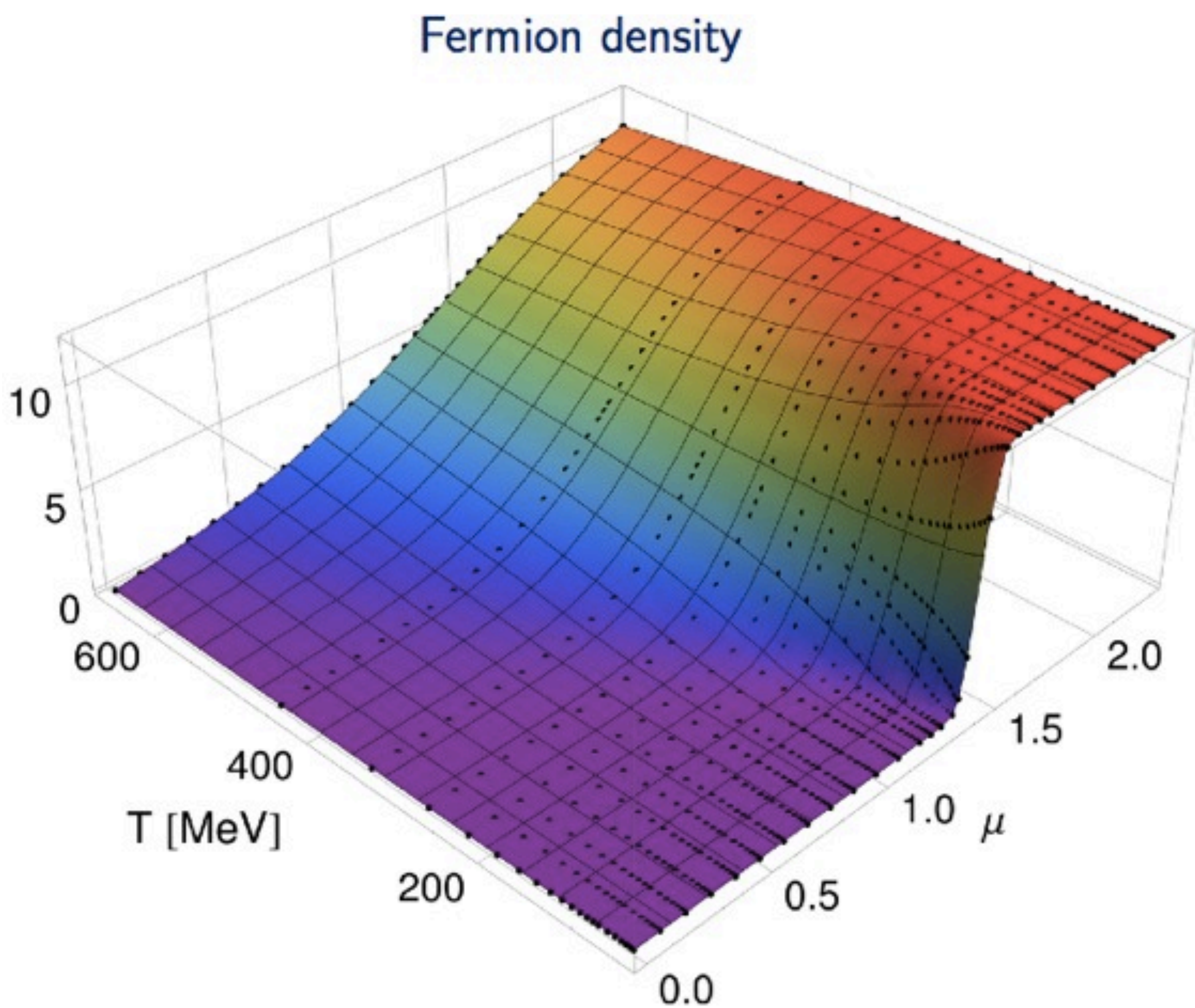

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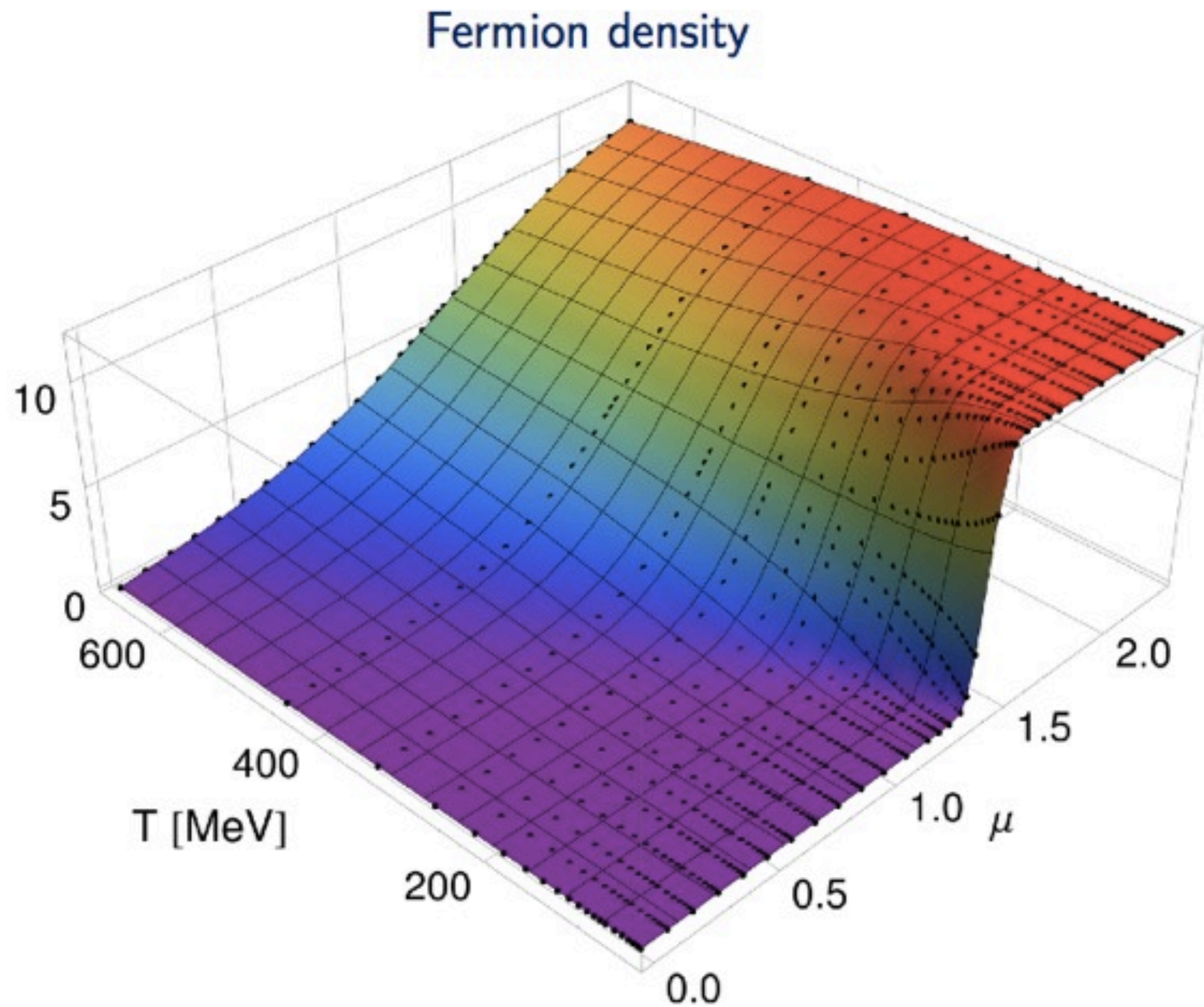

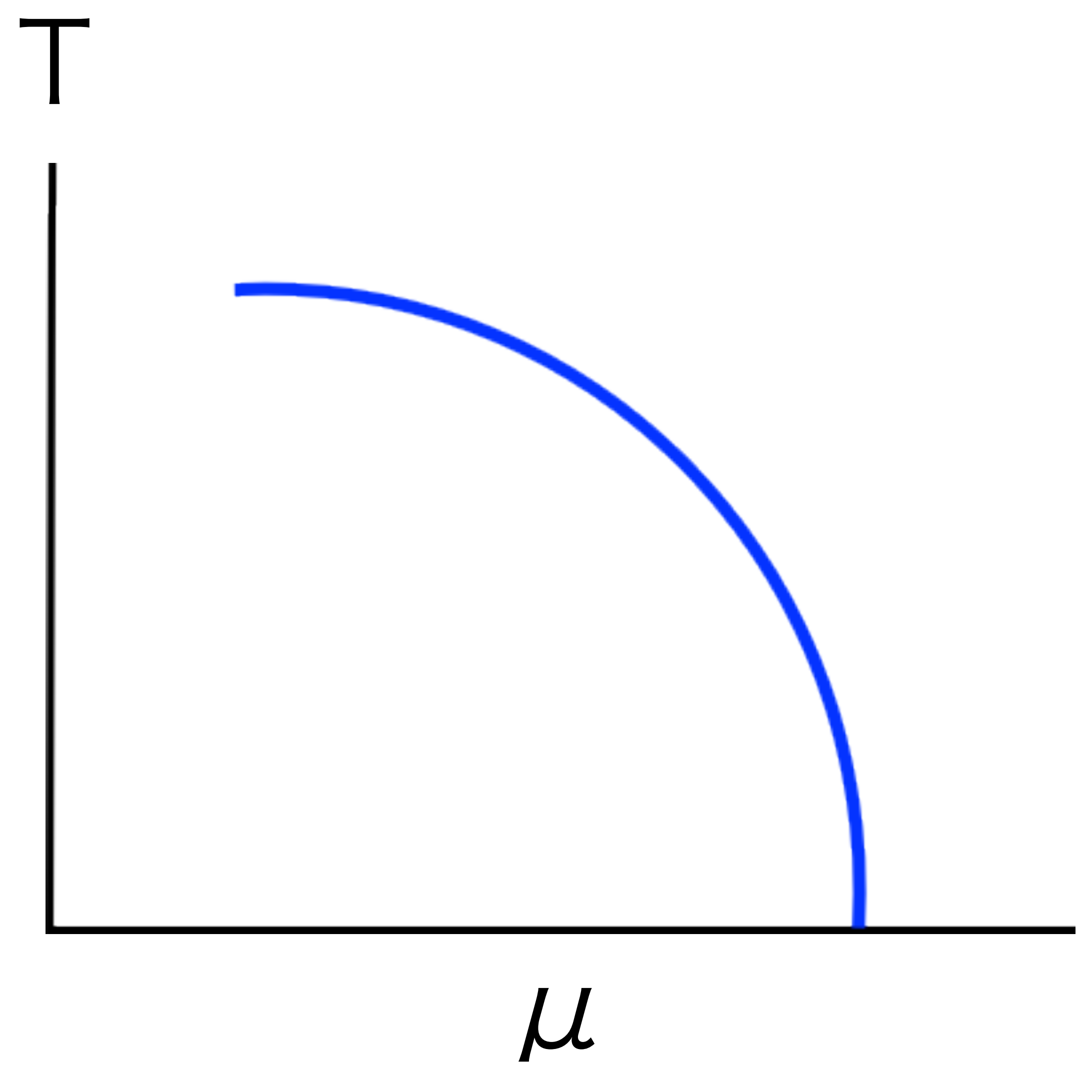

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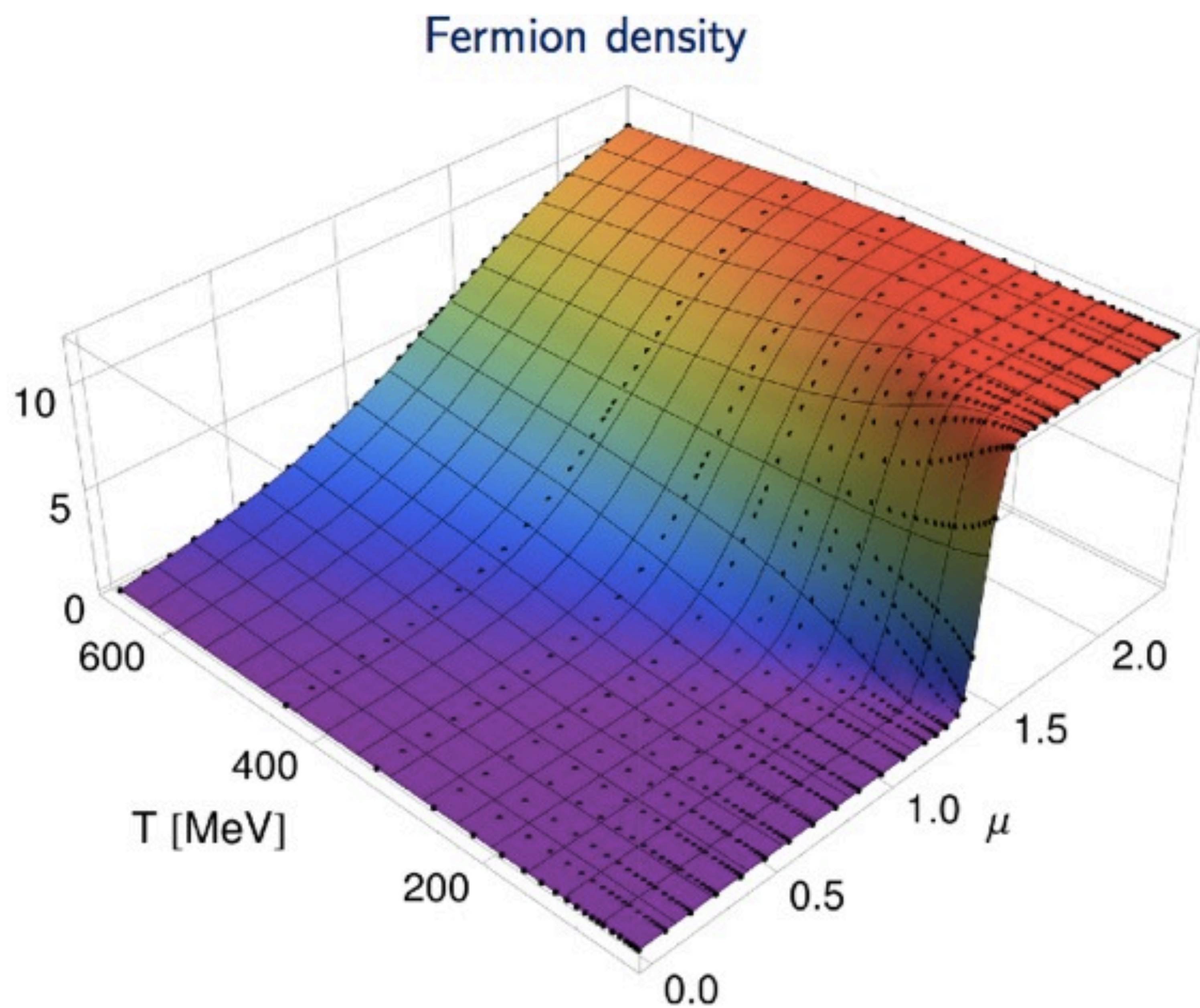

