Thermodynamics of SU(3) Gauge Theory from Gradient Flow

FlowQCD Collab(M.Asakawa et. al.), arXiv:1312.7492

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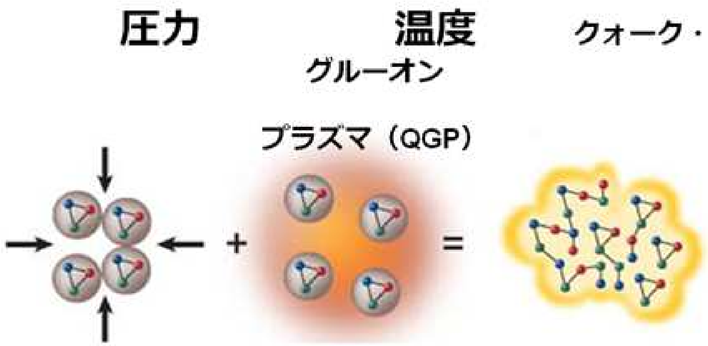
Contents

1 Introduction

Quarks and gluons are deconfined at high temperature and density to be quark-gluon plasma.

- Quark-gluon plasma is characterized by thermodynamic quantities, such as the equation of state(energy density, pressure).



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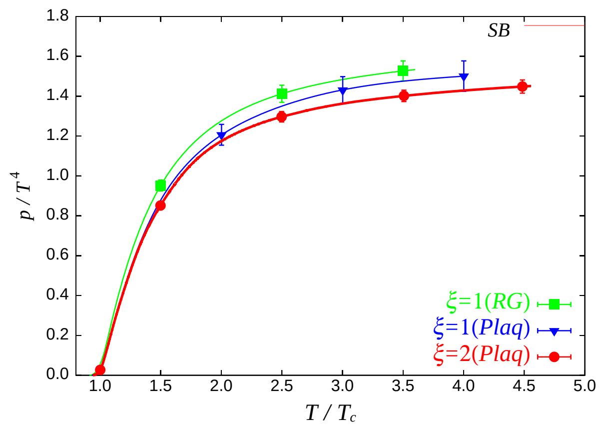


CP-PACS,2001

There are several ways to calculate the equation of state.

- Integral method, $\quad p = -(1/V)\int d\beta \partial_\beta \log Z$
- Operator method
- Energy-momentum tensor $T_{\mu\nu}$ with shifted boundary
- Energy-momentum tensor $T_{\mu\nu}$ with gradient flow $\leftarrow$ This paper.

Energy-momentum tensor is related to the equation of state s.t.

$$\text{energy density} \quad : \quad \epsilon = -\langle T_{00} \rangle$$

$$\text{pressure} \quad : \quad p = \frac{1}{3}\sum_i \langle T_{ii} \rangle$$

2 Formulation

[Gradient flow] cf. Taniguchi-san's journal clubs (2010;2013)
Gradient flow is defined as a deformation of the gauge field $A_\mu(x)$ along a fictitious time t. Atiyah and Bott (1982)

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x), \quad (1)$$

$$B_\mu(t=0,x) = A_\mu(x) \quad (2)$$

$B_\mu(t,x)$: flowed gauge field

$G_{\nu\mu}(t,x)$: flowed field strength

[Energy-momentum tensor from the gradient flow]

- Coefficients $\alpha_U(t), \alpha_E(t)$ are calculated at 1-loop H.Suzuki, 2013.
  - ♢ Small t is needed for perturbations, $a \ll \sqrt{8t} \ll L, 1/T$.
  - ♢ $a, t \to 0$ must be performed satisfying $a \ll \sqrt{8t}$.
    ex. first $a \to 0$, then $t \to 0$.
  - ♢ Non-perturbative determination of $\alpha_U(t), \alpha_E(t)$ has been proposed M.Lüscher,2013; L.Del Debbio et. al., 2013.
- (It corresponds to a smeared or cooled $T_{\mu\nu}$.)

