

Symmetry energy, neutron skin, and neutron star radius From chiral effective field theory interactions

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Key topics:

- ▶ Chiral EFT,
- ▶ Equation of state,
- ▶ Neutron stars

References:

- ▶ *Astrophys. J.* 733 (2013) 11
- ▶ *Phys. Rev. C* 68 (2003) 041001(R)
- ▶ *Nucl. Phys. A* 784 (2007) 79

Introduction

Chiral effective field theory

Given by an infinite series of terms with increasing number of derivatives and/or nucleon fields

- provides a powerful approach to three-nucleon (3N) interactions
- enables controlled calculations

This is especially important for exotic nuclei and neutron-rich matter under extreme conditions in astrophysics

Neutron matter

- only long-range parts of 3N force contributes
- a simpler system to test the chiral EFT power counting and the size of many-body forces for finite density

In general, nuclear force depend on a resolution scale Λ

$$H(\Lambda) = T(\Lambda) + V_{\text{NN}}(\Lambda) + V_{\text{3N}}(\Lambda) + V_{\text{4N}}(\Lambda) \dots$$

The renormalization group is a powerful tool

to systematically change the resolution scale Λ , while preserving low-energy observables

		NN	3N	4N
LO	$\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$		—	—
NLO	$\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$		—	—
N ² LO	$\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$			—
N ³ LO	$\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$			

derived in (2002)

(2011) ... (2006) ...

Phenomenological hierarchy

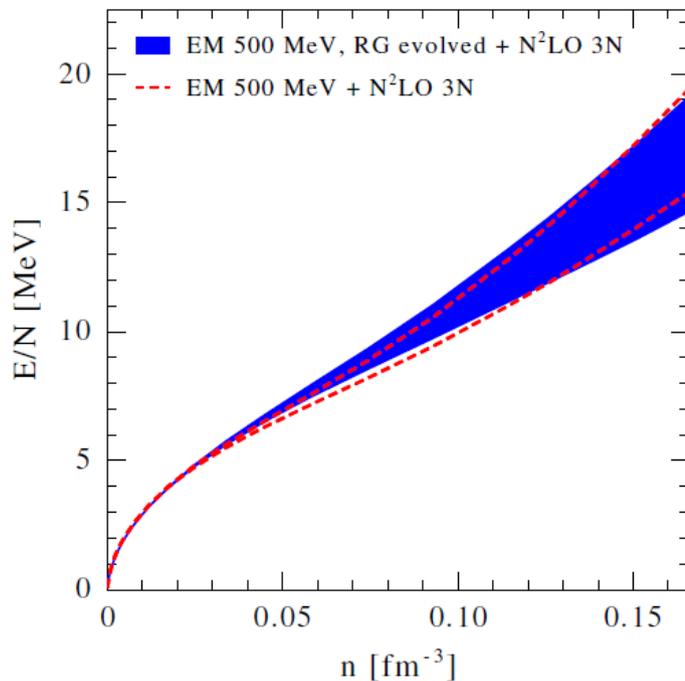
$$V_{\text{NN}}(\Lambda) > V_{\text{3N}}(\Lambda) > \bar{V}_{\text{4N}}(\Lambda)$$

Neutron matter based on chiral EFT interactions

Entem Machleidt $\Lambda=500\text{MeV}$ $N^3\text{LO}$ NN potential

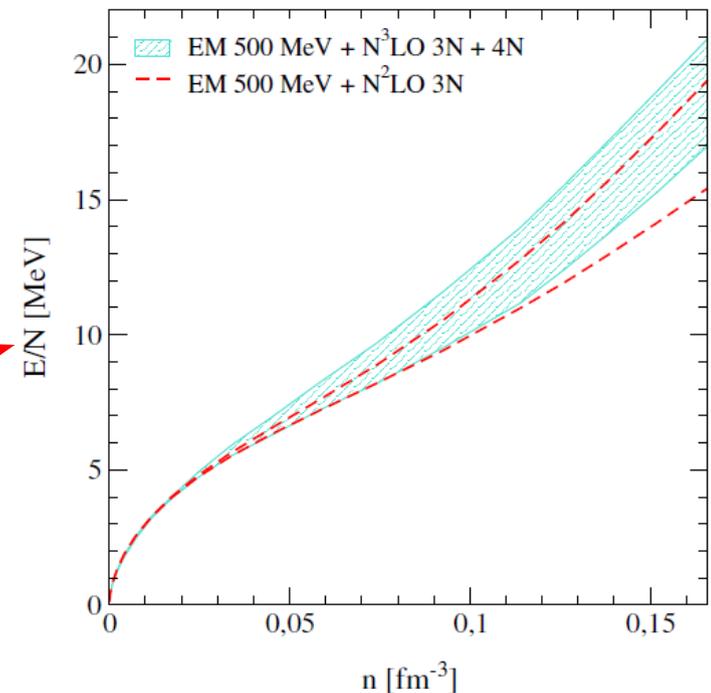
The energy is calculated in a perturbative expansion around the Hartree-Fock energy.