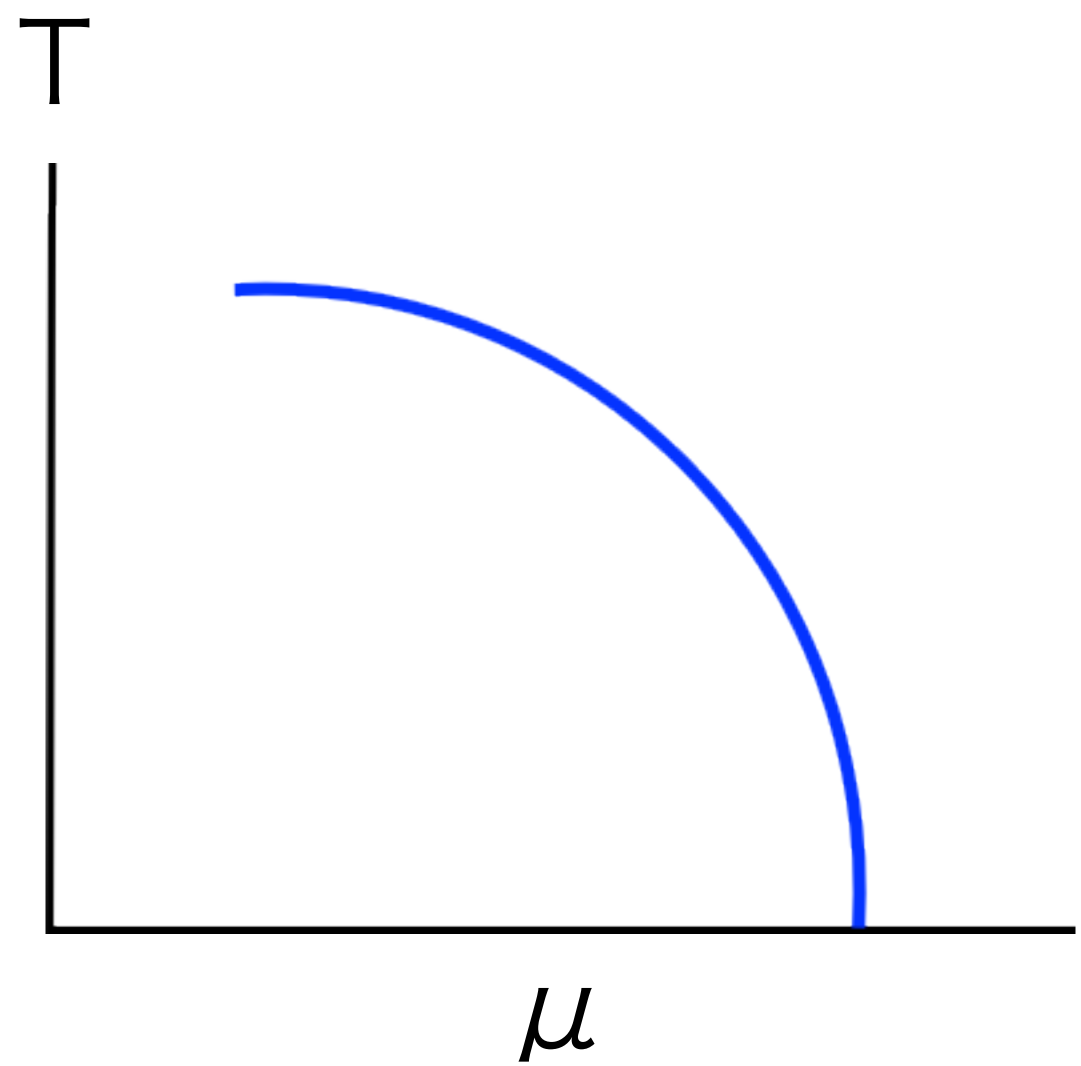

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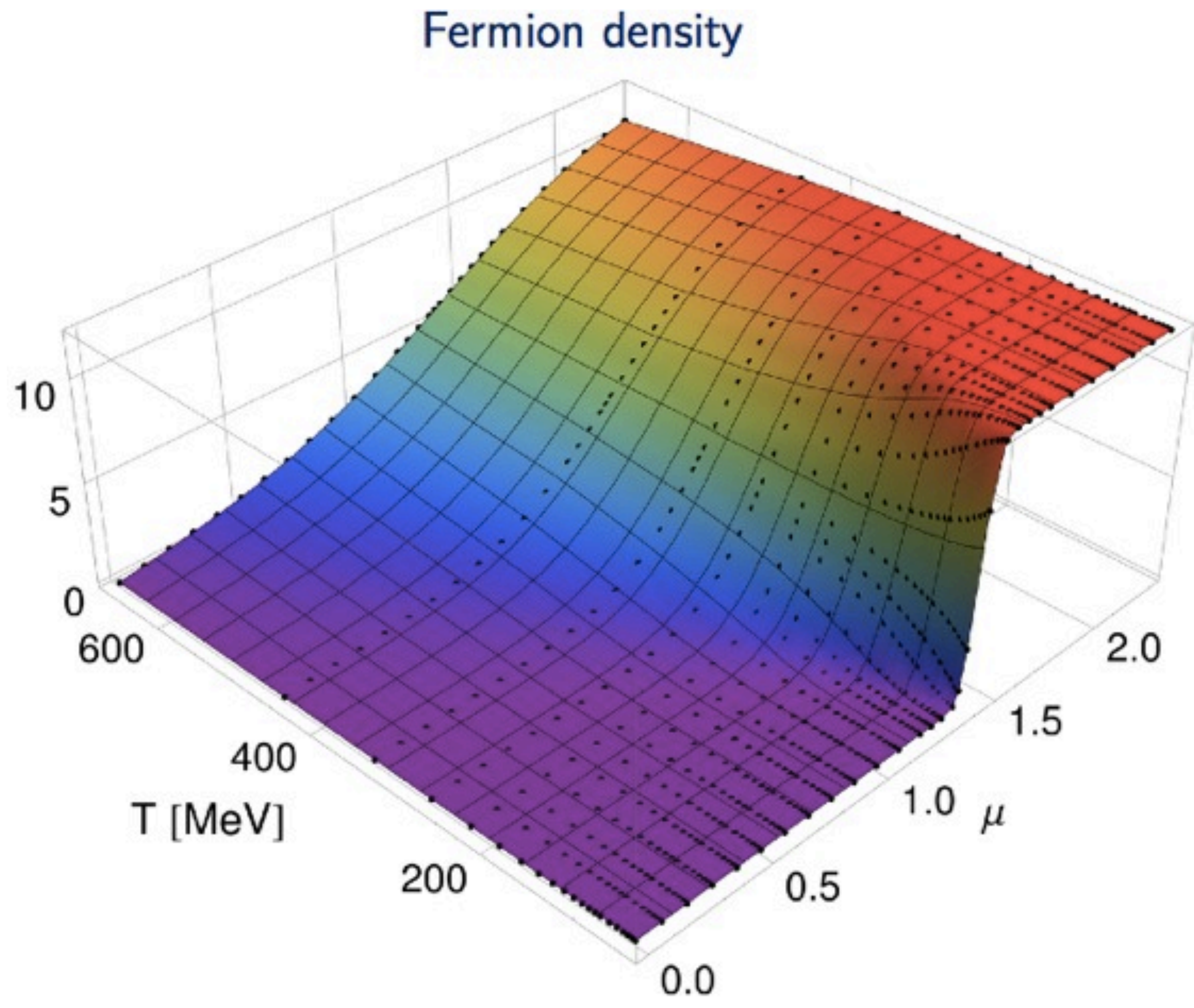

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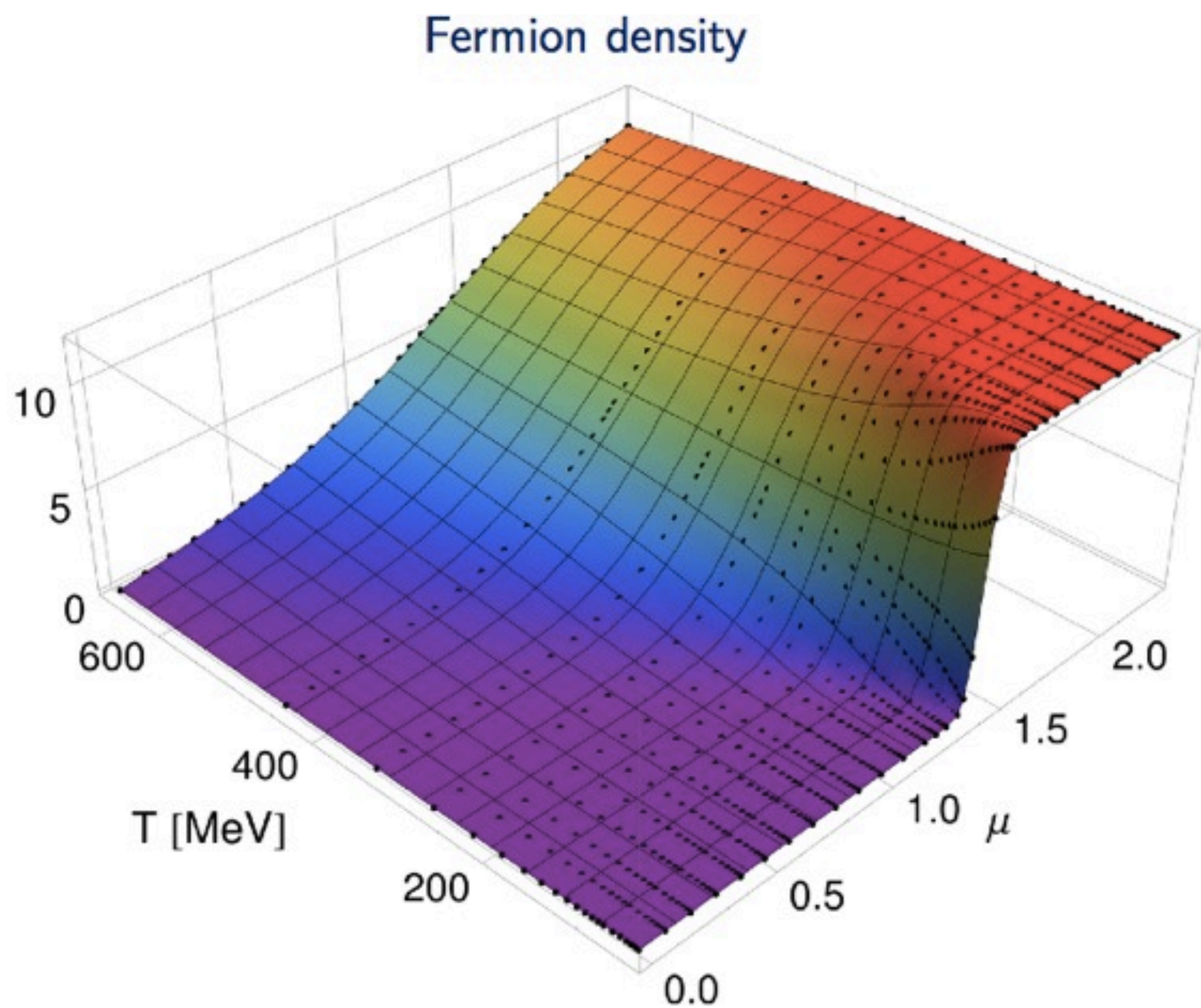

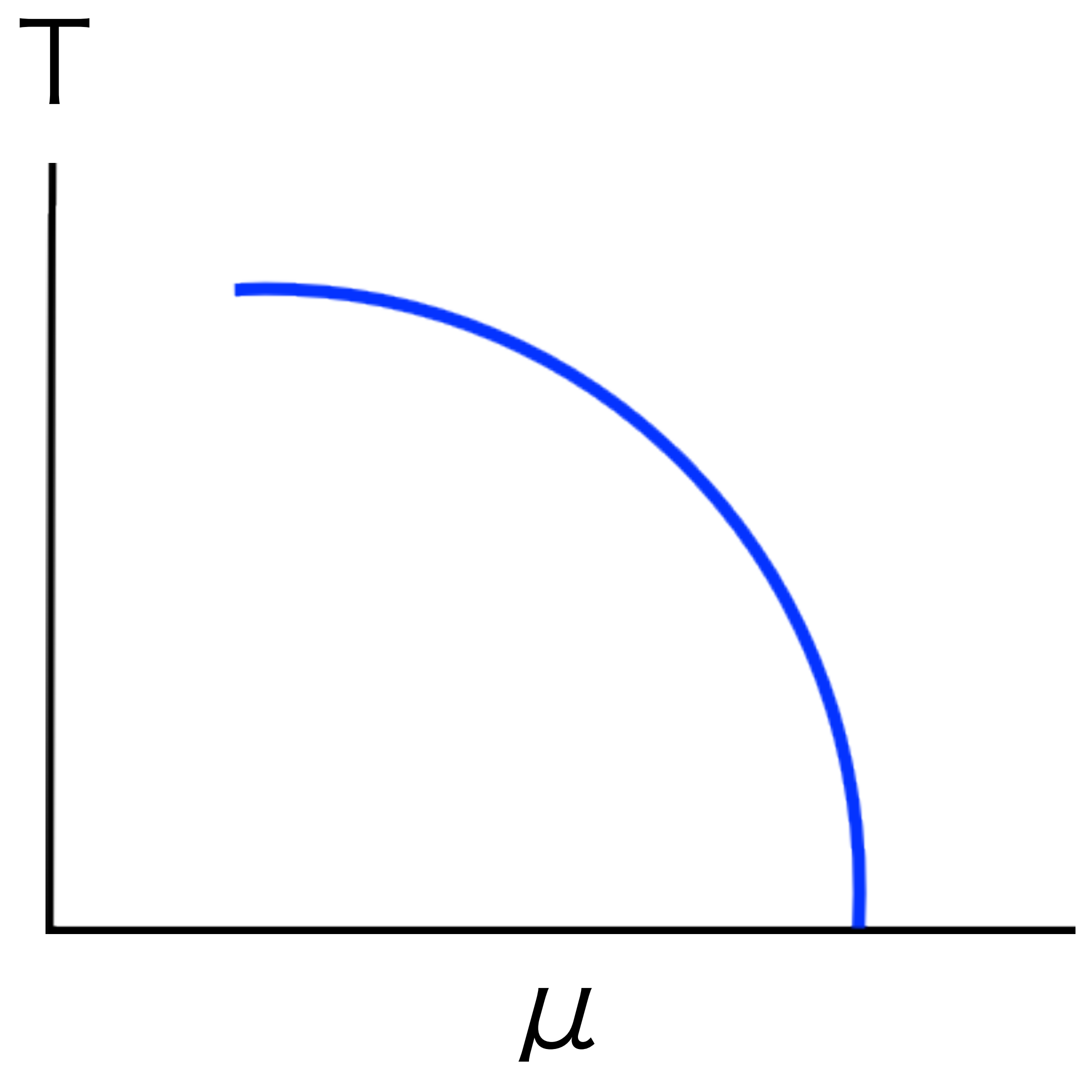

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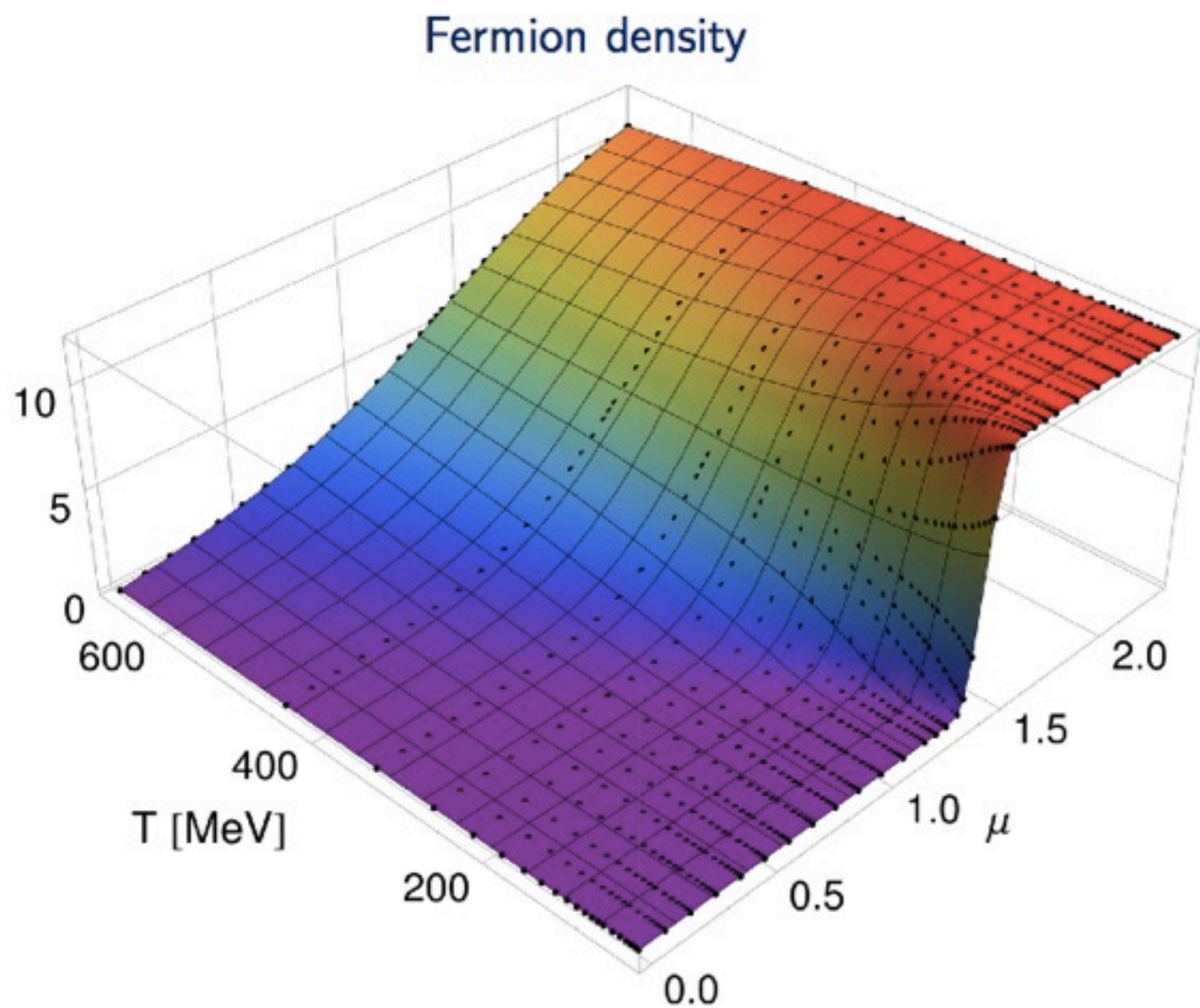

Heavy dense QCD

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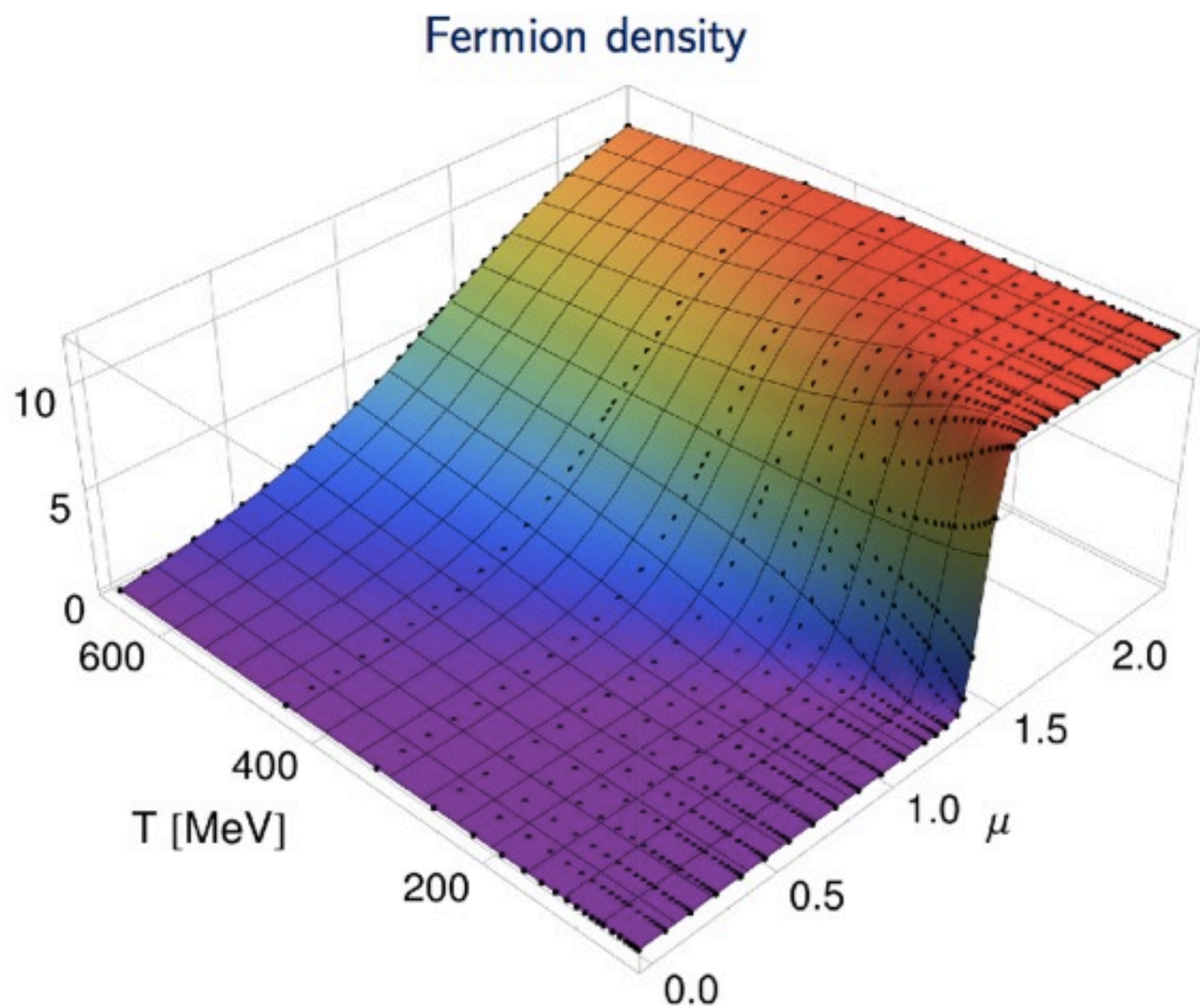