$$T_{\mu\nu} := \lim_{t\to 0}\left(\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{16\alpha_E(t)}\left(G^2(t,x) - \left\langle G^2(t,x)\right\rangle_0\right)\right) \tag{3}$$

$$U_{\mu\nu}(t,x) := G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{\delta_{\mu\nu}}{4}G^2(t,x) \tag{4}$$

$$cf.\ T_{\mu\nu} := F_{\mu\rho}(x)F_{\nu\rho}(x) - \frac{\delta_{\mu\nu}}{4}F^2(x)\ :\ \text{naive definition} \tag{5}$$

3 Simulation setup

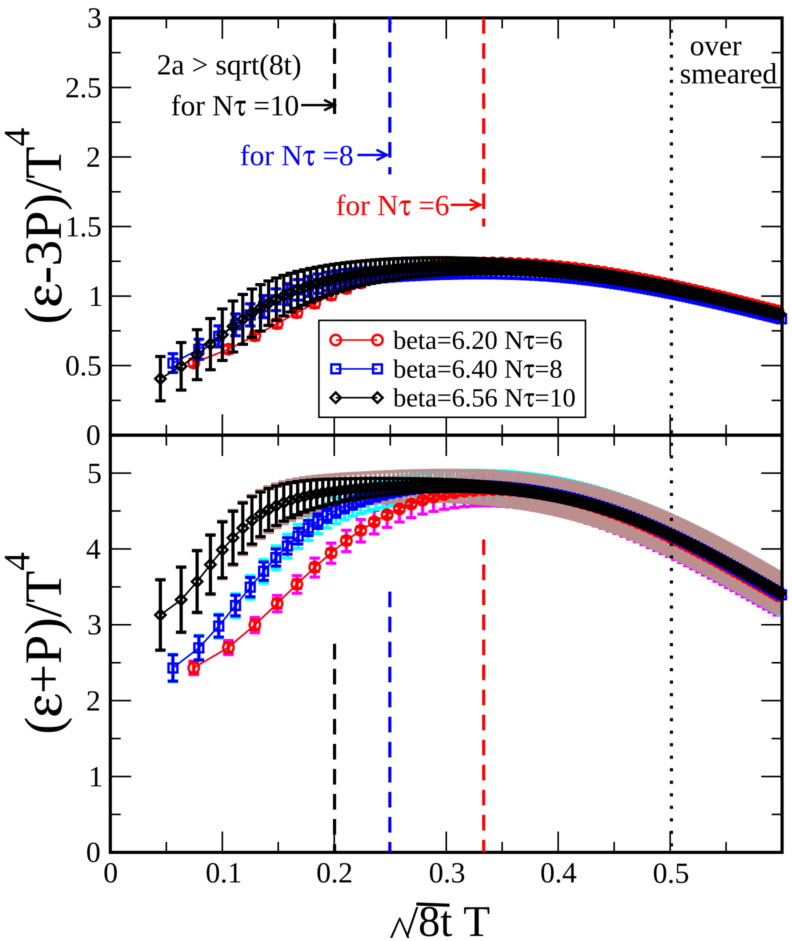
- $N_f = 0$ (pure SU(3) theory)
- Lattice size : $32^3 \times 6, 8, 10, 32$
- plaq action with $\beta = 5.89 - 6.56$ ($a^{-1} = 1.5 - 5.0$ [GeV])
  - $\diamondsuit$ $T/T_c = 0.99 - 1.65$
- #conf = 300

[Calculation procedure]

1. Generate gauge config $A_\mu(x)$
2. Evolve $A_\mu(x)$ to $B_\mu(t,x)$ by the gradient flow
3. Construct $T_{\mu\nu}(t)$ from $B_\mu(t,x)$
4. Perform $\lim_{a,t\to 0} T_{\mu\nu}(a,t)$ satisfying $a \ll \sqrt{8t}$

