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Space-time S-matrix and flux-tube S-matrix at finite coupling

B. Basso, A. Sever and P. Vieira, arXiv: 1303.1396

Introduction

- Integrability of gauge-string duality

  [planar N=4 SYM $\longleftrightarrow$ string on AdS5 x S5]

$\Rightarrow$ quantitative analysis of SYM at strong/finite coupling

  - spectrum
  - scattering amplitudes (SAm)/Wilson loops (WL)
  - correlation functions
  - quark-anti-quark potential/cusp anomalous dim.

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- Also, stimulates weak–coupling analysis

  $\Rightarrow$ N=4 SYM: "harmonic oscillator of gauge theories"

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Basic idea :

- WL are decomposed by "OPE"
- in intermediate channels, flux–tube excitations propagate
- these excitations are well described based on
  su(2|2) x su(2|2) world–sheet (2 dim.) S–matrix
- these pictures + "3 axioms"
  $\Rightarrow$ 4 dim. SAm/WL at $\forall$ coupling

Plan

1. Introduction

2. Wilson loop OPE and flux tube

3. Pentagon transition and 3 axioms

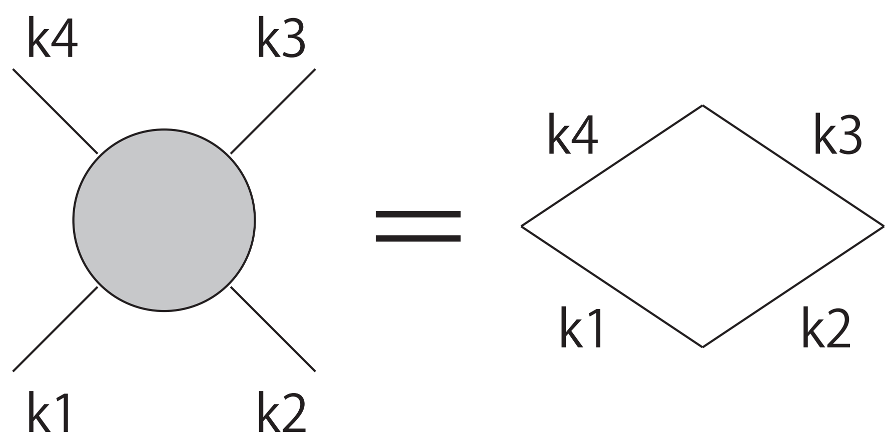
4. Solution for gauge field

5. Summary

Wilson loop OPE and flux tube

[Alday, Gaiotto, Maldacena, Sever and Vieira, arXiv:1006.2788]

