Axial resonances $a_1(1260)$, $b_1(1235)$ and their decay from the lattice

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$a_1(1260)$ and $b_1(1235)$

$a_1(1260)$ [k] $I^G(J^{PC}) = 1^-(1^{++})$

Mass $m = 1230 \pm 40$ MeV [l]
Full width $\Gamma = 250$ to 600 MeV

$a_1(1260)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$(\rho\pi)_{S-\text{wave}}$	seen	353
$(\rho\pi)_{D-\text{wave}}$	seen	353
$(\rho(1450)\pi)_{S-\text{wave}}$	seen	†
$(\rho(1450)\pi)_{D-\text{wave}}$	seen	†
$\sigma\pi$	seen	–
$f_0(980)\pi$	not seen	179
$f_0(1370)\pi$	seen	†
$f_2(1270)\pi$	seen	†
$K\overline{K}^*(892)+$ c.c.	seen	†
$\pi\gamma$	seen	608

axial vector meson $\gamma_i\gamma_5$

$b_1(1235)$ $I^G(J^{PC}) = 1^+(1^{+-})$

Mass $m = 1229.5 \pm 3.2$ MeV (S = 1.6)
Full width $\Gamma = 142 \pm 9$ MeV (S = 1.2)

$b_1(1235)$ DECAY MODES	Fraction (Γ_i/Γ)	Confidence level	p (MeV/c)
$\omega\pi$	dominant		348
$[D/S$ amplitude ratio $= 0.277 \pm 0.027]$			

axial vector(?) meson $\gamma_i\gamma_4\gamma_5$

Calculation of masses and decay widths from S-wave scattering phase

Previous work: Γ_{b_1} from 3-pt function by UKQCD PRD65:094605(2006)

No previous calculation of Γ_{a_1}

Calculation of mass and decay width

$a_1 \to \pi\rho$ in S-wave ($l = 0$)

Scattering phase shift $\delta(p)$ of $\pi(p)\rho(-p)$

$\delta(p) = \pi/2$ @ $\sqrt{s} = m_{a_1} = m^{\text{res}}$

maximum of scattering cross section $\propto \sin^2\delta(p)$

$s = (E_\pi(p) + E_\rho(p))^2, E_H(p) = \sqrt{m_H^2 + p^2}$

The Breit-Wigner parametrization

$$\frac{-\sqrt{s}\,\Gamma(s)}{s - (m^{\text{res}})^2 + i\sqrt{s}\,\Gamma(s)} = \frac{1}{\cot\delta - i}\ , \quad \Gamma_{a_1}(s) \equiv g^2_{a_1\rho\pi}\ \frac{p}{s}$$

Analyze $\dfrac{p}{\sqrt{s}}\cot\delta = \dfrac{m^2_{a_1} - s}{g^2_{a_1\rho\pi}}$ in function of s

1) lhs $= 0 \to m_{a_1}$, 2) slope of $s \to g_{a_1\rho\pi}$, 3) a_0 @ $\sqrt{s} = m_\pi + m_\rho$

$1/a_0 \equiv p\cot\delta|_{p\to 0}$

Calculation of scattering phase shift

Lüscher's finite volume formula

$$\tan\delta(p) = \frac{\sqrt{\pi}\ p\ L}{2\ \mathcal{Z}_{00}\left(1;\left(\frac{pL}{2\pi}\right)^2\right)} \quad \text{with } \mathcal{Z}_{00}(k;q^2) = \sum_{\vec{n}} \frac{1}{\sqrt{4\pi}} \frac{1}{\left(\vec{n}^2 - q^2\right)^k}$$

p from two-particle energy, e.g.) $E_{\pi\rho} = \sqrt{m_\pi^2 + p^2} + \sqrt{m_\rho^2 + p^2}$

Evaluation of two-particle energy

Correlation function matrix with several operators

$$C_{jl}(t) = \frac{1}{N_T} \sum_{t_i} \langle \mathcal{O}_j^\dagger(t_i + t) | \mathcal{O}_l(t_i) \rangle = \sum_n Z_{jn} Z_{ln}^* e^{-E_n t}$$

Generalized eigenvalue problem

$$C(t)\, v_n(t) = \lambda_n(t)\ C(t_0)\ v_n(t) \quad , \ \lambda_n(t) = e^{-E_n(t-t_0)}$$