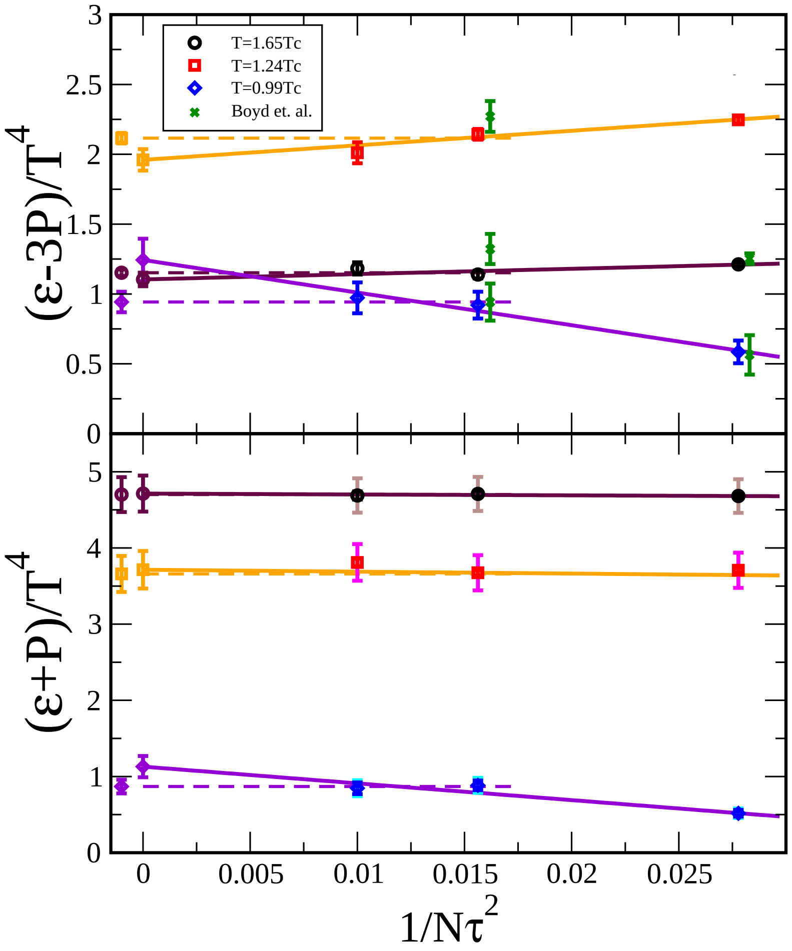
4 Simulation results

[Flow time dependence of observables]



- Data must be in $[t_{min}, t_{max}]$ satisfying $a \ll \sqrt{8t} \ll L, 1/T$.
  - ◇ Set $\sqrt{8t_{min}} = 2a$ to avoid discretization effects.
  - ◇ Set $\sqrt{8t_{max}} = 1/(2T)$, factor 2 is from periodicity.
- Data at $\sqrt{8t}T = 0.4$ is employed.
  - ◇ No $t \to 0$ limit ???
  - ◇ The choice of $\sqrt{8t}T = 0.4$ seems too aggressive.