nuclear matter saturation is predicted with the same hamiltonians.



The width of blue band is dominated by the uncertainties of 3N forces.

At these scale, different NN potentials lead the similar result.



RG-evolution to a low-momentum scale $\Lambda = 2.0\text{fm}^{-1}$

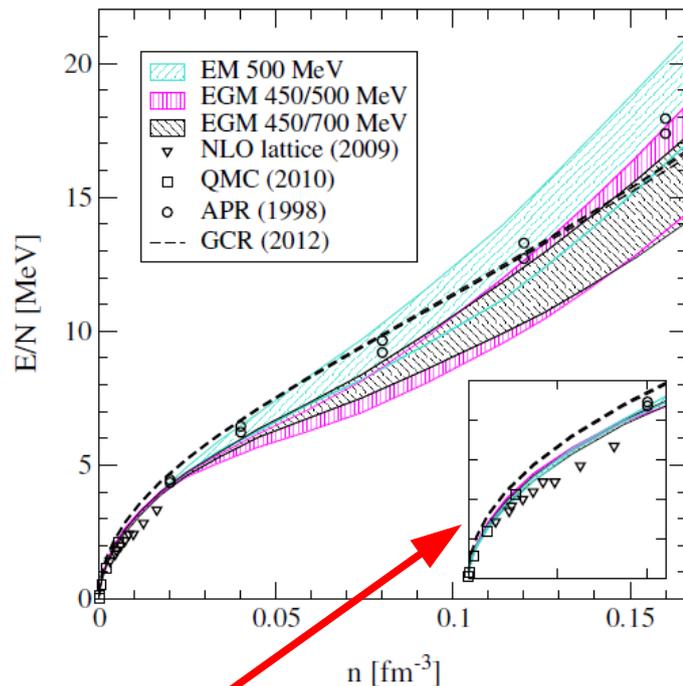
The width of the energy bands based on 3N forces at $N^2\text{LO}$ and $N^3\text{LO}$ are comparable at higher densities

EFT convergence of 3N forces from $N^2\text{LO}$ to $N^3\text{LO}$

Neutron matter based on chiral EFT interactions

The first complete N³LO calculation of neutron matter energy.

All NN, 3N and 4N interactions to N³LO are included



Origins of energy range

- Different NN potentials
- Variation of the couplings c_1 and c_3 in 3NF
- 3N/4N cutoff variation $\Lambda = 2.0 - 2.5 \text{ fm}^{-1}$

Dominant!

N³LO range is in very good agreement

- NLO lattice results
- QMC simulations
- Ab-initio calculation :

(Argonne NN and Urbana 3N)

} at very low densities where the properties are determined by the large scattering length and effective range.

From the neutron matter equation of state to the symmetry energy and neutron skin

Calculations are extended from neutron matter to matter with a finite proton fraction.

The Empirical parametrization

$$\epsilon(\bar{n}, x)T_0 = \frac{3}{5}[x^{5/3} + (1-x)^{5/3}](2\bar{n})^{2/3} - [(2\alpha - 4\alpha_L) \times x(1-x) + \alpha_L]\bar{n} + [(2\eta - 4\eta_L)x(1-x) + \eta_L]\bar{n}^\gamma,$$

T_0 : Fermi energy of symmetric matter
 n : baryon density
 x : proton fraction

This interpolates between the properties of neutron matter and symmetric nuclear matter.