Operators for correlation function matrix

$a_1 \;\; I(J^{PC}) = 1(1^{++})$

$$\mathcal{O}_1^{\bar{q}q} = \sum_{\mathbf{x}} \; \bar{u}(x) \; \gamma_i \; \gamma_5 \; d(x) \; ,$$

$$\mathcal{O}_2^{\bar{q}q} = \sum_{\mathbf{x},j} \; \bar{u}(x) \overleftarrow{\nabla}_j \; \gamma_i \; \gamma_5 \; \overrightarrow{\nabla}_j \; d(x) \; ,$$

$$\mathcal{O}_3^{\bar{q}q} = \sum_{\mathbf{x},j,l} \; \epsilon_{ijl} \; \bar{u}(x) \; \gamma_j \; \frac{1}{2}[\overrightarrow{\nabla}_l - \overleftarrow{\nabla}_l] \; d(x) \; ,$$

$$\begin{aligned}\mathcal{O}^{\rho\pi} &= \frac{1}{\sqrt{2}}[\pi^0(\mathbf{0})\rho^-(\mathbf{0}) - \rho^0(\mathbf{0})\pi^-(\mathbf{0})] \\ &= \frac{1}{2}\Bigg(\sum_{\mathbf{x_1}}[\bar{u}(x_1)\gamma_5 u(x_1) - \bar{d}(x_1)\gamma_5 d(x_1)]\sum_{\mathbf{x_2}}\bar{u}(x_2)\gamma_i d(x_2) \\ &\qquad - \sum_{\mathbf{x_1}}[\bar{u}(x_1)\gamma_i u(x_1) - \bar{d}(x_1)\gamma_i d(x_1)]\sum_{\mathbf{x_2}}\bar{u}(x_2)\gamma_5 d(x_2)\Bigg) \; ,\end{aligned}$$

$b_1 \;\; I(J^{PC}) = 1(1^{+-})$

$$\mathcal{O}_1^{\bar{q}q} = \sum_{\mathbf{x}} \; \bar{u}(x) \; \gamma_i \; \gamma_t \; \gamma_5 \; d(x) \; ,$$

$$\mathcal{O}_2^{\bar{q}q} = \sum_{\mathbf{x},j} \; \bar{u}(x) \overleftarrow{\nabla}_j \; \gamma_i \; \gamma_t \; \gamma_5 \; \overrightarrow{\nabla}_j \; d(x) \; ,$$

$$\mathcal{O}_3^{\bar{q}q} = \sum_{\mathbf{x}} \; \bar{u}(x) \; \gamma_5 \; \frac{1}{2}[\overrightarrow{\nabla}_i - \overleftarrow{\nabla}_i] \; d(x) \; ,$$

$$\mathcal{O}_4^{\bar{q}q} = \sum_{\mathbf{x}} \; \bar{u}(x) \; \gamma_t \; \gamma_5 \; \frac{1}{2}[\overrightarrow{\nabla}_i - \overleftarrow{\nabla}_i] \; d(x) \; ,$$

$$\mathcal{O}^{\omega\pi} = \omega(\mathbf{0})\pi^-(\mathbf{0}) = \frac{1}{\sqrt{2}}\sum_{\mathbf{x_1}}[\bar{u}(x_1)\gamma_i u(x_1) + \bar{d}(x_1)\gamma_i d(x_1)]\sum_{\mathbf{x_2}}\bar{u}(x_2)\gamma_5 d(x_2) \; ,$$

ρ and ω are stable? in $a_1 \to \rho\pi$ and $b_1 \to \omega\pi$

Consider only center-of-mass frame c.f.) $a_1 \to \rho(p)\pi(-p)$

$m_\pi a$	$m_\rho a$	$m_\omega a$
$0.1673(16)$	$0.5107(40)$	$0.514(15)$

$m_\rho > 2m_\pi$ and $m_\omega \sim 3m_\pi$

$m_\pi = 266$MeV

$\rho \to \pi\pi$ in P-wave ($l = 1$) $\to$ prohibited $\pi(0)\pi(0)$ decay

$La = 16$ ($L \sim 2$fm) $\to$ lowest $p_{\text{low}} = 2\pi/La = 0.3927$

$m_\rho < 2E_\pi(p_{\text{low}}) \to \rho(p = 0)$ is stable

However, $E_\rho(p_{\text{low}}) \sim m_\pi + E_\pi(p_{\text{low}}) \to \rho(p_{\text{low}})$ might decay

PRD:84:054503(2011)

Higher states than $\rho(p_{\text{low}})\pi(-p_{\text{low}})$ are not considered

$\omega \to \pi\pi\pi$ in P-wave ($l = 1$) $\to$ prohibited $\pi(0)\pi(0)\pi(0)$ decay