Heavy dense QCD

$\kappa \to 0, \ \mu \to \pm\infty$

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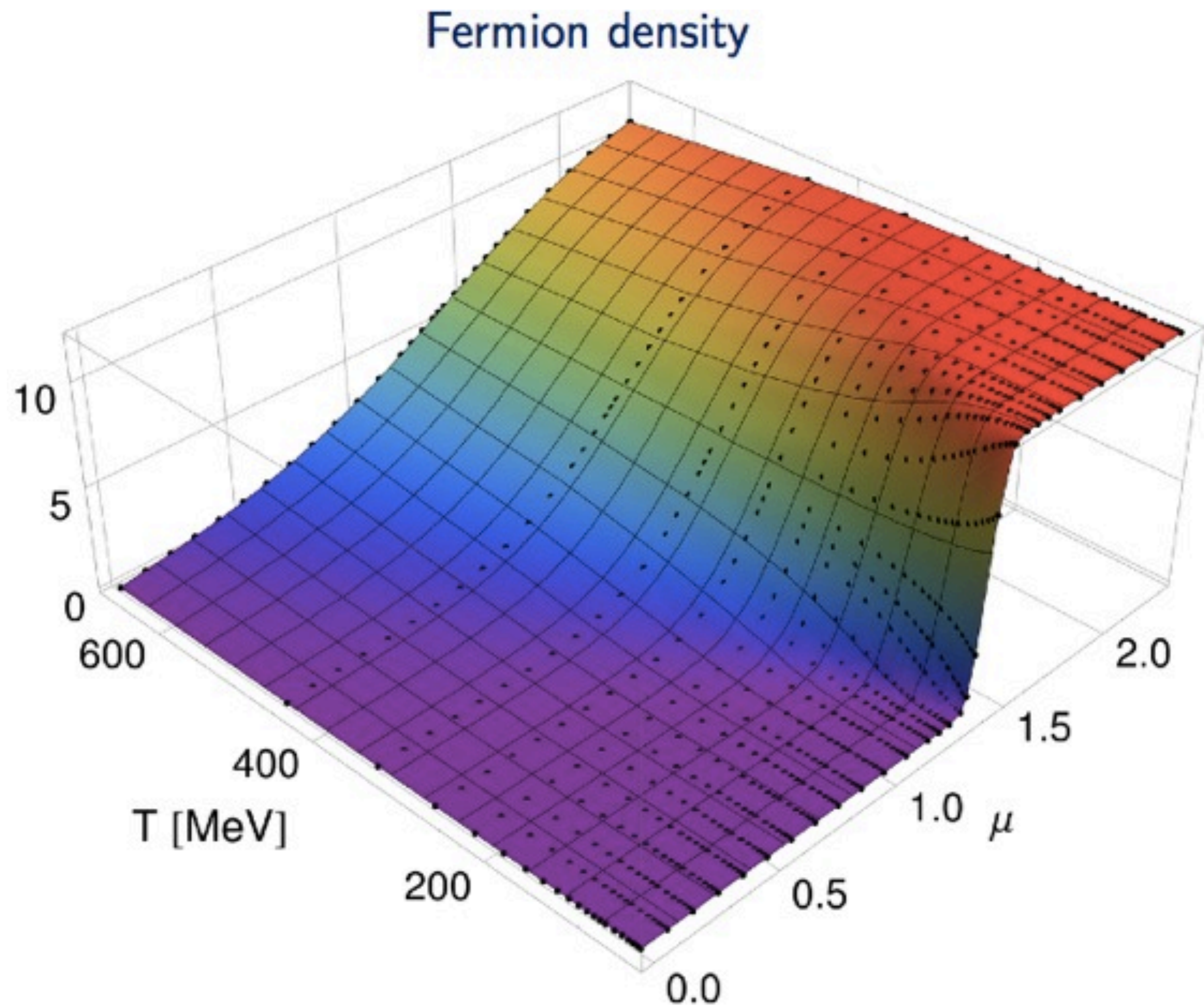

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$$\kappa \to 0,\ \mu \to \pm\infty$$

$$\kappa e^{\pm\mu a} = \mathrm{const}$$

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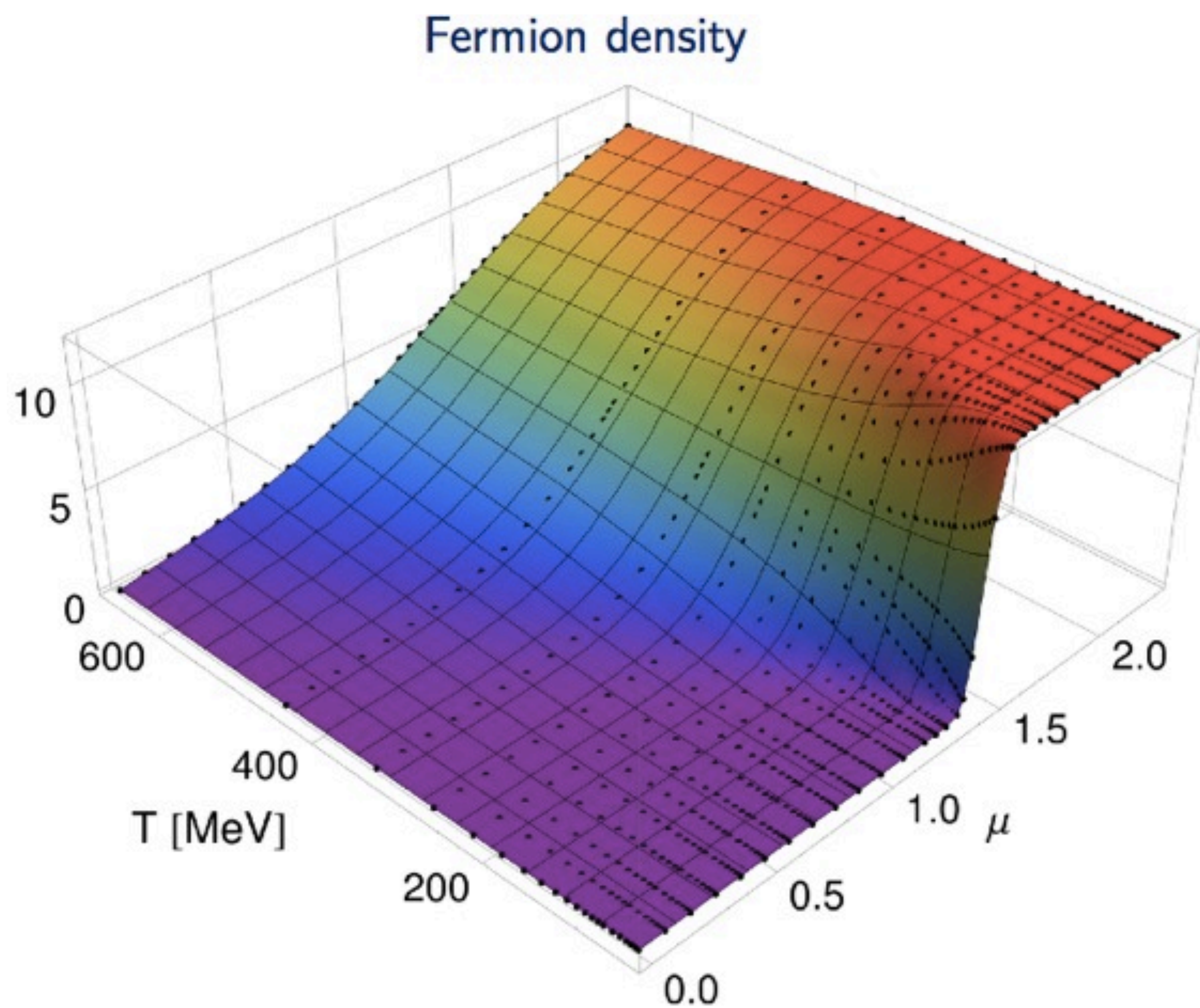

Heavy dense QCD

$$\kappa \to 0,\ \mu \to \pm\infty$$

$$\kappa e^{\pm\mu a} = \text{const}$$

$$\det D(\mu) = \prod_{\vec{x}} \det\left(1 + C\, P_{\vec{x}}\right)^2 \det\left(1 + C'\, P_{\vec{x}}^{-1}\right)^2$$

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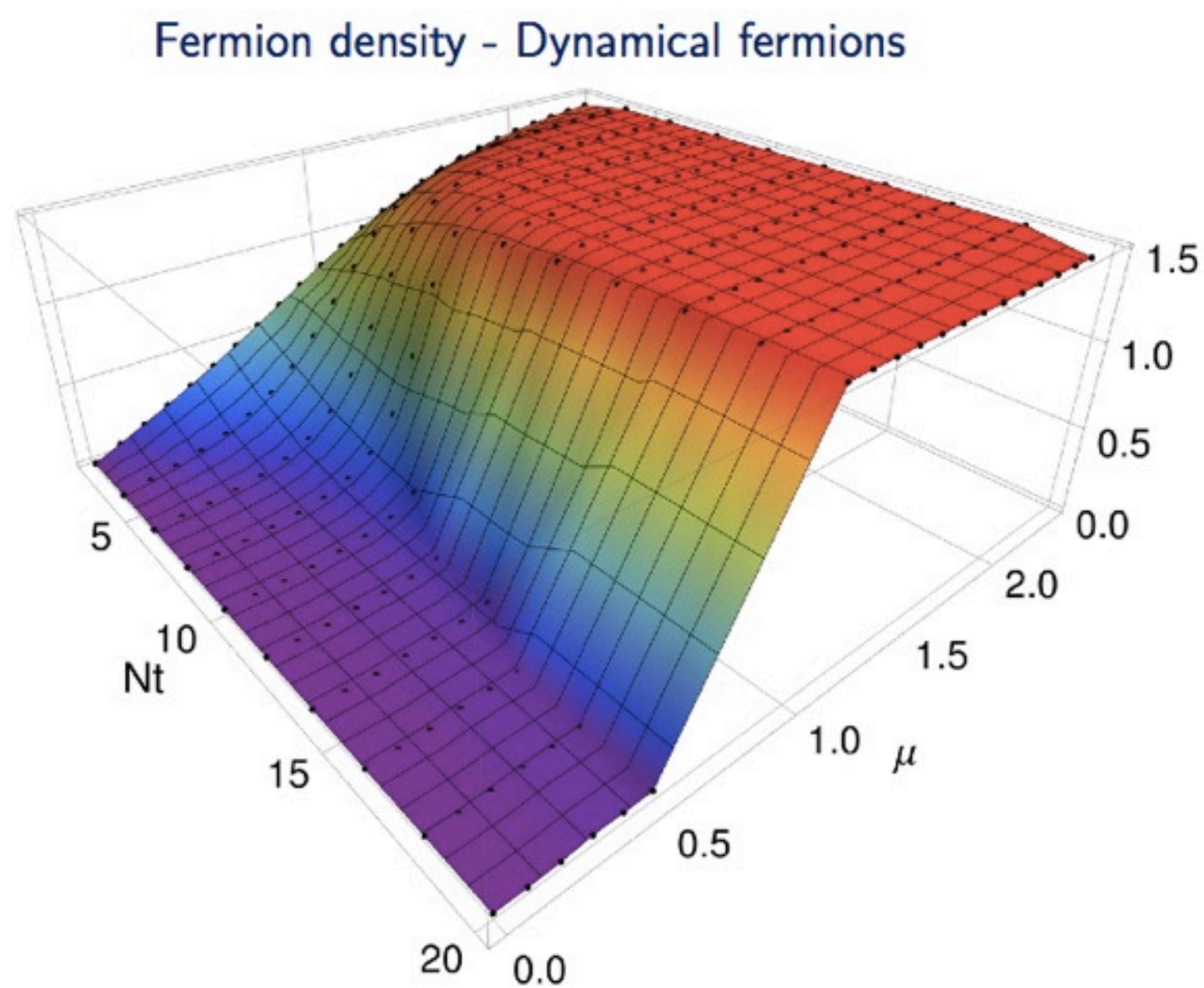

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Plenary talk, Fri 27/6 by Denes Sexty

New algorithms for finite density QCD

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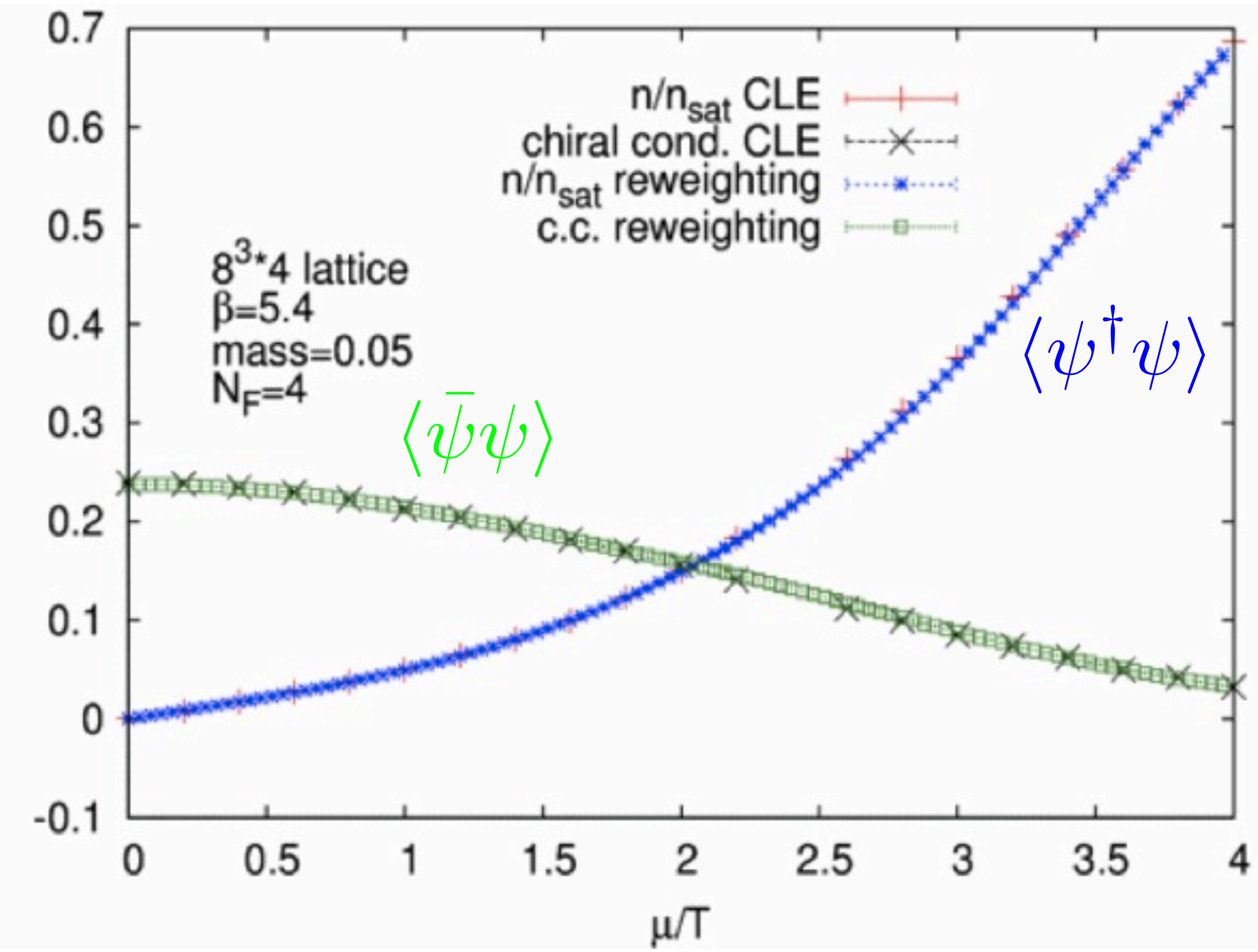

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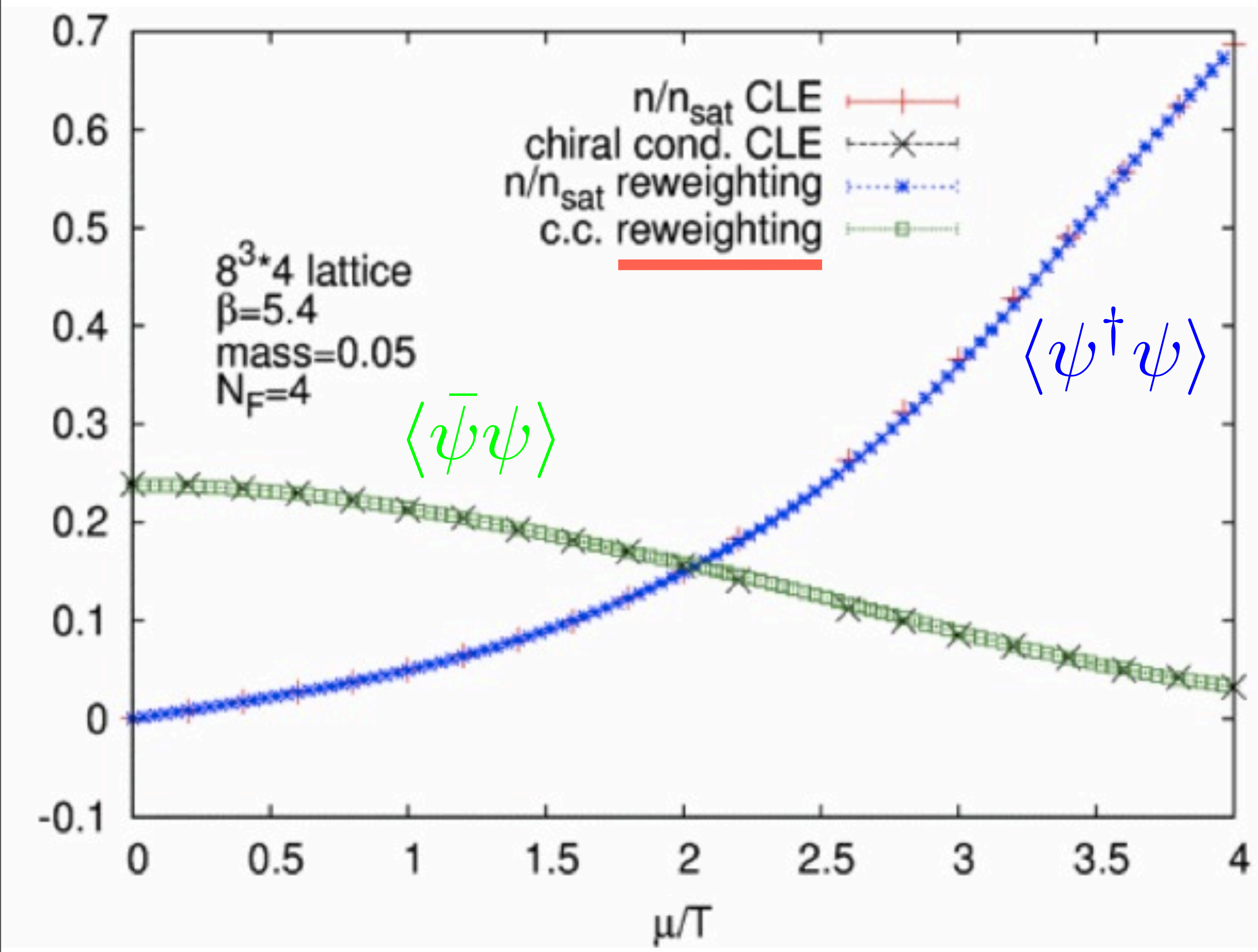

