Black Holes: Complementarity or Firewalls

Almheiri, Marolf, Polchinski and Sully, arXiv:1207.3123[hep-th]

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Black Holes

Schwarzschild solution

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{1}{1 - \frac{2M}{r}} dr^2 - r^2 d\Omega^2$$

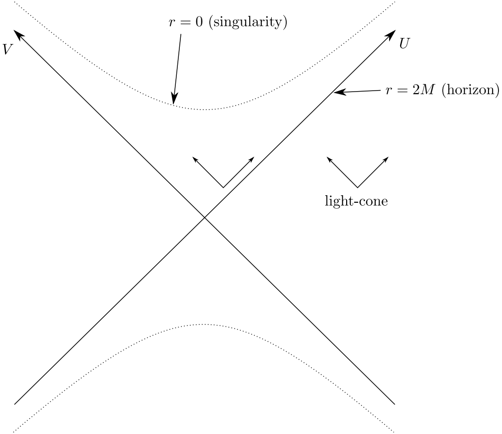
Kruskal coordinates

$$\begin{aligned} U &= -\left(\frac{r}{2M} - 1\right)^{\frac{1}{2}} e^{\frac{r-t}{4M}} \\ V &= \left(\frac{r}{2M} - 1\right)^{\frac{1}{2}} e^{\frac{r+t}{4M}} \end{aligned}$$

$$ds^2 = \frac{32M^3}{r} e^{-\frac{r}{2M}} dU dV - r^2 d\Omega^2$$

Black Holes

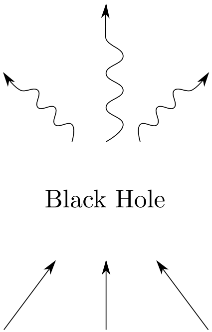
$$ds^2 = \frac{32M^3}{r} e^{-\frac{r}{2M}} dUdV - r^2 d\Omega^2$$



Nothing can come out from the black hole classically.

The information paradox

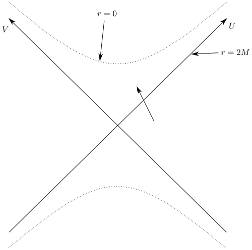
Quantum mechanically black holes radiate. If the black hole evaporates away



Even if one starts from a pure state, one ends up with a mixed state. (Hawking)
The time evolution should be unitary. ('tHooft, Susskind)

Mixed state

Freely falling observers (FFO)



- For FFO, nothing special is happening at the horizon. (equivalence principle) For big black holes, the horizon region looks like a Minkowski space with

$$T = \frac{U+V}{2}$$
$$X = \frac{U-V}{2}$$

Mixed state

- For FFO, the quantum field theory in the black hole background is like the ordinary one and

$$|\psi\rangle \sim |0\rangle$$

For outside observers

$$|\psi\rangle = \sum \sqrt{\rho_i}\, |i\rangle_{\text{inside}} \otimes |i\rangle_{\text{outside}}$$

and the system is described by the density matrix

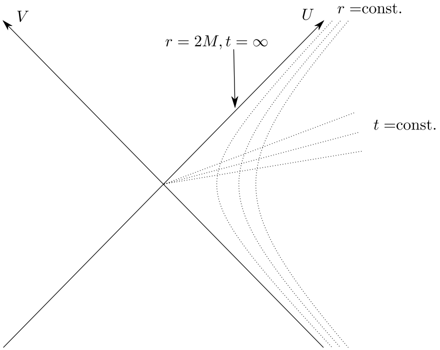
$$\rho = \mathrm{Tr}_{\text{inside}} |\psi\rangle \langle\psi| = \sum \rho_i |i\rangle \langle i|$$

- Radiation with temperature

$$T = \frac{1}{8\pi M}$$

Unitarity

Fiducial observer (FIDO) (t, r, θ, φ)



- For FIDO, it takes infinite time for matter to reach the horizon. There exists a membrane just outside the horizon.
- The Hawking radiation is from this membrane and the time evolution is unitary.

Complementarity

FIDO$\longleftrightarrow$FFO

- These two points of view should be comlementary.
- Time evolution is unitary and there is nothing special at the horizon for FFO.
- acceleration of FIDO relative to FFO

$$a(r) = \frac{M}{r^2}\left(1 - \frac{2M}{r}\right)^{-\frac{1}{2}}$$

- In order to show this, one needs a theory of quantum gravity.

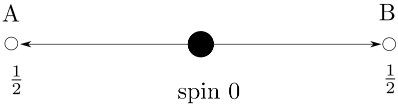
Complementarity

Is complementarity consistent with the principles of quantum mechanics?
In this paper, the authors argue that this is not the case.