$m_\omega < 2E_\pi(p_{\text{low}}) + m_\pi \to \omega(p = 0)$ is stable

Higher states than $\omega(p_{\text{low}})\pi(-p_{\text{low}})$ are not considered

Result of effective energy

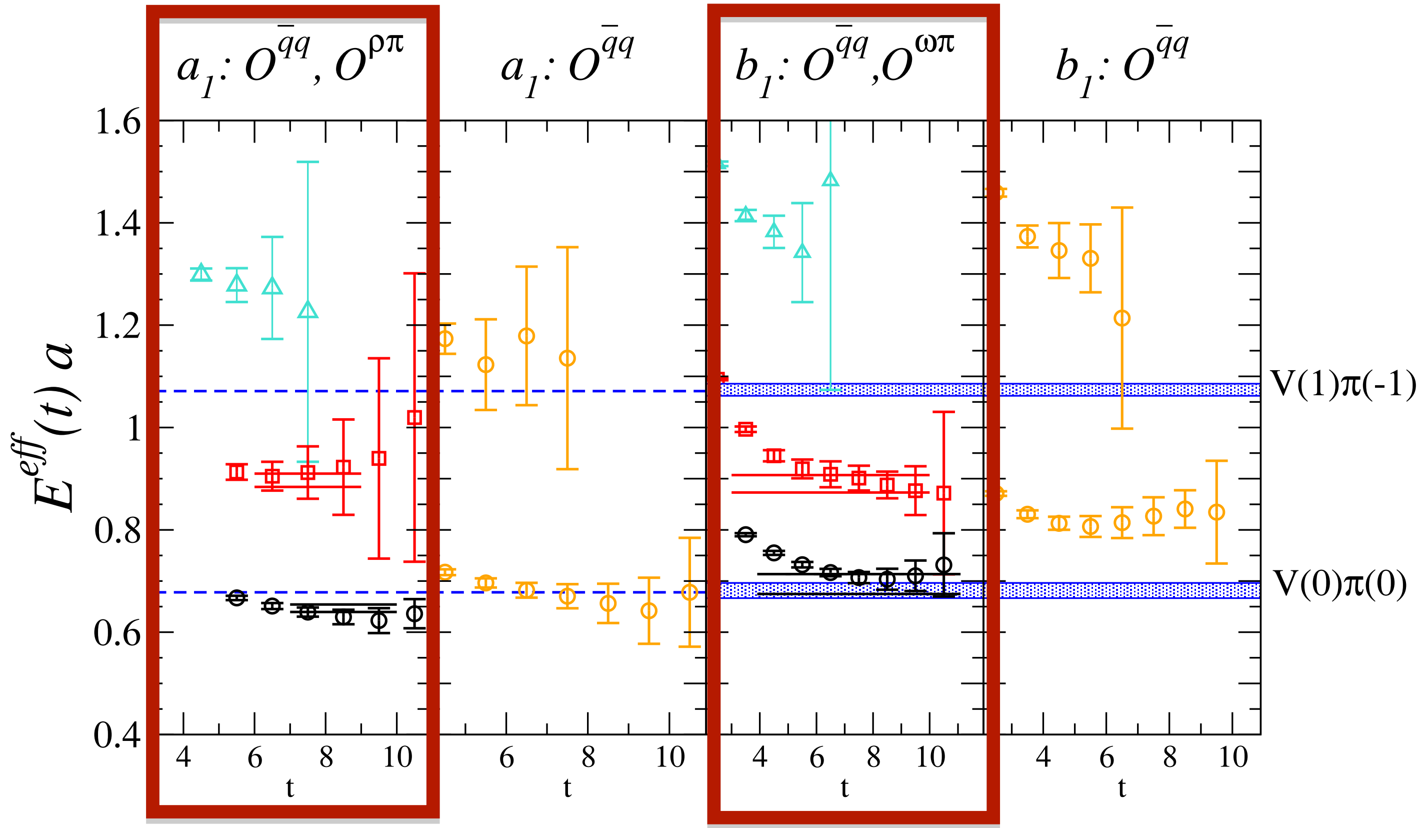
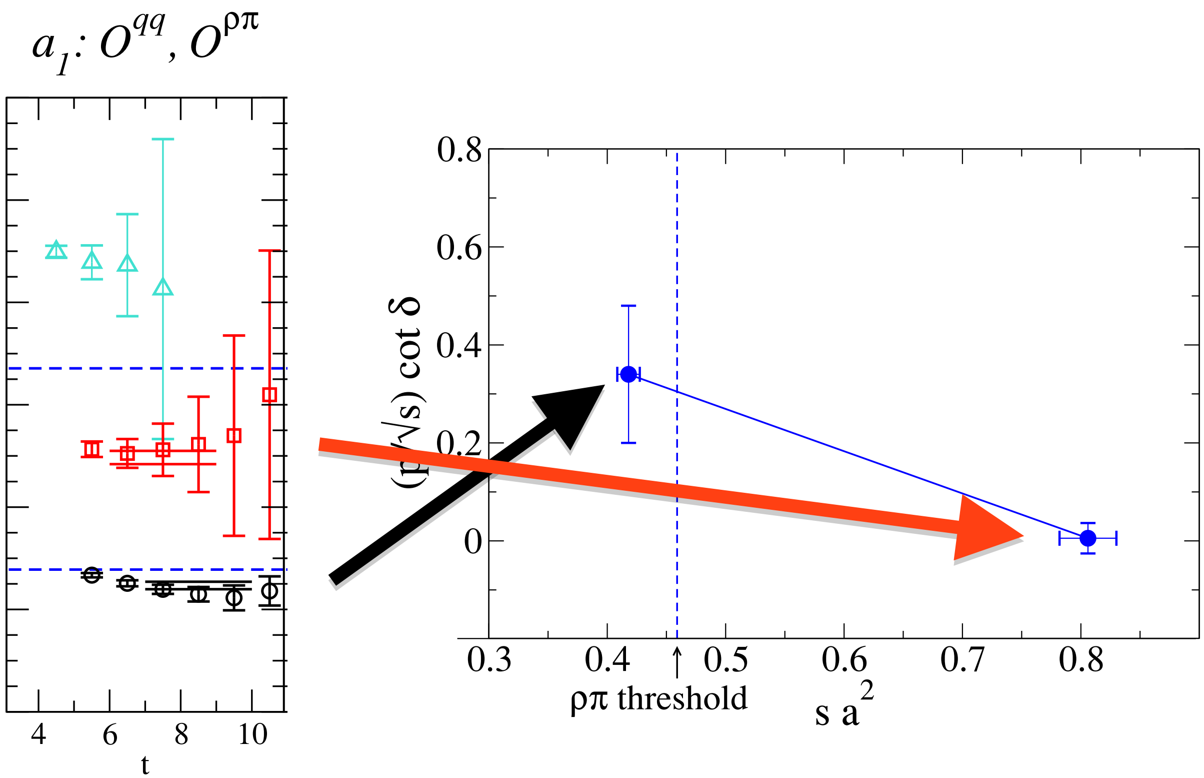