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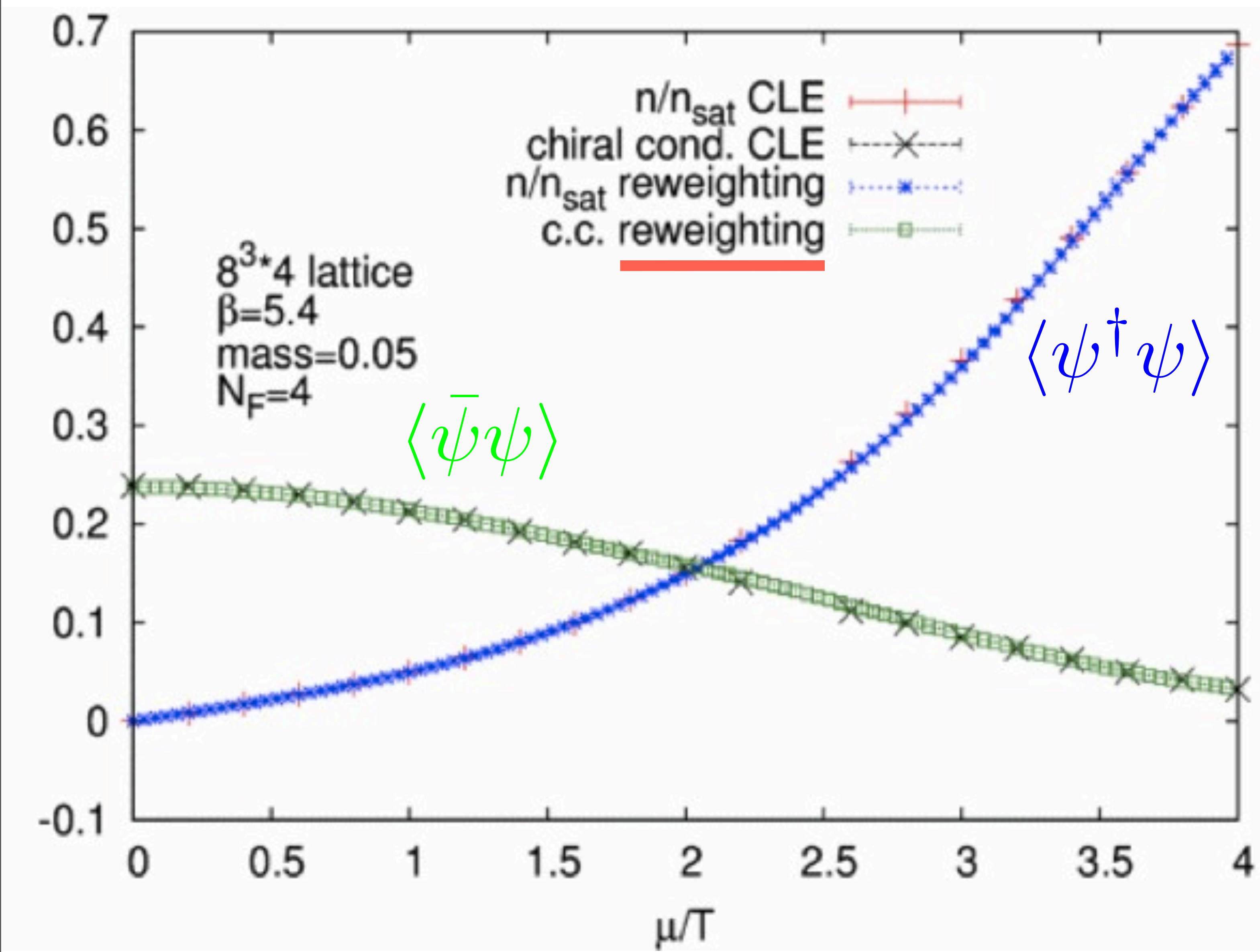

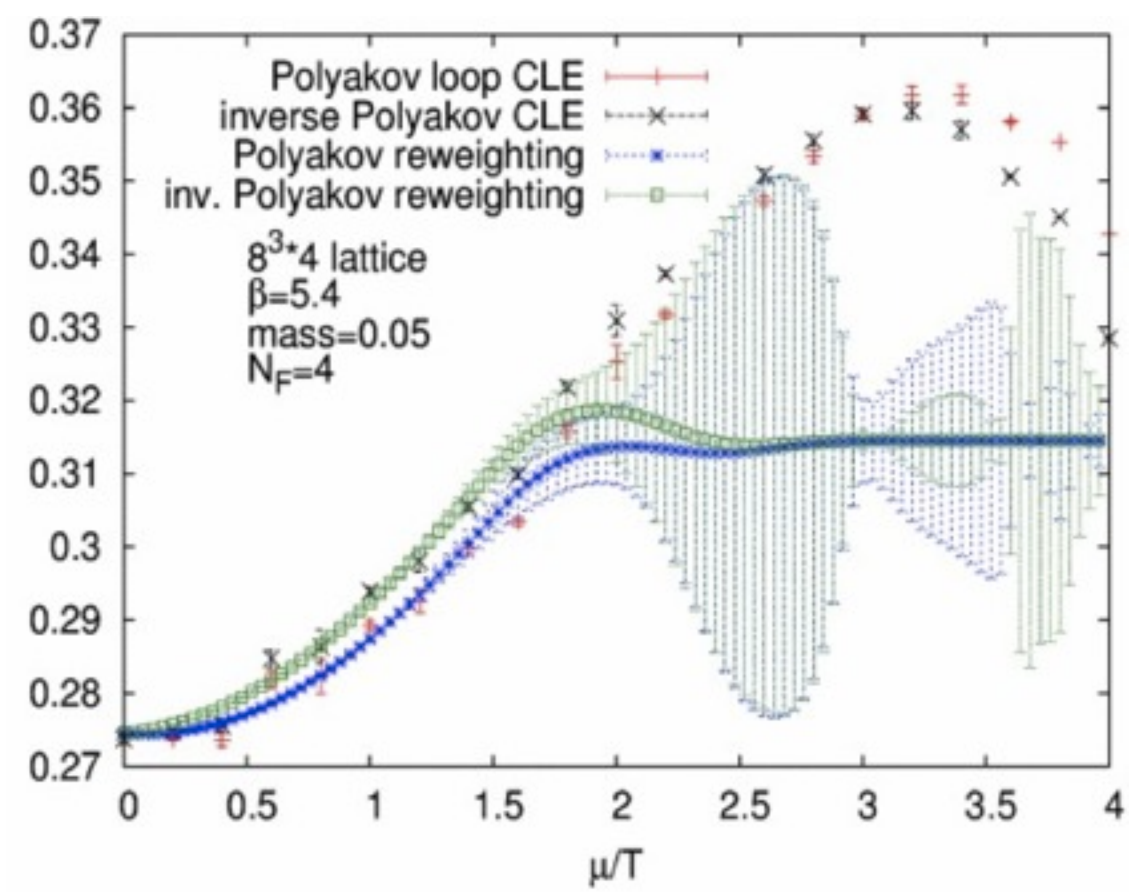

Langevin algorithm

(Montvay-Munster)

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Path integralの実行

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Path integralの実行

$$Z = \int D\phi e^{-S[\phi]}$$

Langevin algorithm

(Montvay-Munster)

Path integralの実行

$$Z = \int D\phi e^{-S[\phi]} = \int D\phi D\pi e^{-\frac{1}{2}\pi^2 - S[\phi]}$$

Langevin algorithm

(Montvay-Munster)

Path integralの実行

$$Z = \int D\phi e^{-S[\phi]} = \int D\phi D\pi e^{\underbrace{-\frac{1}{2}\pi^2 - S[\phi]}_{H[\pi,\phi]}}$$

Langevin algorithm

(Montvay-Munster)

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H[π, ϕ]

Micro canonical ensemble

Langevin algorithm

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決められたエネルギーで現れる配位を全て足す

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Langevin algorithm

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離散化

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離散化

$$\phi(\tau_{n+1}) = \phi(\tau_n) + (\tau_{n+1} - \tau_n)\,\pi(\tau_n) - \frac{1}{2}(\tau_{n+1} - \tau_n)^2 \frac{\partial S[\phi]}{\partial \phi}$$

Langevin algorithm

Langevin algorithm

Canonical ensemble

Langevin algorithm

Canonical ensemble

エネルギーの出入りを許す

Langevin algorithm

Canonical ensemble

エネルギーの出入りを許す $\Leftrightarrow$ π をGaussianで振る

Langevin algorithm

Canonical ensemble

エネルギーの出入りを許す ⟺ π をGaussianで振る

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↓

Langevin algorithm

Canonical ensemble

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$$\Downarrow$$

$$\phi' = \phi - \epsilon \frac{\partial S[\phi]}{\partial \phi} - \sqrt{\epsilon}\eta$$ Langevin equation

Langevin algorithm

Canonical ensemble

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ϕ の出現確率：P[ϕ]

Langevin algorithm

Canonical ensemble

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ϕ の出現確率：P[ϕ] ← Fokker-Planck eq.

Langevin algorithm

Canonical ensemble

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$$P'[\phi'] = \left\langle \int D\phi P[\phi] \delta\left(\phi' - \phi + \epsilon \frac{\partial S[\phi]}{\partial \phi} + \sqrt{\epsilon}\eta\right)\right\rangle_\eta$$

Langevin algorithm

Canonical ensemble

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→ $$\frac{\partial P[\phi]}{\partial \tau} = \frac{\partial}{\partial \phi}\left(\frac{\partial P[\phi]}{\partial \phi} + \frac{\partial S[\phi]}{\partial \phi}P[\phi]\right) + \mathcal{O}(\epsilon)$$

Langevin algorithm

Fokker-Planck equation

$$\frac{\partial P[\phi]}{\partial \tau} = \frac{\partial}{\partial \phi}\left(\frac{\partial P[\phi]}{\partial \phi} + \frac{\partial S[\phi]}{\partial \phi}P[\phi]\right) = L^T P[\phi]$$

Langevin algorithm

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一般解 $P[\phi](\tau) \propto e^{L^T \tau}$

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If L is negative semi-definite.

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If L is negative semi-definite.

$$P[\phi](\tau \to \infty) \to e^{-S[\phi]}$$

Complex Langevin

(Parisi)

Complex action S_c

Complex Langevin

(Parisi)

Complex action S_c

Langevin equation $$\phi' = \phi - \epsilon \frac{\partial S_c[\phi]}{\partial \phi} - \sqrt{\epsilon}\eta$$

Complex Langevin

(Parisi)

Complex action Sc

Langevin equation $\phi' = \phi - \epsilon \frac{\partial S_c[\phi]}{\partial \phi} - \sqrt{\epsilon}\eta$

複素数になる

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複素数になる

場の量も複素化する $\phi \to \phi_c = x + iy$

Complex Langevin

(Parisi)

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複素数になる

場の量も複素化する $\phi \to \phi_c = x + iy$

Complex Langevin equation

$$\frac{dx}{dt} = K_x + \sqrt{N_R}\eta_R, \qquad \frac{dy}{dt} = K_y + \sqrt{N_I}\eta_I$$

$$K_x = \mathrm{Re}\left.\frac{dS(x)}{dx}\right|_{x\to x+iy}, \qquad K_y = \mathrm{Im}\left.\frac{dS(x)}{dx}\right|_{x\to x+iy}$$

Complex Langevin

Fokker-Planck equation

$$\frac{\partial}{\partial t}P(x,y;t) = L^T P(x,y;t)$$

$$L^T = \nabla_x[N_\mathrm{R}\nabla_x - K_x] + \nabla_y[N_\mathrm{I}\nabla_y - K_y]$$

Complex Langevin

Fokker-Planck equation

$$\frac{\partial}{\partial t}P(x,y;t) = L^T P(x,y;t)$$ ← 出現確率（実数）

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名状し難き方程式

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complex Fokker-Planck equation?

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名状し難き方程式

complex Fokker-Planck equation?

$$\frac{\partial}{\partial t}\rho(x;t) = L_0^T\rho(x;t) \qquad L_0^T = \nabla_x[\nabla_x + \nabla_x S(x)]$$

Complex Langevin

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密度（複素数）

Complex Langevin

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密度（複素数）

定常解として $\rho(x) \propto e^{-S_c[x]}$ を持つ

Complex Langevin

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→ 数値計算可能な時には確かめられている

Complex Langevin

Parisiの期待

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$$\langle O \rangle_{P(t)} = \frac{\int O(x+iy)P(x,y;t)dxdy}{\int P(x,y;t)dxdy}$$

Complex Langevin

Parisiの期待

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complex Langevinによる期待値

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complex Langevinによる期待値

名状し難き方程式による期待値

complex actionによる期待値

Complex Langevin

Parisiの期待

$$\langle O\rangle_{P(t)} = \frac{\int O(x+iy)P(x,y;t)dxdy}{\int P(x,y;t)dxdy} \qquad \langle O\rangle_{\rho(t)} = \frac{\int O(x)\rho(x;t)dx}{\int \rho(x;t)dx}$$

$$\langle O\rangle_{P(t)} = \underline{\langle O\rangle_{\rho(t)} \to \langle O\rangle}$$

期待できそう

complex Langevinによる期待値

名状し難き方程式による期待値

complex actionによる期待値

Complex Langevin

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non-trivialな部分 期待できそう

complex Langevinによる期待値

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Complex Langevin

Criteria $\langle O \rangle_{P(t)} = \langle O \rangle_{\rho(t)}$? (Aarts, James, Seiler, Stamatescu)

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t-dependent operator

$$\frac{\partial}{\partial t} O(z;t) = \tilde{L} O(z;t) \qquad \tilde{L} = [\nabla_z - (\nabla_z S(z))]\nabla_z$$

一般解 $O(z;t) = \exp[t\tilde{L}]O(z)$

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定義

$$F(t,\tau) = \int P(x,y;t-\tau) O(x+iy;\tau) dxdy$$

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$$\frac{\partial}{\partial \tau} F(t,\tau) = 0 \quad \rightarrow \quad \langle O \rangle_{P(t)} = \langle O \rangle_{\rho(t)}$$

Complex Langevin

$$\frac{\partial}{\partial \tau}F(t,\tau) = -\int \left(L^T P(x,y;t-\tau)\right) O(x+iy;\tau)dxdy + \int P(x,y;t-\tau)LO(x+iy;\tau)dxdy$$

Complex Langevin

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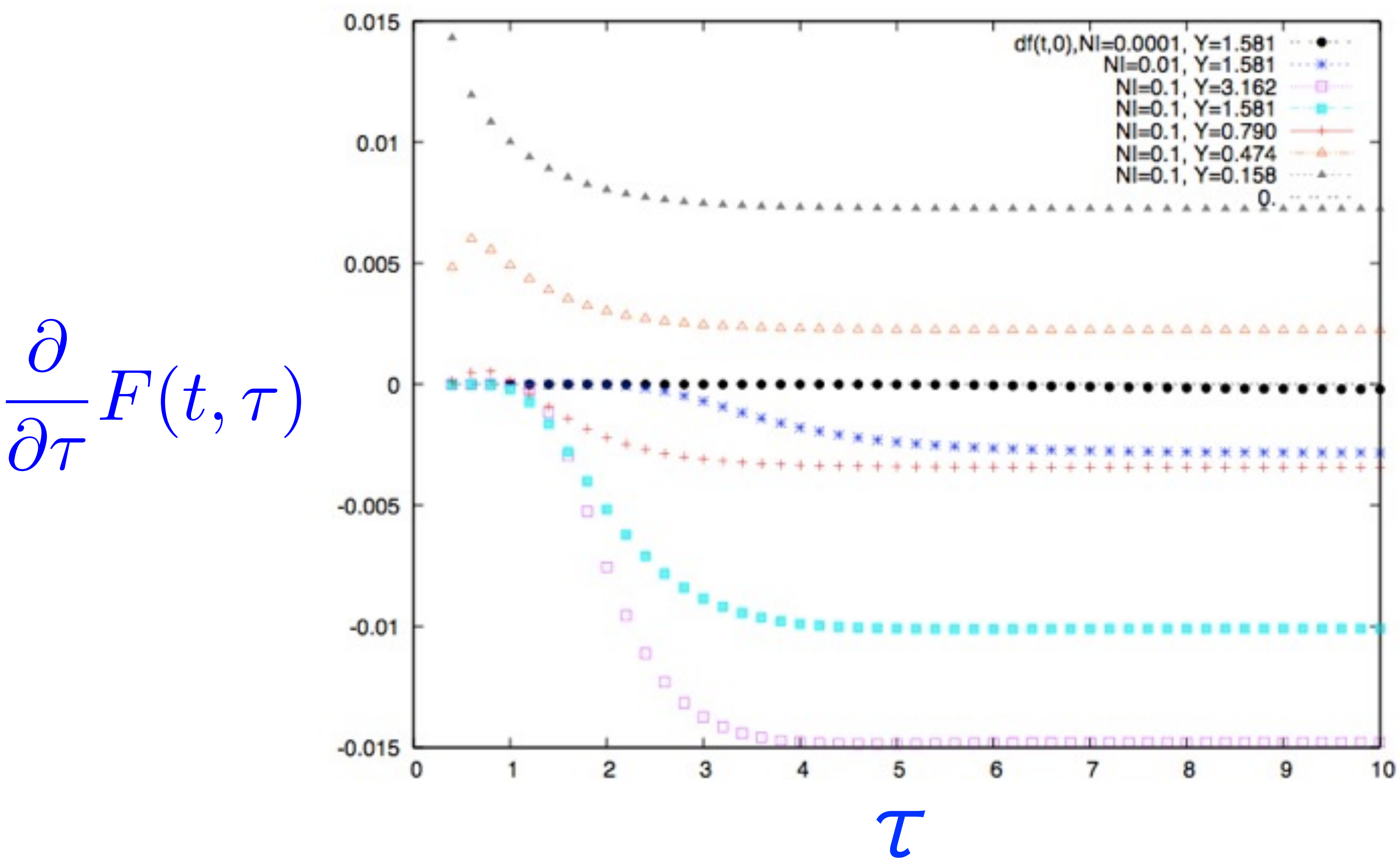
U(1) one link model