Parameters can be determined from the empirical saturation properties and from the microscopic calculation of neutron matter.

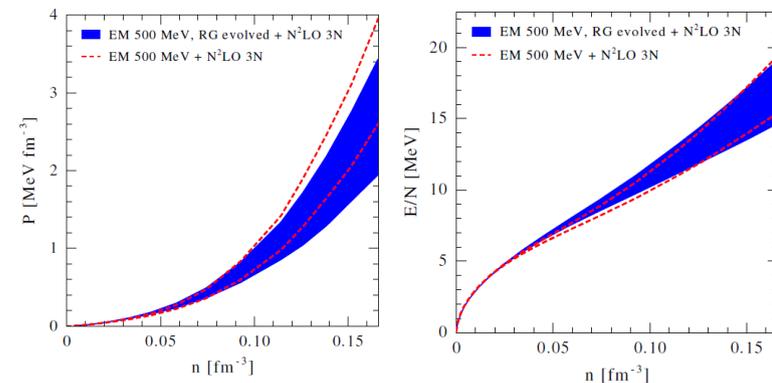
Symmetric nuclear matter

$$\epsilon(\bar{n} = 1, x = 1/2) = -B = -16 \text{ MeV}$$

$$P(\bar{n} = 1, x = 1/2) = 0$$

$$\longrightarrow \alpha = 5.87, \eta = 3.81$$

neutron matter



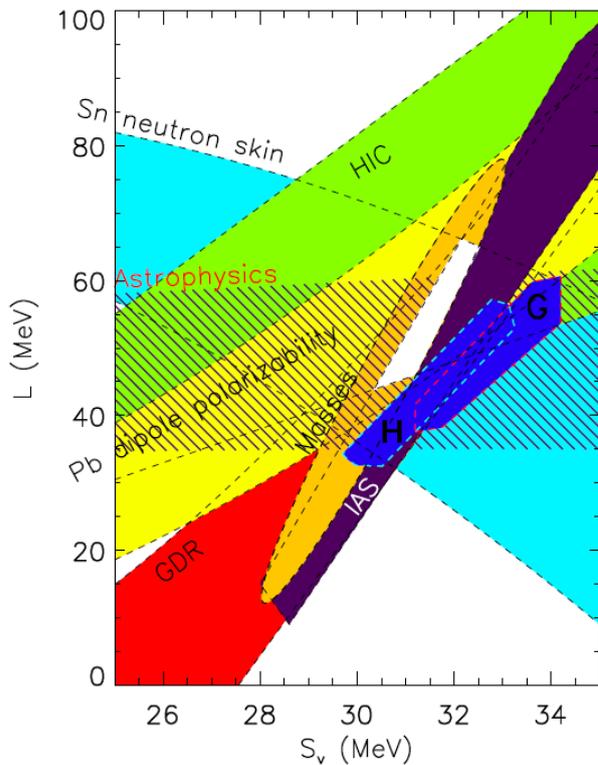
$$\longrightarrow \alpha_L = 1.18-1.59 \text{ and } \eta_L = 0.64-1.11$$

This parametrization provides excellent global fit for the energy up to a density $n=1.1n_0$

From the neutron matter equation of state to the symmetry energy and neutron skin

The parametrization allows to predict the symmetry energy and its density derivative

$$S_v = \frac{1}{8} \frac{\partial^2 \epsilon(\bar{n}, x)}{\partial x^2} \Big|_{\bar{n}=1, x=1/2} \quad \text{and} \quad L = \frac{3}{8} \frac{\partial^3 \epsilon(\bar{n}, x)}{\partial \bar{n} \partial x^2} \Big|_{\bar{n}=1, x=1/2}$$



S_v -L region predicted by neutron matter results

H : Neutron matter constraints

G : AFDMC calculation (Av8' and Urbana IX)