Figure 1. Effective energies $E_n^{\rm eff}a$ in the a_1 and b_1 channels, that correspond to the energy levels $E_n a$ in the plateau region. The horizontal lines indicate the $m_V + m_\pi$ threshold and the energy of a non-interacting $V(1)\pi(-1)$ state, where $V=\rho$ for a_1 and $V=\omega$ for b_1. We compare the results when $\mathcal{O}^{V\pi}$ is included in or excluded from the interpolator basis.

Result of a_1

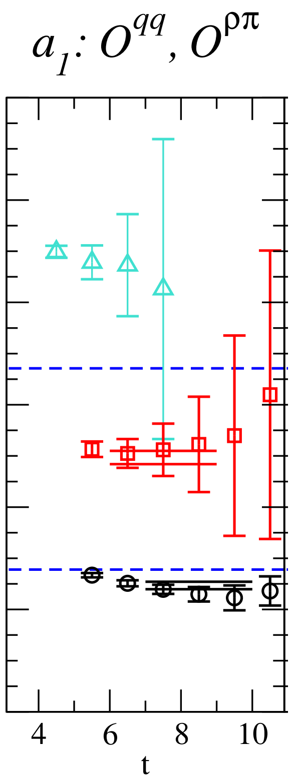


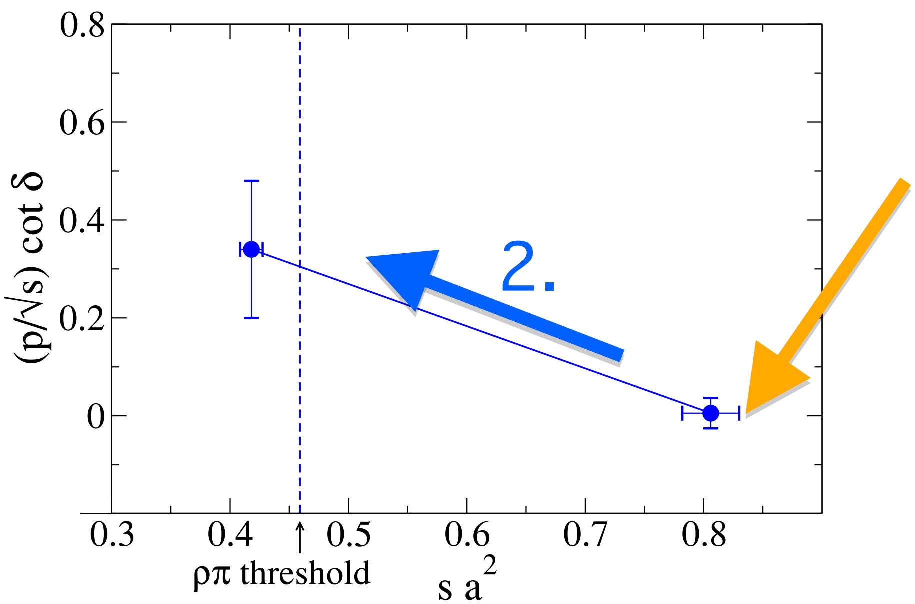
1. m_{a_1} @ $y = 0$
2. $g_{a_1 \rho\pi}$ from slope
3. a_0 @ $\rho\pi$ thershold
   $(\sqrt{s} = m_\rho + m_\pi)$

resonance	$a_1(1260)$		
quantity	$m^{\text{res}}_{a_1}$ [GeV]	$g_{a_1 \rho\pi}$ [GeV]	$a^{\rho\pi}_{l=0}$ [fm]
lat	$1.435(53)(^{+0}_{-109})$	$1.71(39)$	$0.62(28)$
exp	$1.230(40)$	$1.35(30)$	-

PDG: $\Gamma_{a_1} = 250$ to 600MeV $\rightarrow 425(175)$MeV

Result of a_1



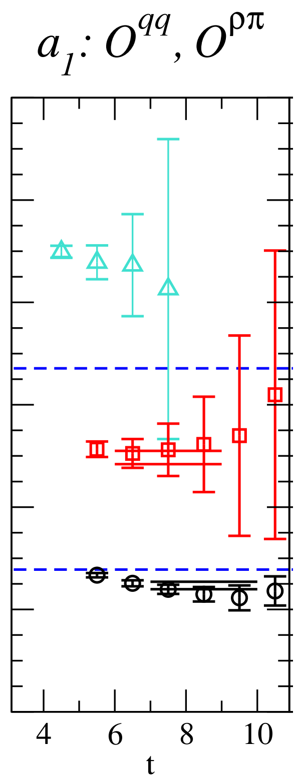


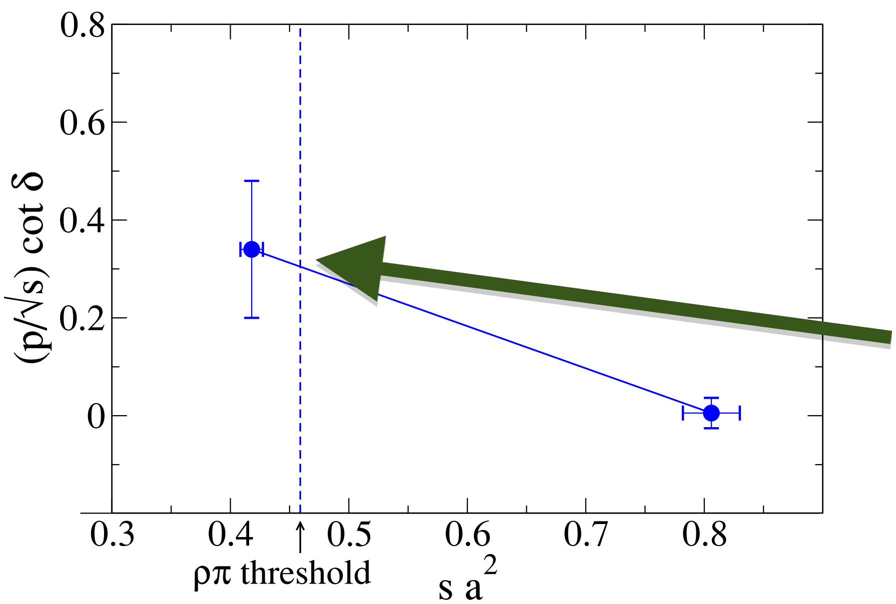
1. m_{a_1} @ $y = 0$
2. $g_{a_1\rho\pi}$ from slope
3. a_0 @ $\rho\pi$ thershold $(\sqrt{s} = m_\rho + m_\pi)$