Complex Langevin

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weaker condition

$$\lim_{t\to\infty} \frac{d}{d\tau}F(t,\tau)\Big|_{\tau=0} = 0$$

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Criterion

Complex Langevin

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weaker condition

$$\lim_{t\to\infty} \frac{d}{d\tau}F(t,\tau)\bigg|_{\tau=0} = 0$$

定常解になり０

Criterion

$$E_O \equiv \int P(x,y;\infty)LO(x+iy,0)dxdy = \langle LO \rangle = 0$$

Complex Langevin

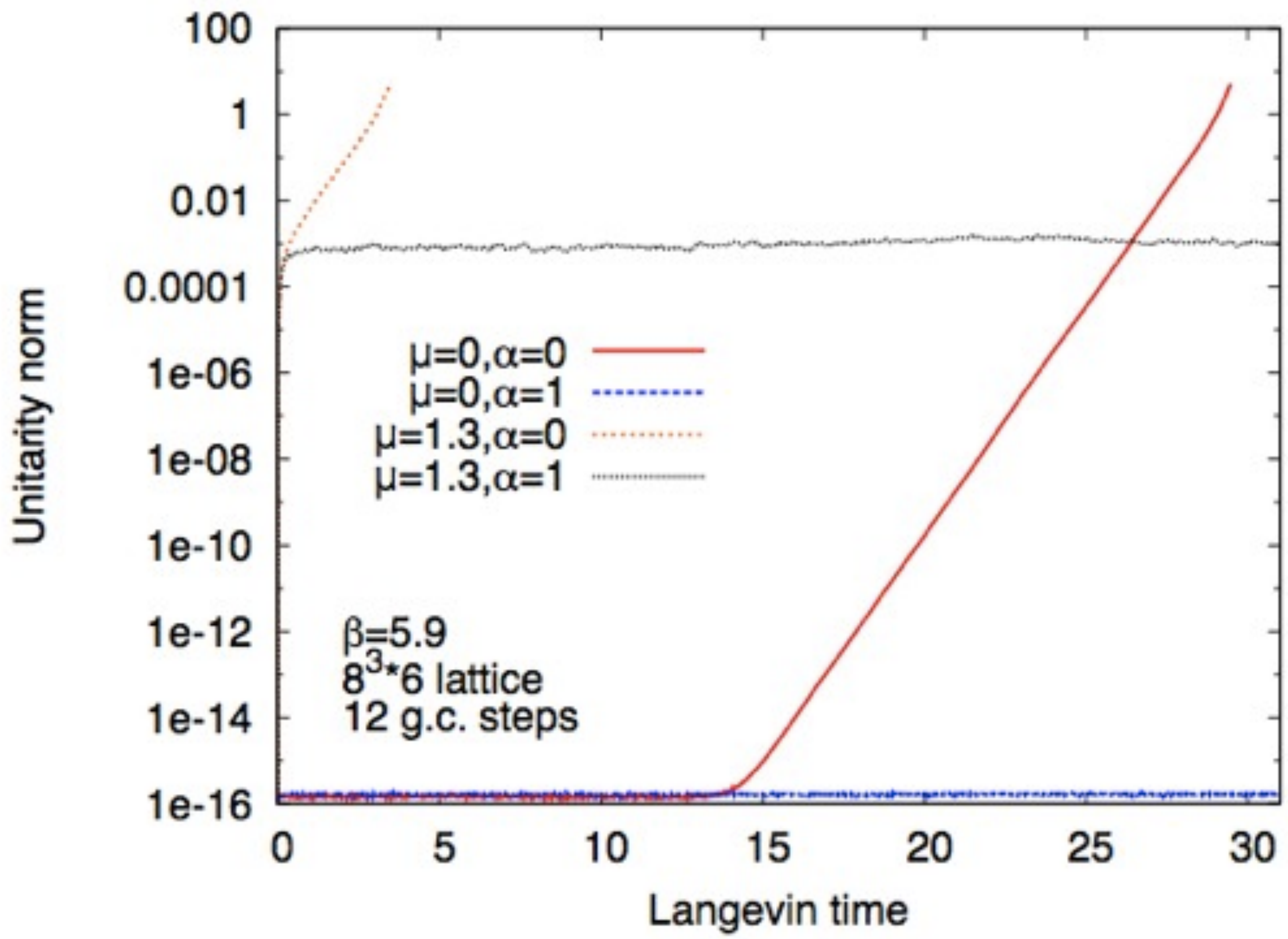
問題点

(Aarts, James, Seiler, Stamatescu)

Complex Langevin

問題点

Runaway problem

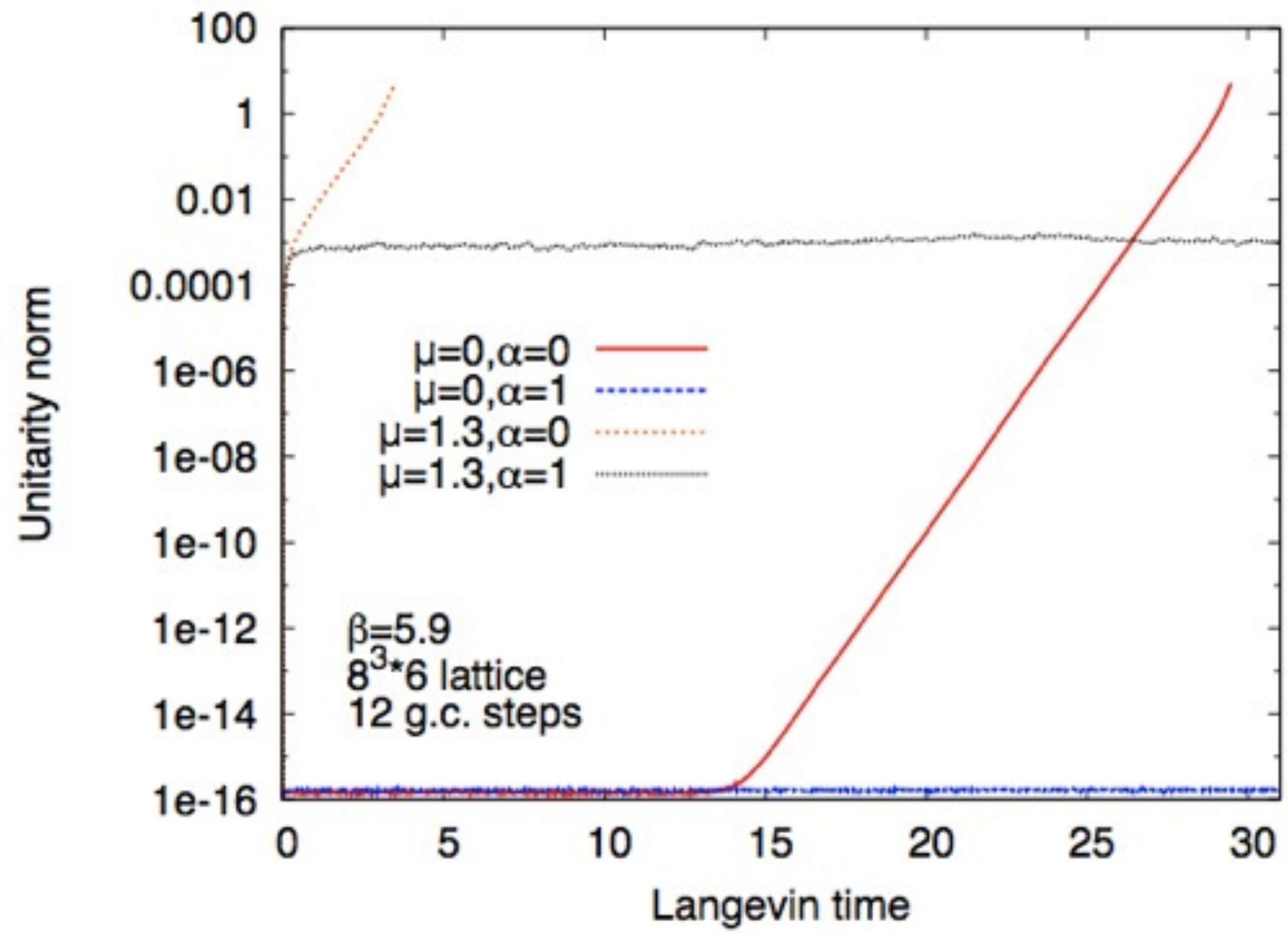


(Aarts, James, Seiler, Stamatescu)

Complex Langevin

問題点

Runaway problem



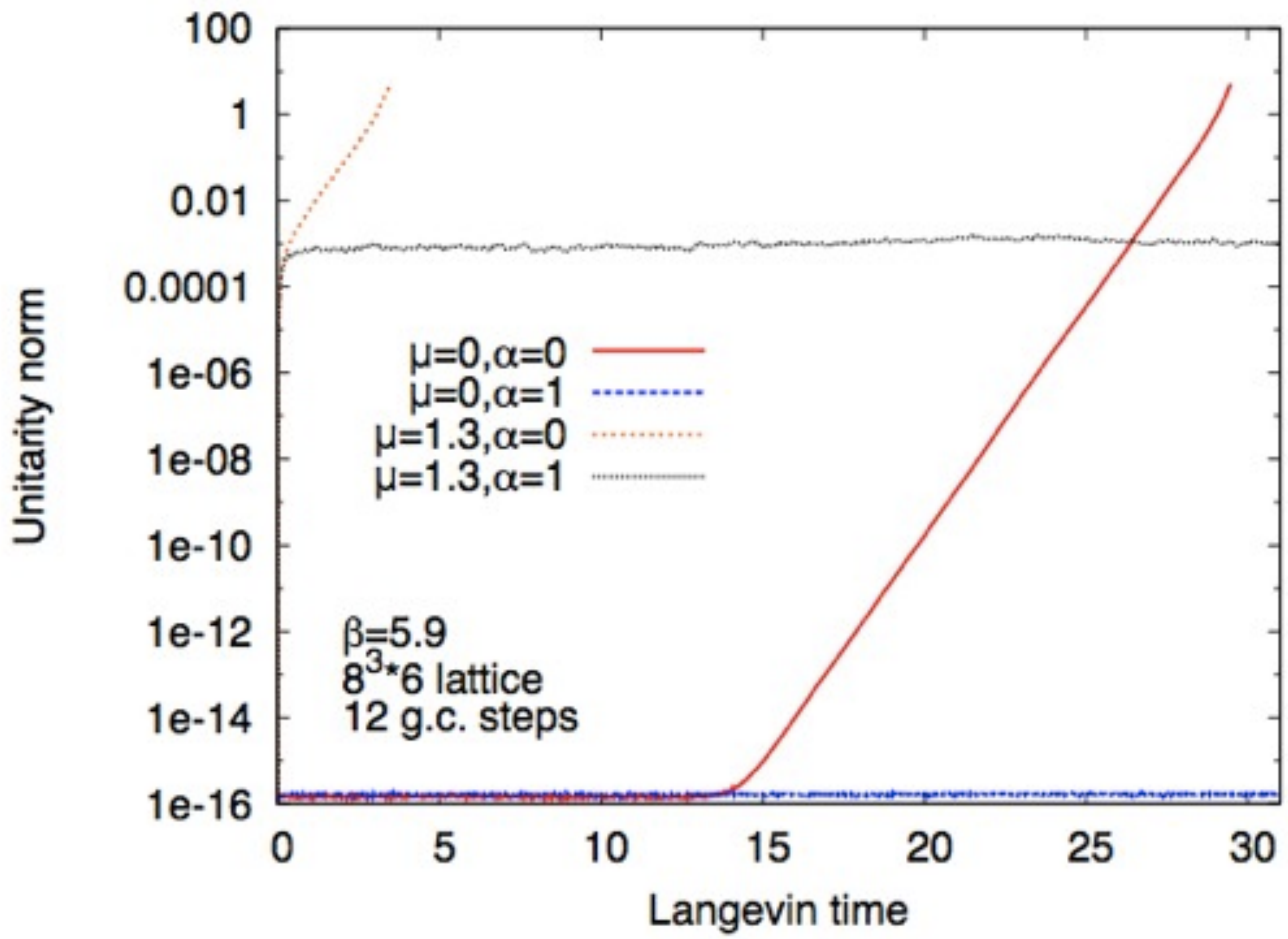
離散化Langevin方程式のstep sizeが大きすぎる

(Aarts, James, Seiler, Stamatescu)

Complex Langevin

問題点

Runaway problem



離散化Langevin方程式のstep sizeが大きすぎる

→ adaptive step size

(Aarts, James, Seiler, Stamatescu)