H and G are different hamiltonian but good agreement

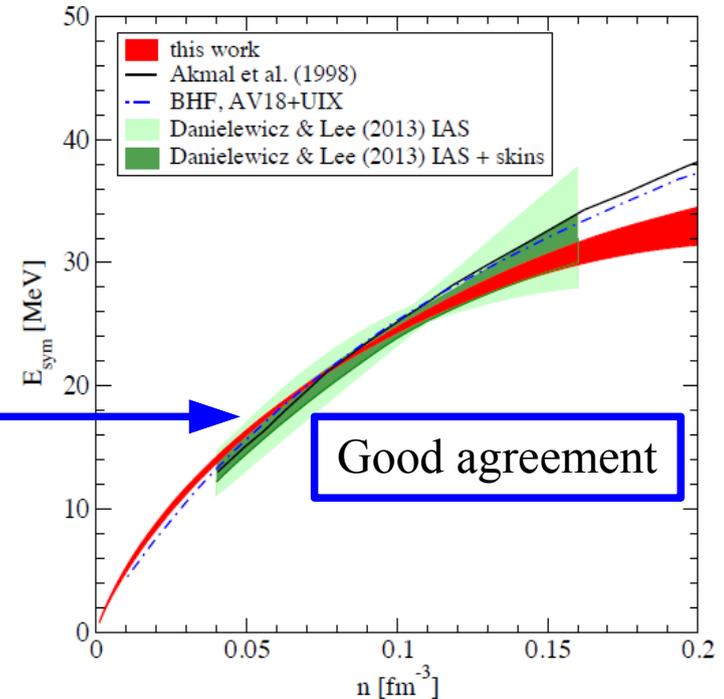
Neutron matter results provide the tightest constraints

The results are based on 3N forces fit only to light nuclei, Without adjustments to empirical nuclear matter properties.

Neutron skin of ^{208}Pb is in excellent agreement

$$0.17 \pm 0.03 \text{ fm} \quad \longleftrightarrow \quad 0.156^{+0.025}_{-0.021} \text{ fm}$$

from dipole polarizability



Good agreement

Constraints on neutron star radii

Further application of the parametrization to extend the neutron matter to neutron star matter.

Proton fraction in beta equilibrium

$$\frac{\partial \epsilon(\bar{n}, x)}{\partial x} + \mu_e(\bar{n}, x) - (m_n - m_p)c^2 = 0$$

To describe the EoS of neutron star matter, we use **the BPS outer crust EoS** for densities below $n_0/2$.

Without 3N forces, the calculated EoSs would not match on to a standard crust EoS.

For $n > 1.1n_0$, **general piecewise polytropic extensions** are used.