1. No cloning theorem
2. Monogamy

1. No cloning theorem

It is impossible to clone a quantum mechanical state.
EPR



$$|\psi\rangle = \frac{|+\rangle_A |-\rangle_B - |-\rangle_A |+\rangle_B}{\sqrt{2}}$$

- If one mesures the z-component of the spin of A, the wave function of B becomes $|\pm\rangle_B$. The density matrix of B does not change by doing this, and no information is sent.
- If one can clone B, one can see that the mearurement is made for A.

No cloning theorem

Suppose we can clone states:

$$
\begin{aligned}
U\left(|\psi\rangle \otimes |s\rangle\right) &= |\psi\rangle \otimes |\psi\rangle \\
U\left(|\phi\rangle \otimes |s\rangle\right) &= |\phi\rangle \otimes |\phi\rangle
\end{aligned}
$$

Taking the inner product ot these

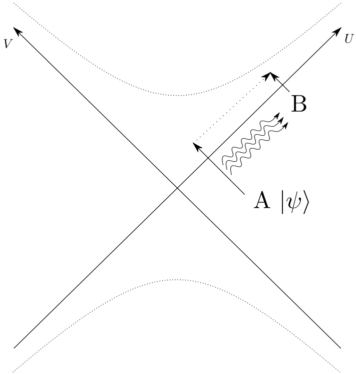
$$\langle\phi|\psi\rangle = \left(\langle\phi|\psi\rangle\right)^2$$

and we get

$$\langle\phi|\psi\rangle = 0, 1$$

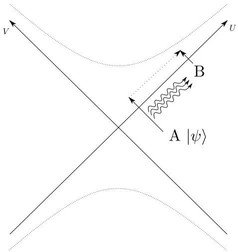
Black Holes

It looks possible to clone states by using black holes, if the complementarity is correct.



- Since the Hawking radiation is unitary, B can recover $|\psi\rangle$ from the Hawking radiation.
- B jumps into the black hole and A sends $|\psi\rangle$. B gets two copies of $|\psi\rangle$.

No cloning theorem



- B can get the information of $|\psi\rangle$ at $t \sim M^3$ and $U \sim e^{M^2}$.
- In order for A to send information before B hits the singularity, it should be with energy $\sim Me^{M^2} \gg M$. This will completely deform the geometry.

2. Monogamy

- If $|\psi\rangle_{AB}$ is a pure state and not in the form $|a\rangle_A |b\rangle_B$, for example
$$|\psi\rangle_{AB} = \frac{|+\rangle_A |-\rangle_B - |-\rangle_A |+\rangle_B}{\sqrt{2}} \ ,$$
A and B are said to be maximally entangled.
- Maximal entanglement is monogamous. If B is maximallyentangled with A, B cannot be maximally entangled with C.

Monogamy

- For a system with density matrix ρ, the entropy is defined to be

$$S \equiv -\mathrm{Tr}\rho \ln \rho$$

- For a system which consists of subsystems A, B and with the density matrix ρ_{AB}, we define

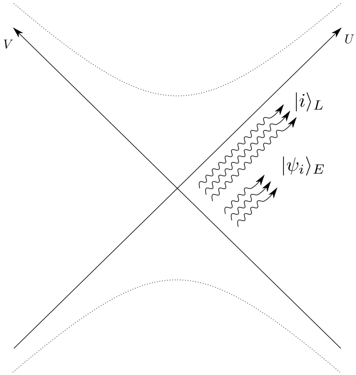
$$S_{AB} \equiv -\mathrm{Tr}_{AB}\rho_{AB} \ln \rho_{AB}\,, \rho_A \equiv \mathrm{Tr}_B \rho_{AB}\,, S_A \equiv -\mathrm{Tr}_A \rho_A \ln \rho_A\,, \text{etc.}$$

- The following inequalities hold

$$\begin{aligned} S_{ABC} + S_B \quad &\leq \quad S_{AB} + S_{BC} && (1) \\ |S_A - S_B| \leq \quad S_{AB} \quad &\leq S_A + S_B && (2) \end{aligned}$$

- If A and B are maximally entangled, $S_{AB} = 0$. From (2), we get $S_C \leq S_{ABC} \leq S_C$ and therefore $S_{ABC} = S_C$. Substituting these into (1), we get $S_C + S_B \leq S_{BC}$. If B and C are also maximally entangled, $S_B = S_C = 0$ and A,B,C are all pure.

Monogamy



- Early radiation and late radiation are maximally entangled.

$$|\psi\rangle = \sum_i |\psi_i\rangle_E \otimes |i\rangle_L$$

- Therefore $|i\rangle_L$ cannot be maximally entangled with the modes inside. For FFO, this means that they do not see vaccuum at the horizon but a firewall.