Complex Langevin

問題点

Complex Langevin

問題点

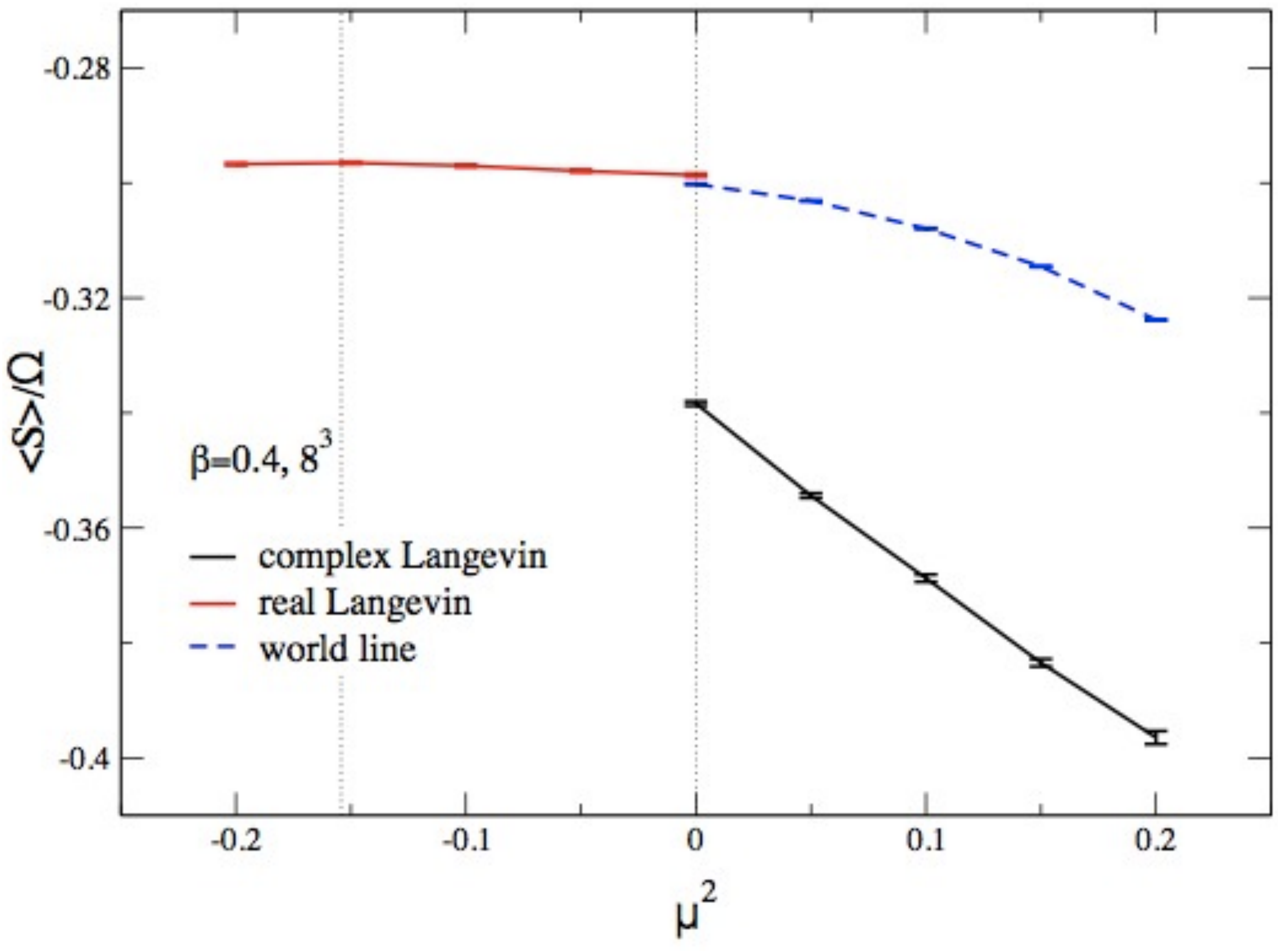
Convergence to the wrong limit

Complex Langevin

問題点

Convergence to the wrong limit

3D XY model

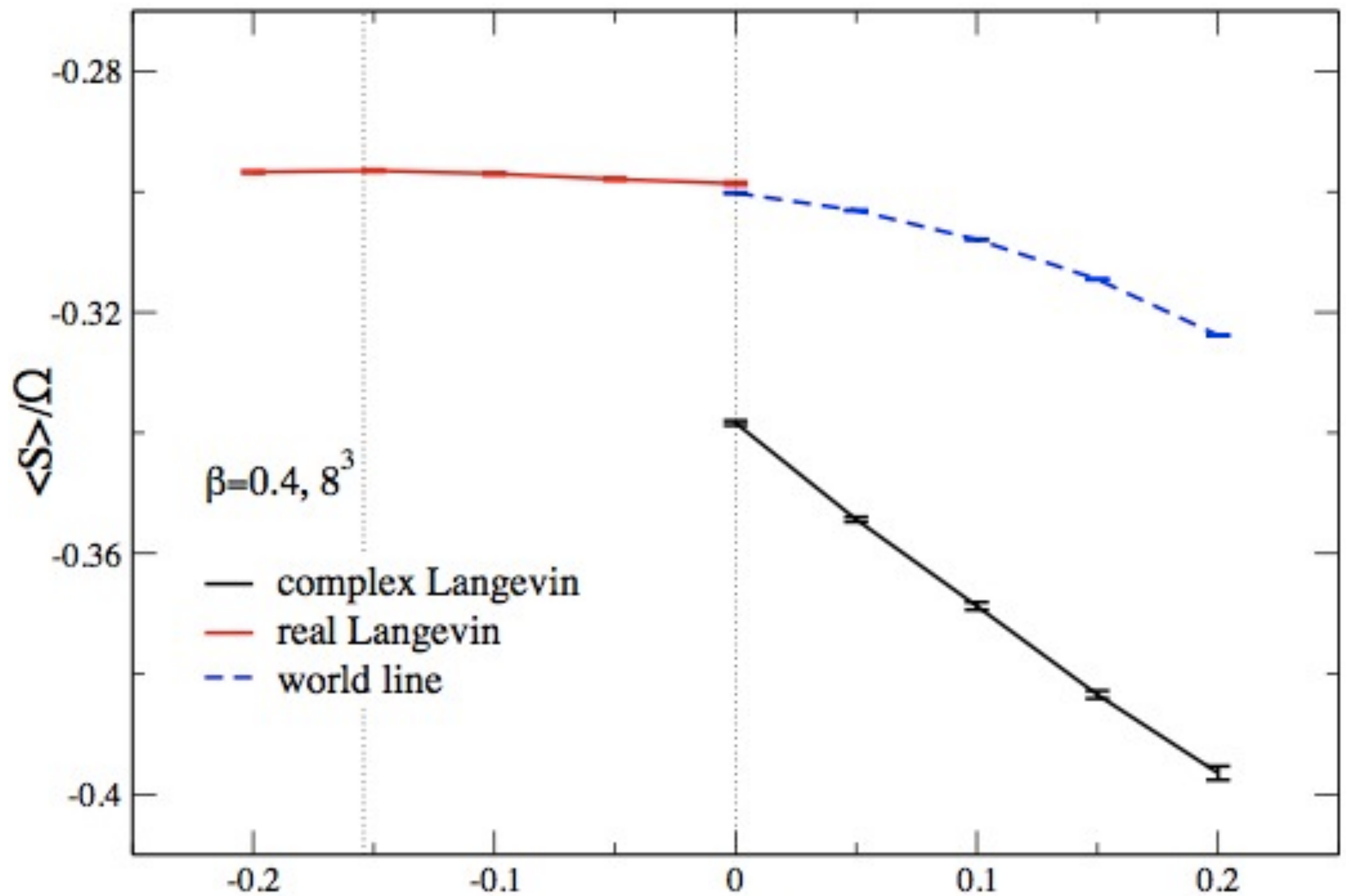


Complex Langevin

問題点

Convergence to the wrong limit

3D XY model



imaginary chemical

Complex Langevin

問題点

Convergence to the wrong limit

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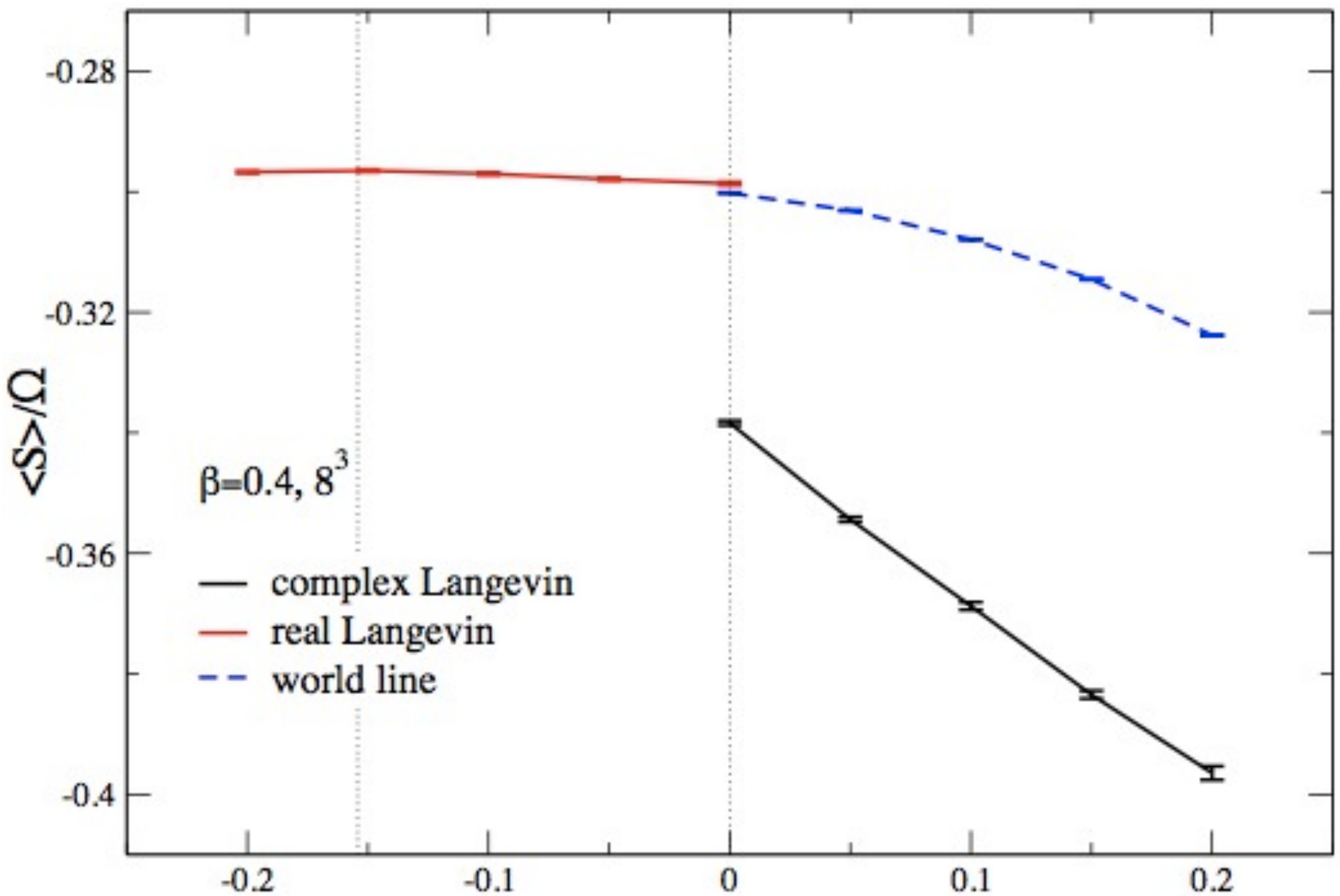


imaginary chemical μ^2 real chemical

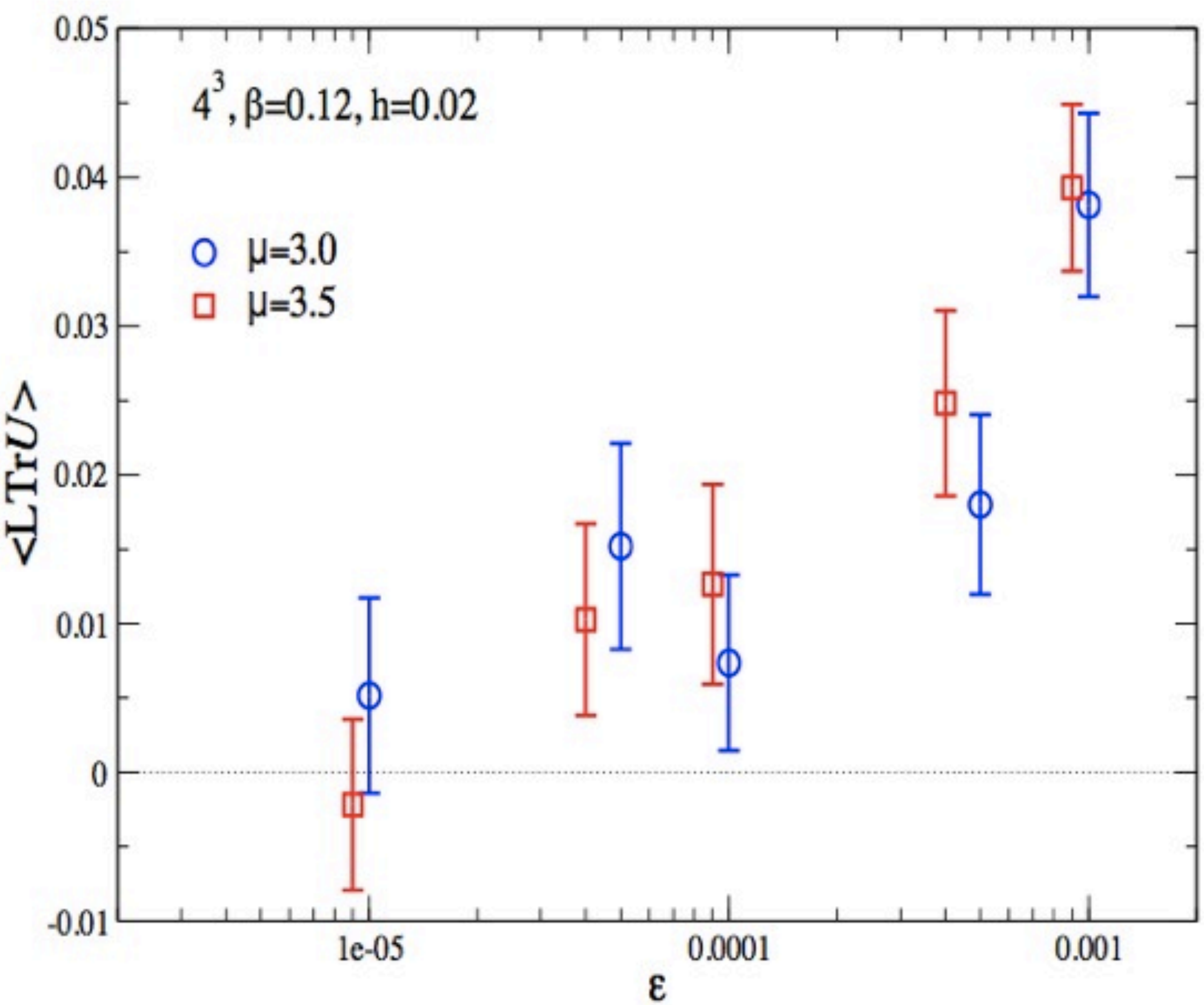
Complex Langevin

Criterionを見てみると

Complex Langevin

Criterionを見てみると

3D SU(3) spin model

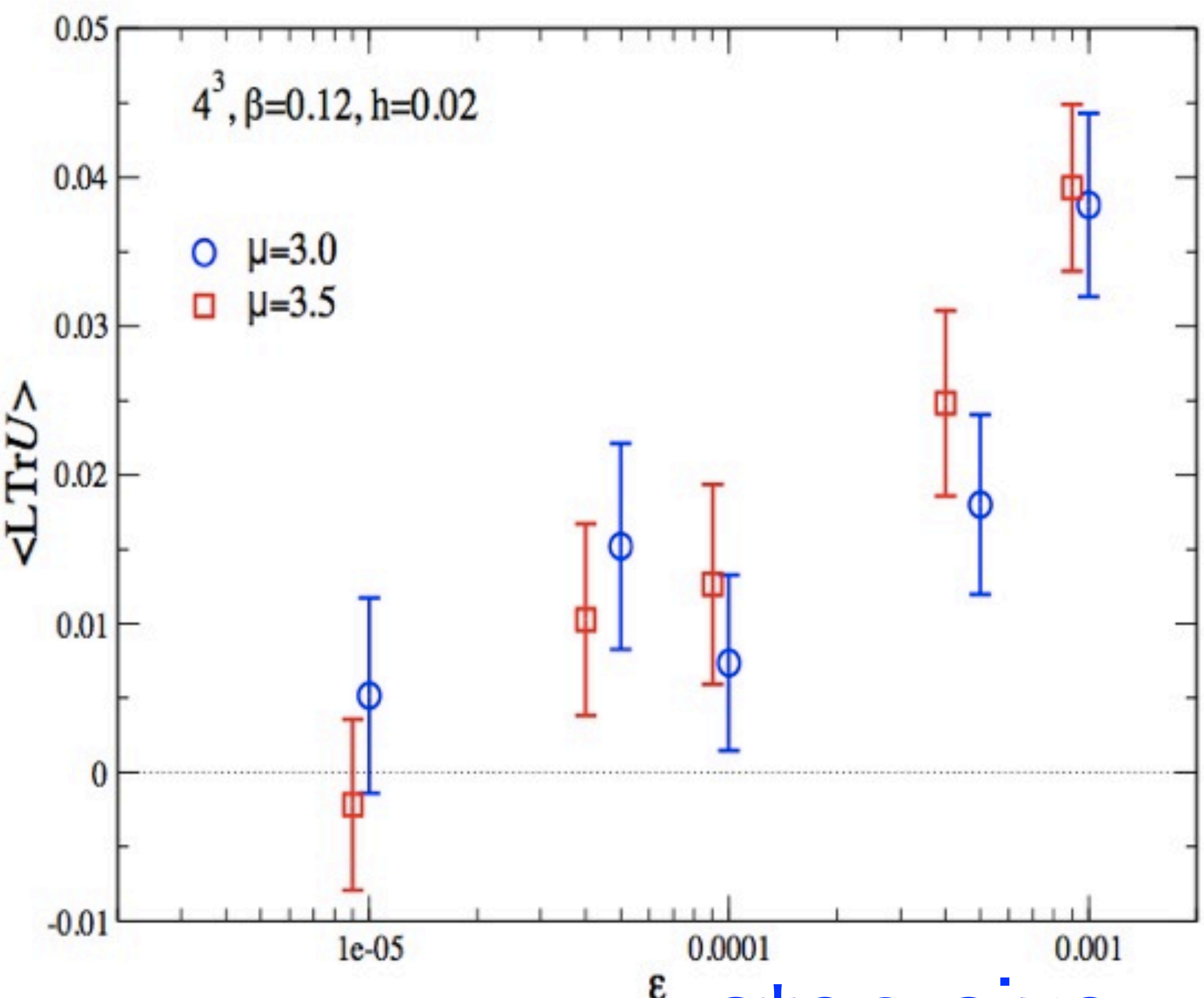


Complex Langevin

Criterionを見てみると

3D SU(3) spin model

$\langle L\mathrm{Tr}U \rangle$



step size

Complex Langevin

Criterionを見てみると

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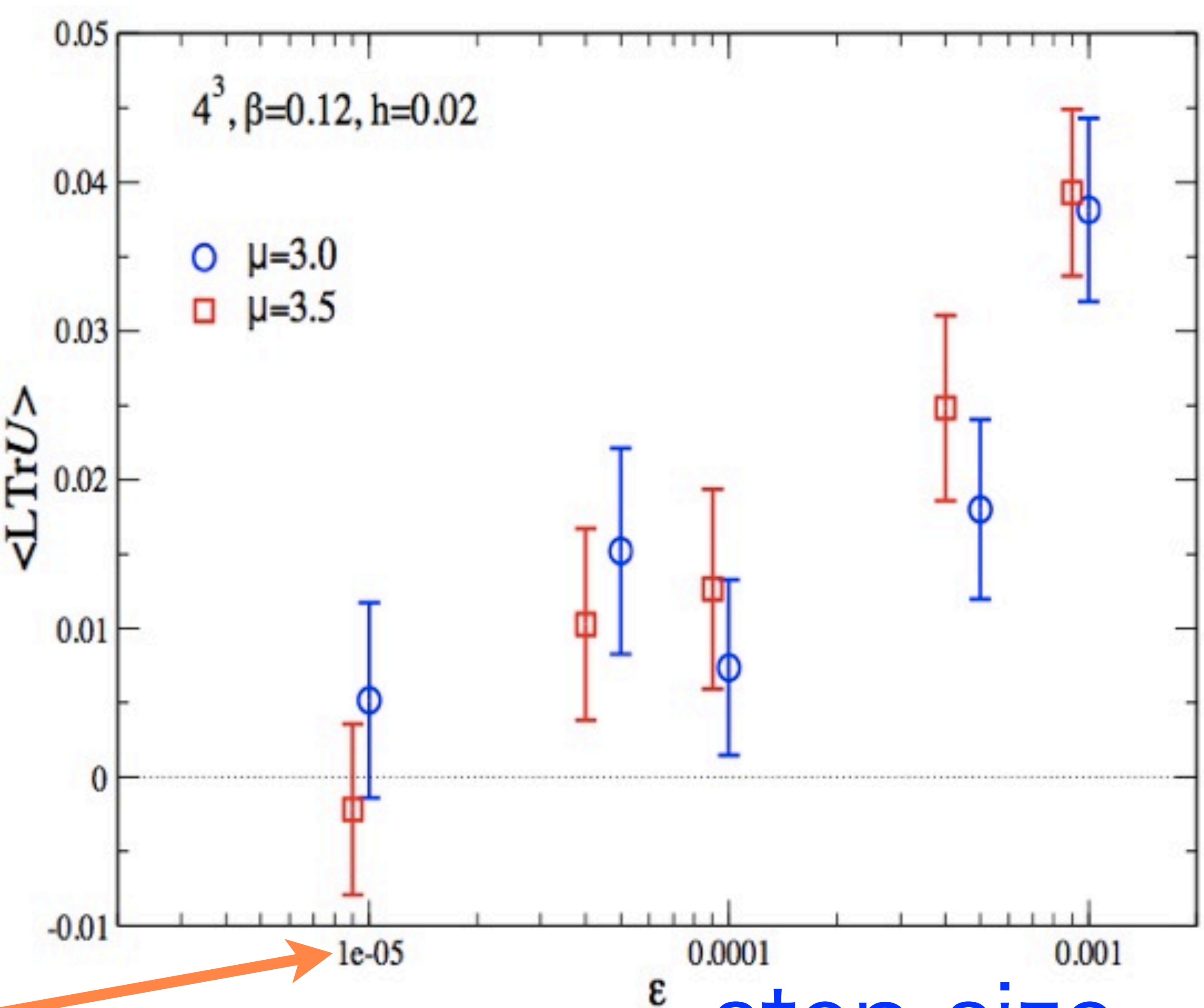