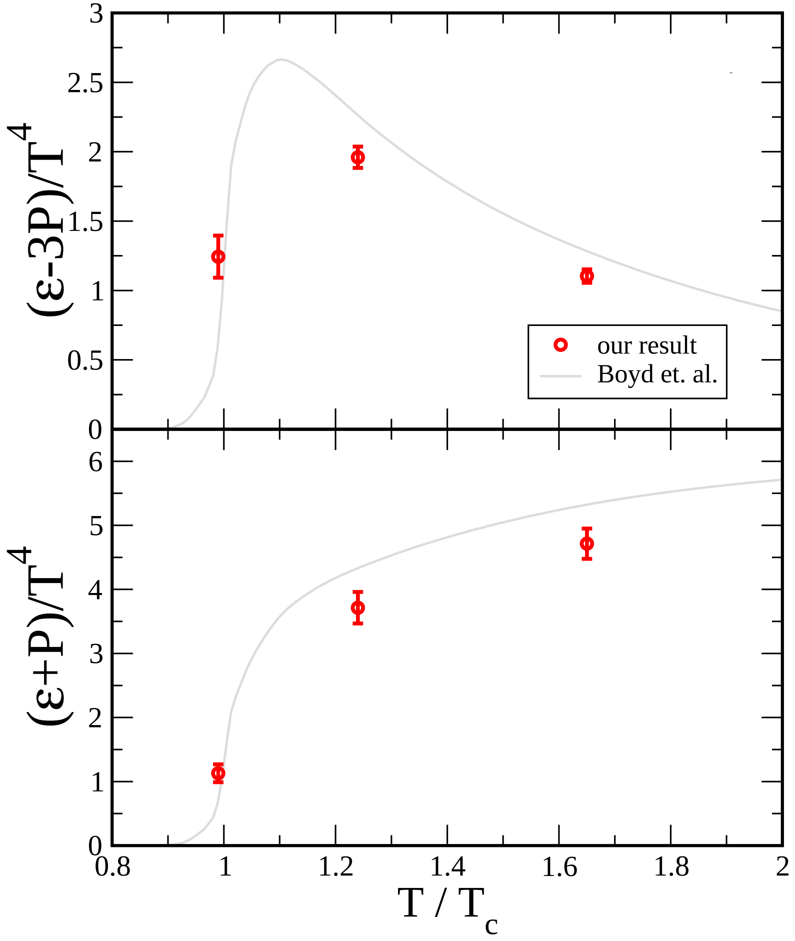
[Continuum extrapolations with $\sqrt{8t}T = 0.4$ data]



- The data of $(\epsilon - 3p)/T^4$ is consistent with the previous result at a finite lattice spacing.
  $\leftarrow$ No comparison for $(\epsilon + p)/T^4$???

- Finite size effects must be visible on the finest lattice $(1/N_t^2 = 0.01)$, where $N_s/N_t = 3.2$.

[Equation of state]



- The authors claim "The results are qualitatively consistent".
  ← i.e. the authors fail to reproduce the previous results.

- No reason is given. It may be

  ◇ $t \to 0$ extrapolation
  No $t \to 0$ extrapolation seems to be performed.

  ◇ Finite size effects
  The smallest aspect ratio is $N_s/N_t = 3.2$.

5 Conclusion

The energy density and pressure of quark-gluon plasma (strictly, those of SU(3) gauge theory) are calculated by use of the gradient flow.

- The results are ”qualitatively consistent with the previous works”.
  $\leftarrow$ i.e. the authors fail to reproduce the previous results.
- Although the authors do not show the origins of the discrepancies, they seem to be the followings.
  - ◊ $t \to 0$ extrapolation
  - ◊ Finite size effects
- The method is interesting. More detailed analysis is needed.

Appendix

[Energy-momentum tensor]
The energy-momentum tensor is a Noether current associated with the space-time translational symmetry.

$$\text{For } x \to x' = x + \xi, \phi'(x') = \phi(x). \tag{6}$$

Then, the energy-momentum tensor $T_{\mu\nu}$ is defined as,

$$T_{\mu\nu} := \frac{\partial L}{\partial(\partial_\nu \phi)} \partial_\mu \phi - \delta_{\mu\nu} L. \tag{7}$$

[ex. $T_{\mu\nu}$ for electromagnetic field]

$$L_{EM} = \frac{1}{16\pi} F^2 \tag{8}$$

$$T_{\mu\nu} = \frac{1}{4\pi} \left(F_{\mu\rho} F_{\nu\rho} - \frac{\delta_{\mu\nu}}{4} F^2 \right). \tag{9}$$