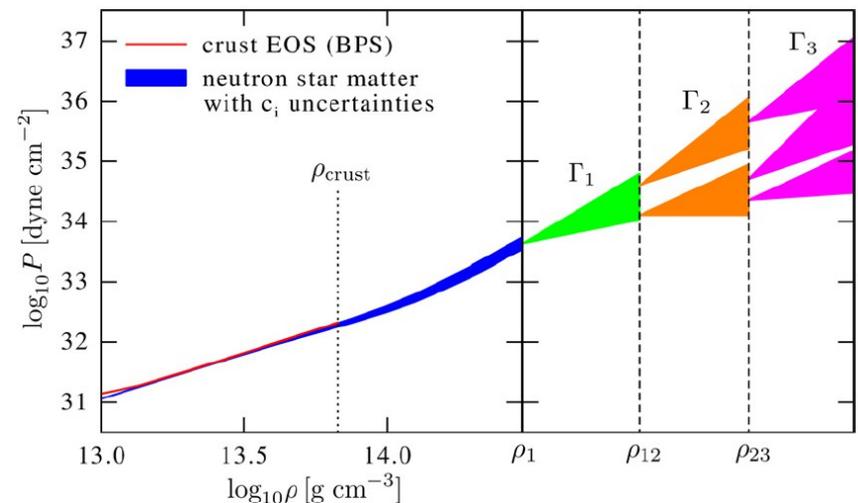
$$P(\rho) = \kappa \dot{\rho}^\Gamma$$



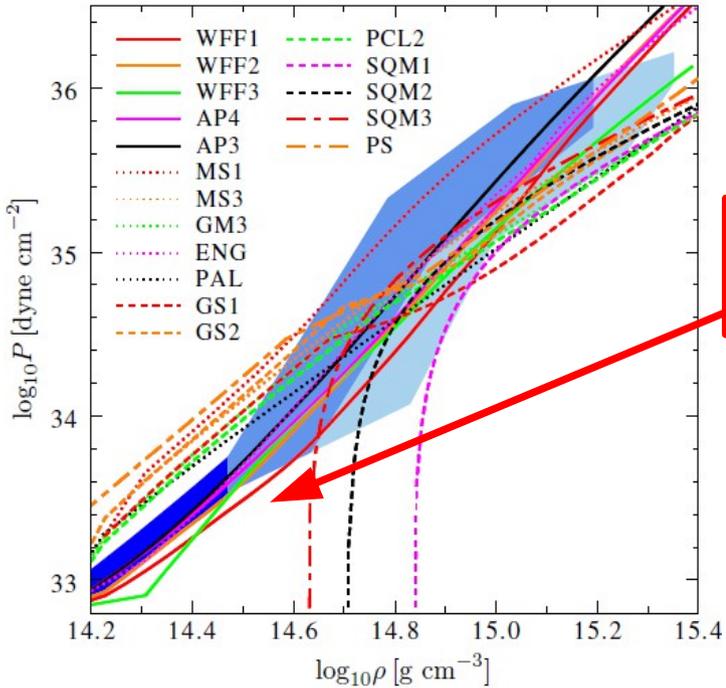
This strategy generates
a very large number of EoSs

After solving TOV equation,

- 1) remain causal for all relevant densities
- 2) able to support a neutron star mass M
(the mass of the heaviest neutron star
observed or potential heavier candidates)



Constraints on neutron star radii



Light blue band : support a mass $M=1.97M_{\odot}$
 Dark blue band : $M=2.4M_{\odot}$
 Blue : predicted by the neutron matter results

This demonstrates that chiral EFT interactions provide strong constraints ruling out many model EoSs at low densities.

AP3 and AP4 are consistent within the uncertainty bands.

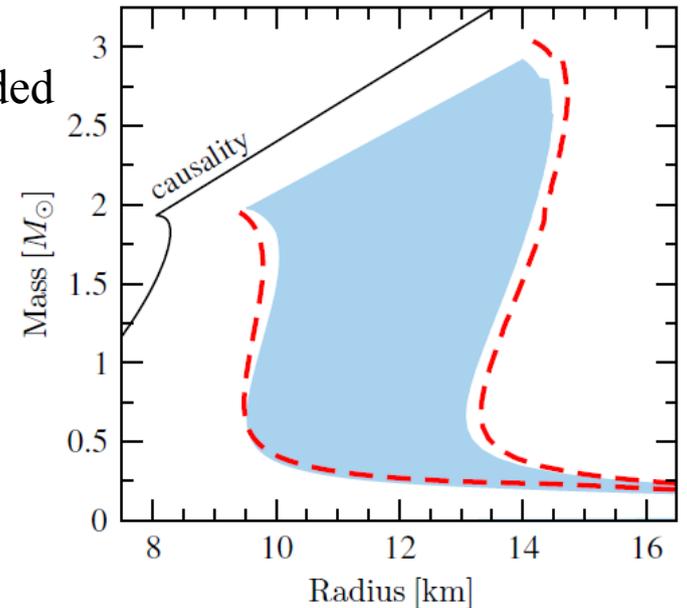
	$\widehat{M} = 1.97 M_{\odot}$		$\widehat{M} = 2.4 M_{\odot}$	
	min	max	min	max
$\rho_c/\rho_0 (1.4 M_{\odot})$	1.8	4.4	1.8	2.7
$\rho_c/\rho_0 (1.97 M_{\odot})$	2.0	7.6	2.0	3.4
$\rho_c/\rho_0 (2.4 M_{\odot})$			2.2	5.4



Radius constraints

Typical 1.4M neutron star : $R = 9.7 \sim 13.9\text{km}$

RG-evolved : Blue-shaded
 Unevolved : Red-dash



Summary and outlook

- The properties of neutron-rich matter at nuclear densities are well constrained by chiral EFT interactions
- This results in tight constraints for the symmetry energy, the neutron skin of ^{208}Pb , and the radius of neutron stars.
- The theoretical uncertainties are dominated by the uncertainties in 3N forces.

Therefore developments in 3N forces will be important next steps

Representative EoSs : soft (green), intermediate (orange), stiff (red) *Astrophys. J. 733 (2013) 11*