10^{-5}

step size

Complex Langevin

教訓

- step sizeは小さく！とにかく小さく！
- 他と比較せよ！
  - 一致するまで信用するな！
- criterionが計算できるときは嬉しい！

QCD using CLE

QCD using CLE

link variable SU(3) → SL(3,C)

QCD using CLE

link variable SU(3) → SL(3,C)

complex Langevin equation

$$U_{x,\mu} \mapsto \exp\left\{ -\sum_a i\lambda_a (\epsilon K_{a,x,\mu} + \sqrt{\epsilon}\eta_{a,x,\mu}) \right\} U_{x,\mu}$$

QCD using CLE

link variable SU(3) → SL(3,C)

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force: sl(3,C)

QCD using CLE

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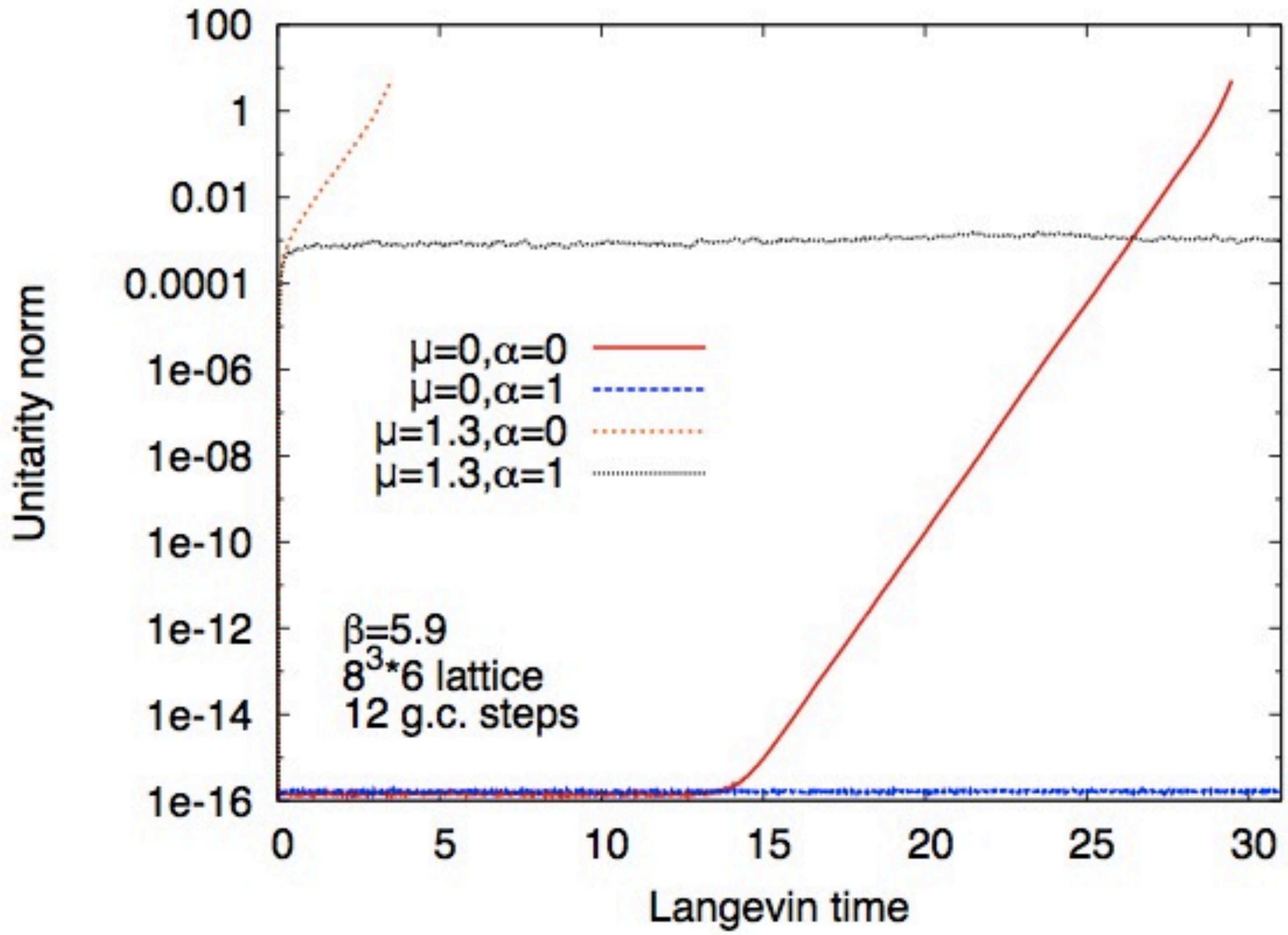
force: sl(3,C)

unitarityが危ない！

$$F(\{U\}) \equiv \sum_{x,\mu} \mathrm{tr}\left[U_{x,\mu}^\dagger U_{x,\mu} + (U_{x,\mu}^\dagger)^{-1} U_{x,\mu}^{-1} - 2\right] \geq 0$$

QCD using CLE



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QCD using CLE

gauge cooling (Seiler, Sexty, Stamatescu)

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gauge変換の自由度 $U_{x,\hat{\mu}} \mapsto \exp\left(-\sum_a \tilde{\alpha}\lambda_a G_{a,x}\right) U_{x,\hat{\mu}}$

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unitarity normを減らす方向

$$G_{a,y} \equiv D_{a,y}F = 2\mathrm{tr}\lambda_a \left[U_{y,\mu}U^\dagger_{y,\mu} - U^\dagger_{y-\hat{\mu},\mu}U_{y-\hat{\mu},\mu}\right] + 2\mathrm{tr}\lambda_a \left[-(U^\dagger_{y,\mu})^{-1}U^{-1}_{y,\mu} + (U^\dagger_{y-\hat{\mu},\mu})^{-1}U^{-1}_{y-\hat{\mu},\mu}\right]$$

QCD using CLE

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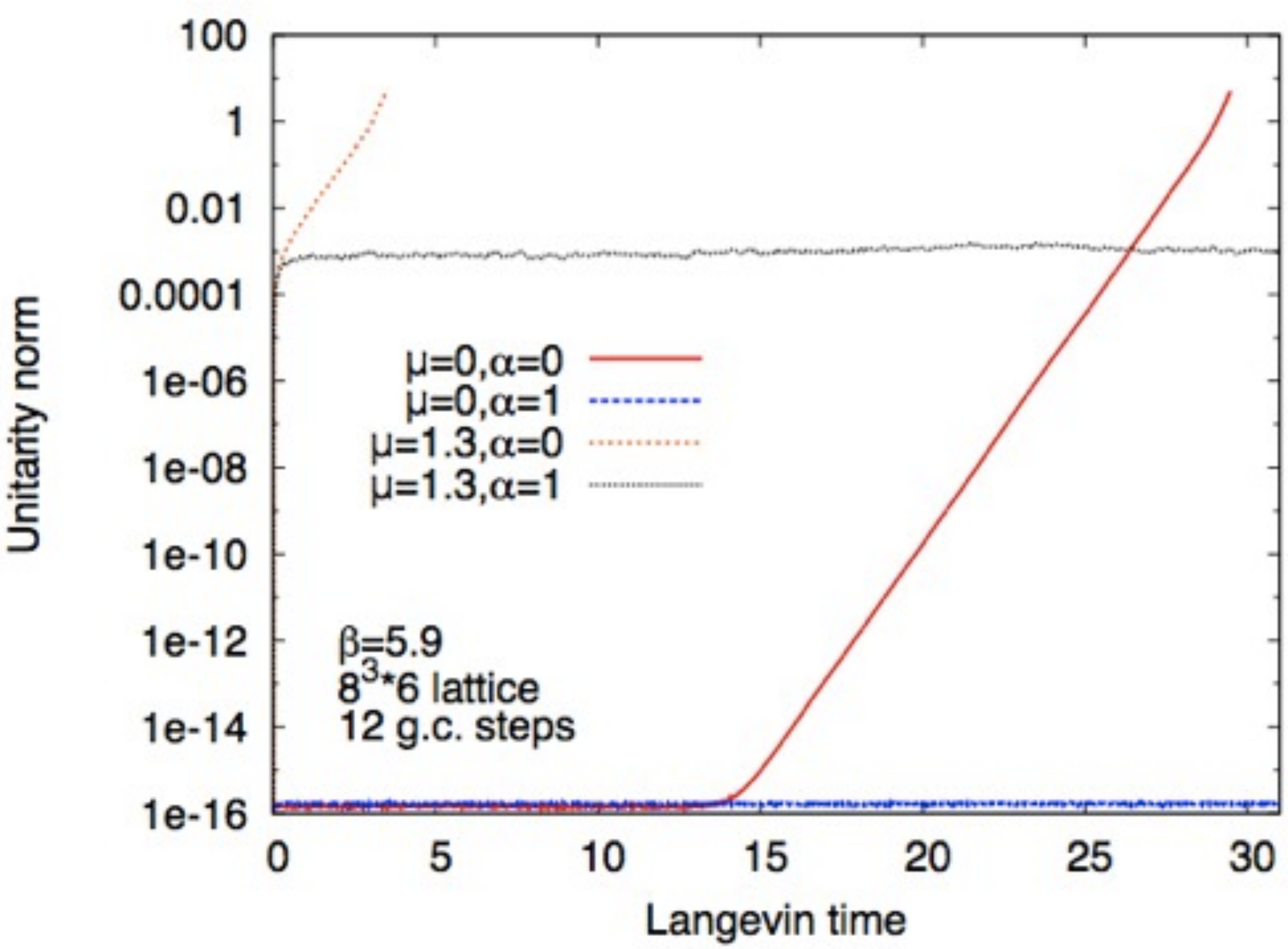

QCD using CLE

gauge coolingはがっつりやるべし！

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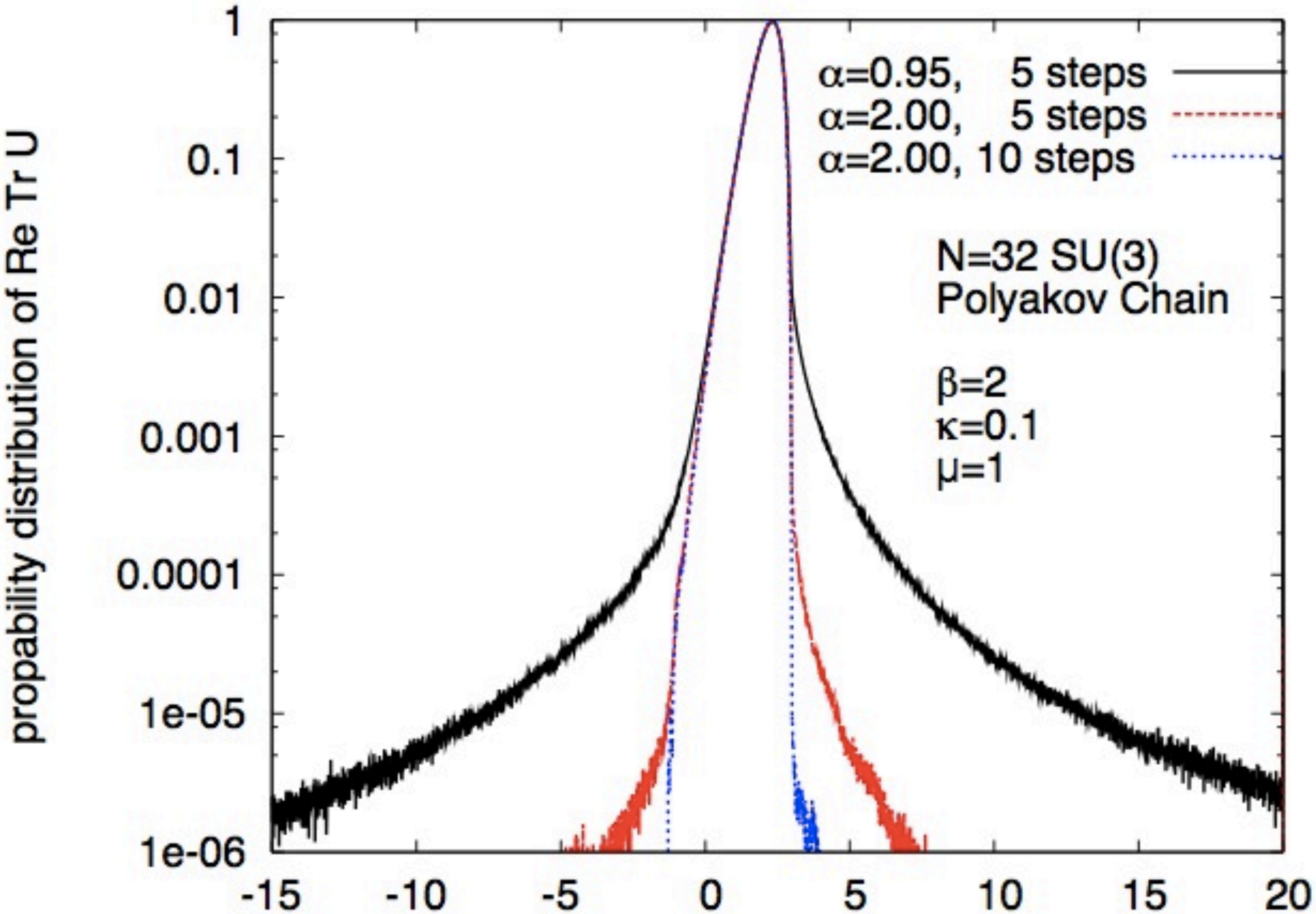

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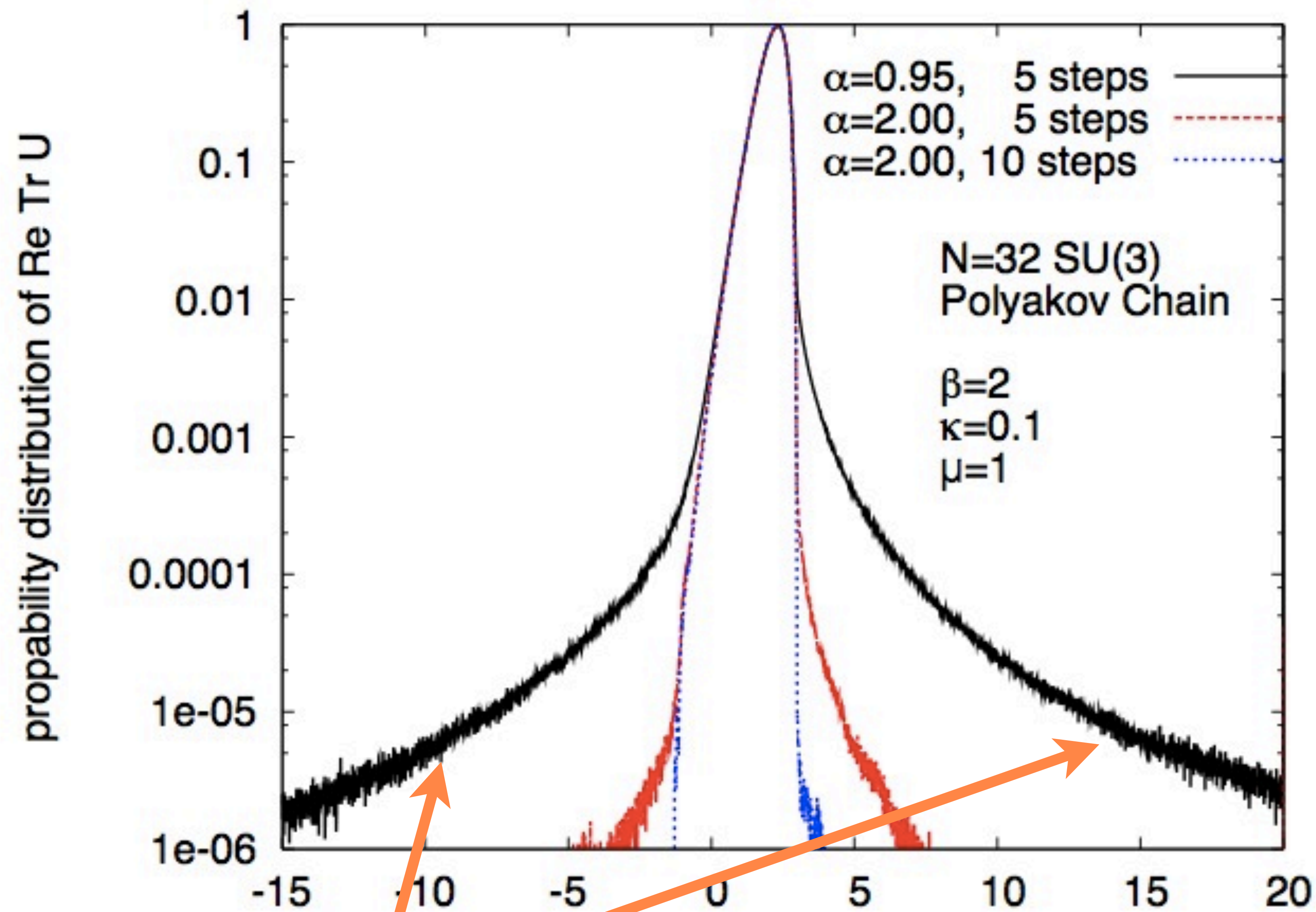