resonance	$a_1(1260)$		
quantity	$m_{a_1}^{res}$ [GeV]	$g_{a_1\rho\pi}$ [GeV]	$a_{l=0}^{\rho\pi}$ [fm]
lat	$1.435(53)(^{+0}_{-109})$	1.71(39)	0.62(28)
exp	1.230(40)	1.35(30)	-

PDG: $\Gamma_{a_1} = 250$ to $600\text{MeV} \to 425(175)\text{MeV}$

Result of a_1



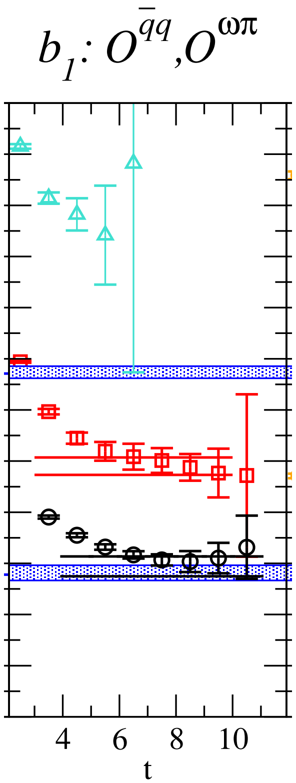


1. m_{a_1} @ $y = 0$
2. $g_{a_1\rho\pi}$ from slope
3. a_0 @ $\rho\pi$ thershold
   $(\sqrt{s} = m_\rho + m_\pi)$

resonance	$a_1(1260)$		
quantity	$m^{res}_{a_1}$ [GeV]	$g_{a_1\rho\pi}$ [GeV]	$a^{\rho\pi}_{l=0}$ [fm]
lat	$1.435(53)(^{+0}_{-109})$	1.71(39)	0.62(28)
exp	1.230(40)	1.35(30)	-

PDG: $\Gamma_{a_1} = 250$ to 600MeV $\rightarrow$ 425(175)MeV

Result of b_1



n	t_0	interp.	fit range	$\frac{\chi^2}{\text{d.o.f}}$	Ea	$E=\sqrt{s}$ [GeV]	pa	δ [°]	$\frac{p\cot(\delta)}{\sqrt{s}}$
1	3	$\mathcal{O}^{\bar{q}q}_{1,2,4}, \mathcal{O}^{\omega\pi}$	4-11	0.12	0.694(19)	1.105(31)	0.057(45)	−3.0(6.3)	−1.6(2.2)
2	2	$\mathcal{O}^{\bar{q}q}_{1,3}, \mathcal{O}^{\omega\pi}$	3-10	0.049	0.890(17)	1.418(27)	0.264(13)	93.5(7.5)	−0.018(38)

$E_1 \sim m_\omega + m_\pi$ not good result

$$m^{\text{res}}_{b_1} = \left[E_2^2 + (g^{\text{exp}}_{b_1\omega\pi})^2 \left(\frac{p\cot\delta}{\sqrt{s}} \right)^2 \right]^{1/2} = 1.414(36)(^{+0}_{-83})\ \text{GeV}$$

$\approx E_2 = 1.418(27)$GeV due to $\delta \sim \pi/2$

$g^{\text{exp}}_{b_1\omega\pi} = 0.787(25)$GeV

Summary

Calculation at $m_\pi = 266$MeV on $L \sim 2$fm

resonance	$a_1(1260)$			$b_1(1235)$	
quantity	$m_{a_1}^{\rm res}$ [GeV]	$g_{a_1\rho\pi}$ [GeV]	$a_{l=0}^{\rho\pi}$ [fm]	$m_{b_1}^{\rm res}$ [GeV]	$g_{b_1\omega\pi}$ [GeV]
lat	$1.435(53)(^{+0}_{-109})$	1.71(39)	0.62(28)	$1.414(36)(^{+0}_{-83})$	input
exp	1.230(40)	1.35(30)	-	1.2295(32)	0.787(25)

First calculations for m_{a_1} and m_{b_1} with two-particle states

Possible systematic errors

1. Small volume
2. Effects of ρ and ω decays
3. Chiral extrapolation to physical m_π