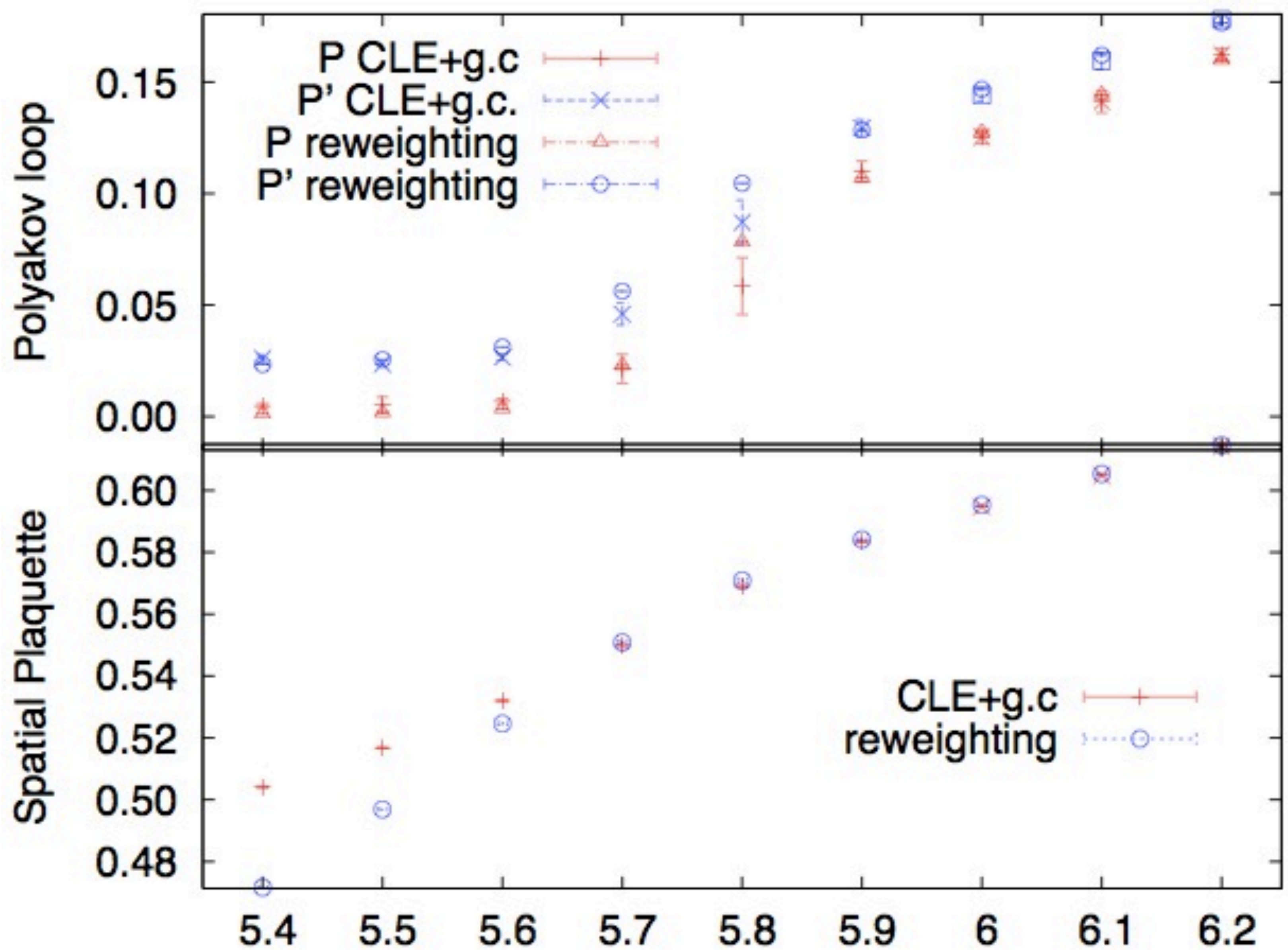
経験則

skirtが現れたらwrong limitへ行ってしまう

QCD using CLE

strong coupling側は苦手

HDQCD



QCD using CLE

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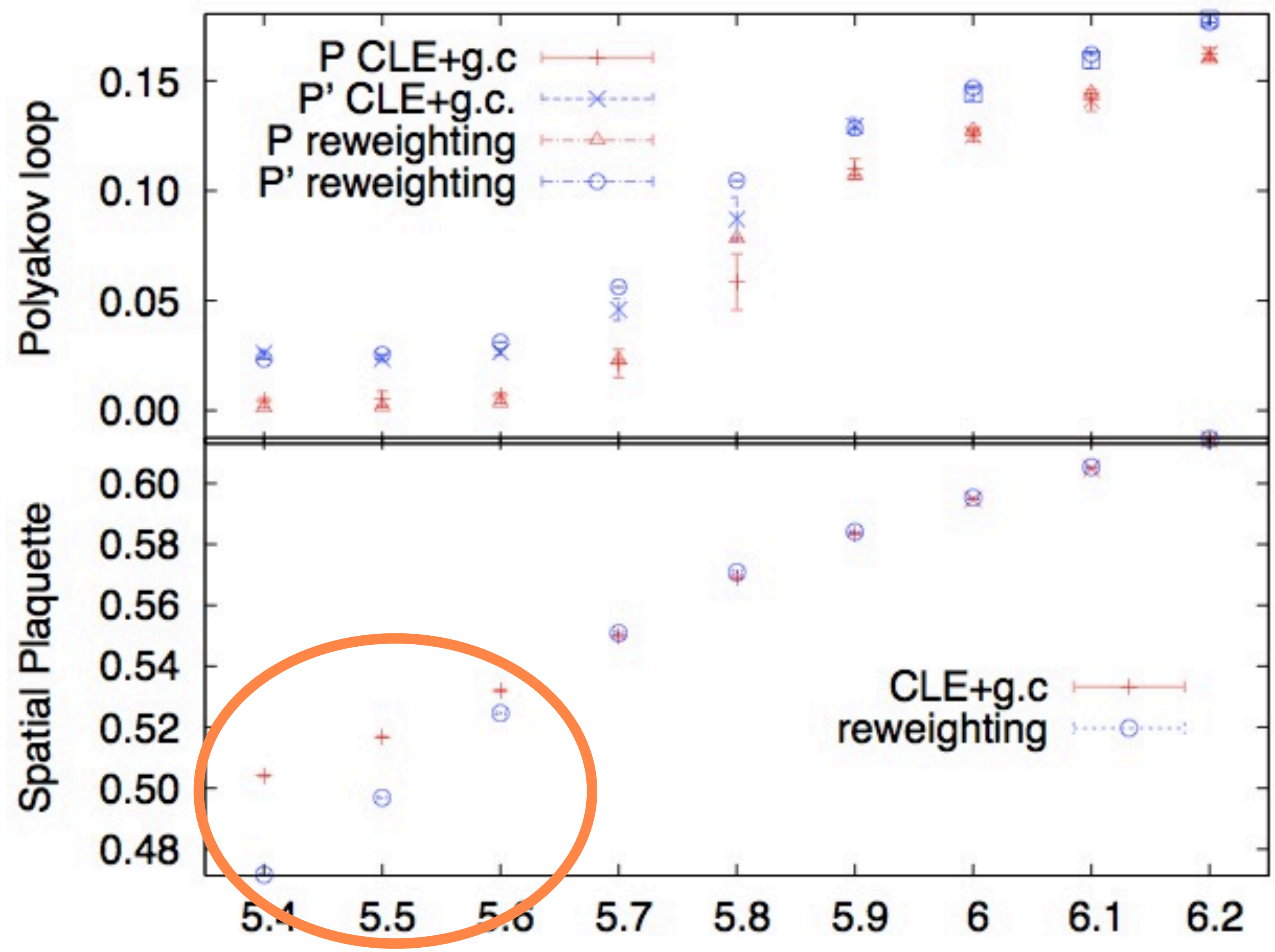
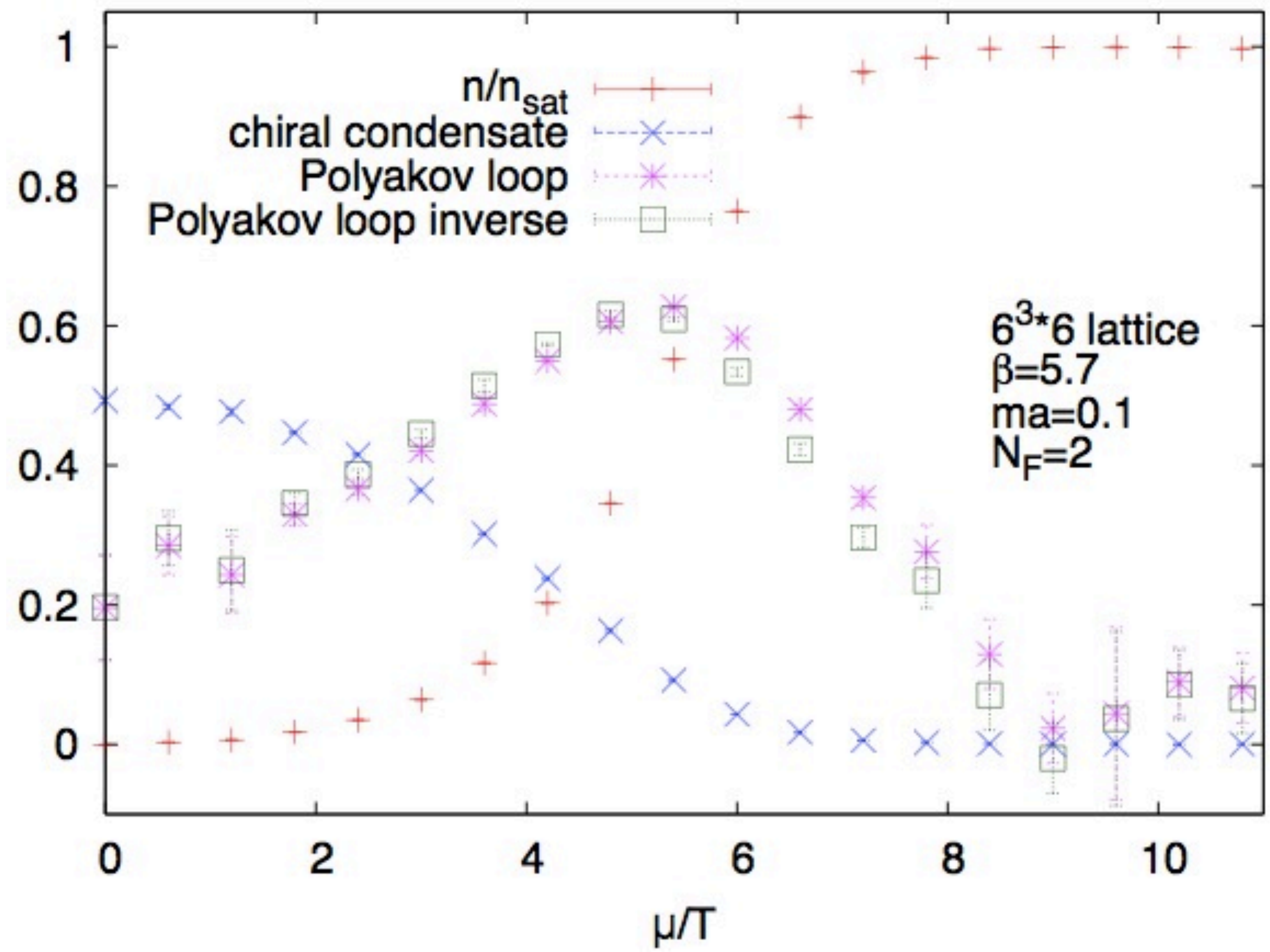


QCD using CLE

Results



QCD using CLE

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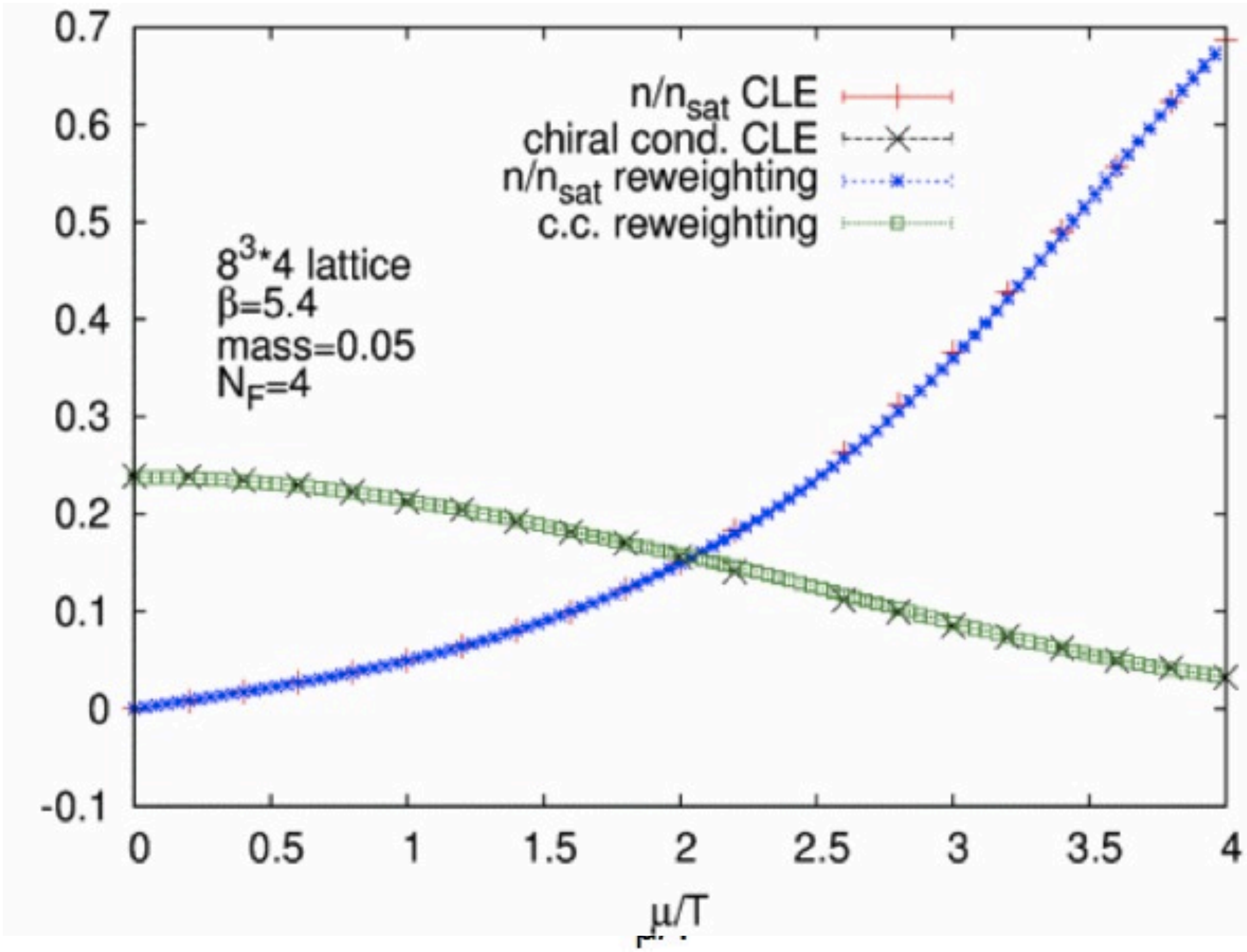

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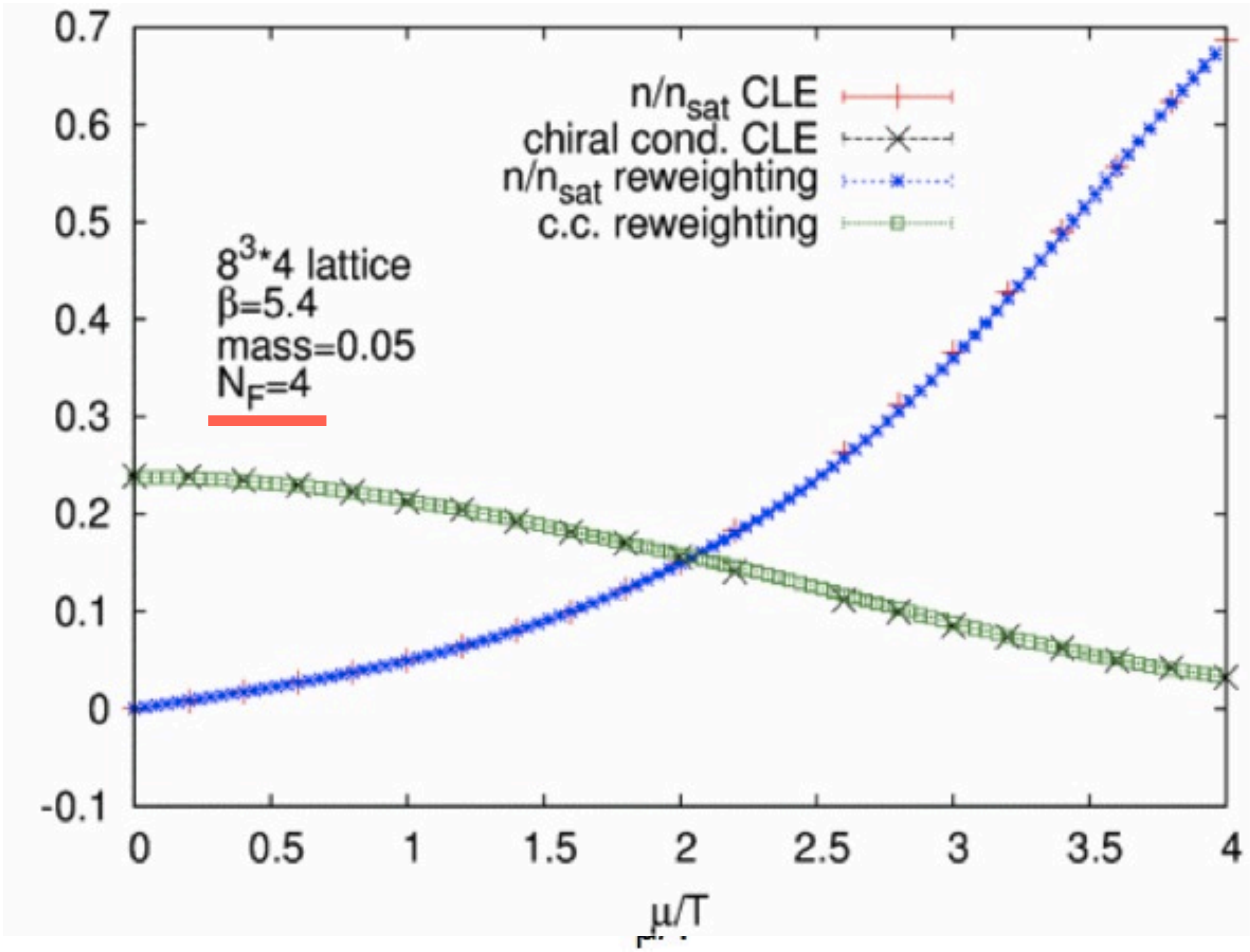

Complex Langevin

結論

- step sizeは小さく！とにかく小さく！
- 要gauge cooling.
- 他と比較せよ！
  - 比較対象：HDQCD, reweighting
  - 概ね一致している
- full QCDの計算が始まっている
  - コストはμ=0の100〜1000倍