- scattering amplitude of N=4 SYM ≈ null polygonal WL

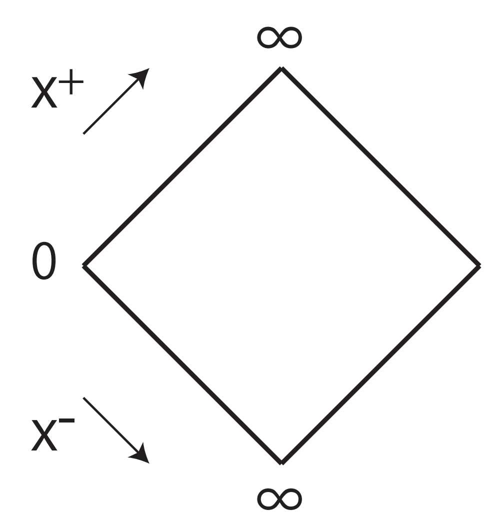


- null square has 3 commuting symmetries

$$E := D + M^{+-} \sim x^{+}\partial_{+} = \partial_{\tau}$$

$$p := D - M^{+-} \sim x^{-}\partial_{-} = \partial_{\sigma}$$

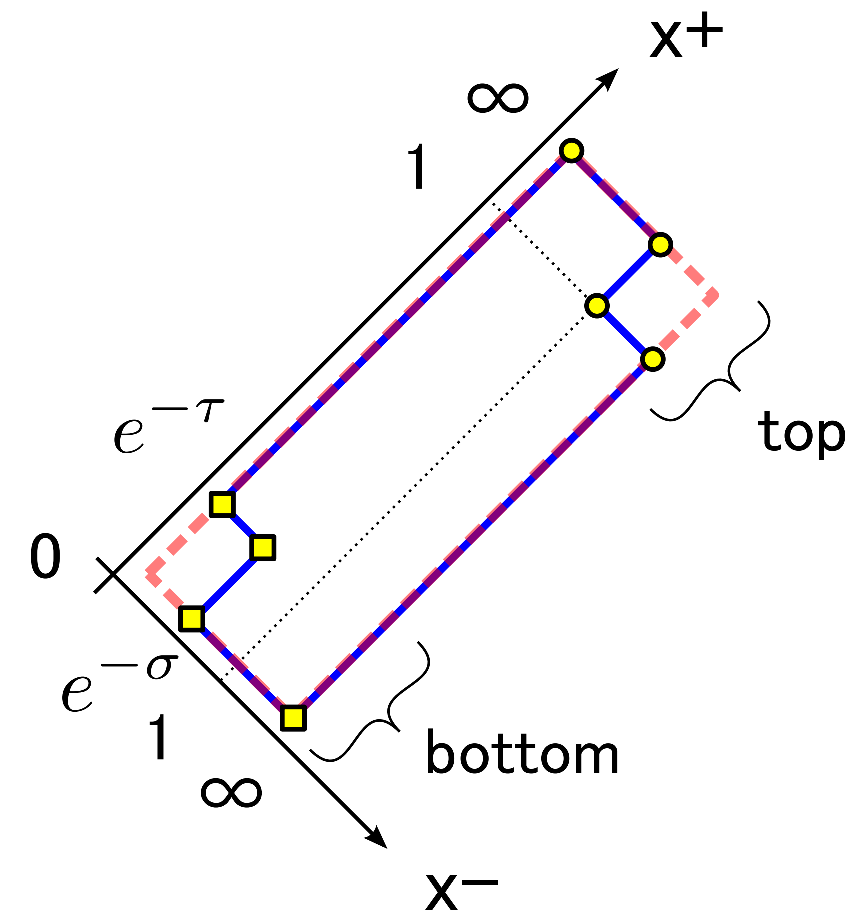
$$m := M^{12} = \partial_{\phi}$$

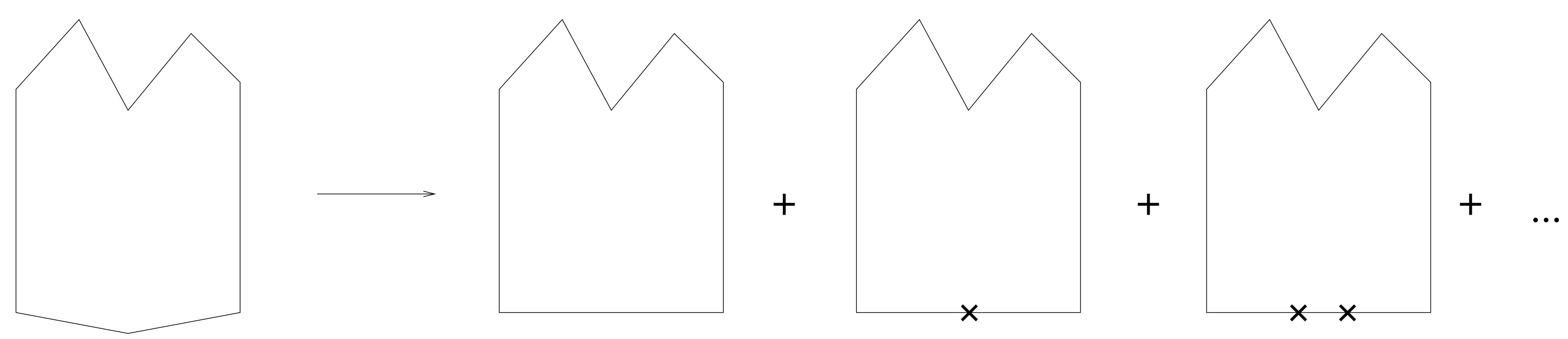


- WL can be split into "top" and "bottom" part

- acting w/ E on either part, top or bottom edge is flatten for large τ

- deformation corresponds to insertion of operators, e.g., $F_{+\mu}$

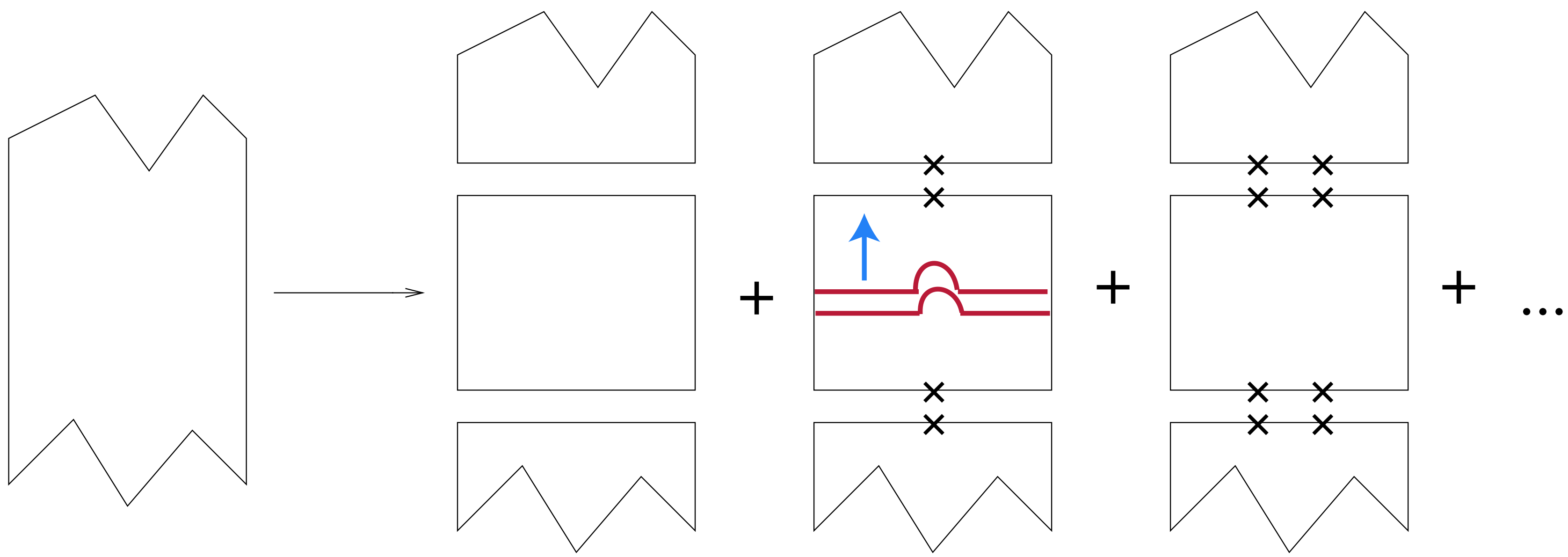




- these considerations

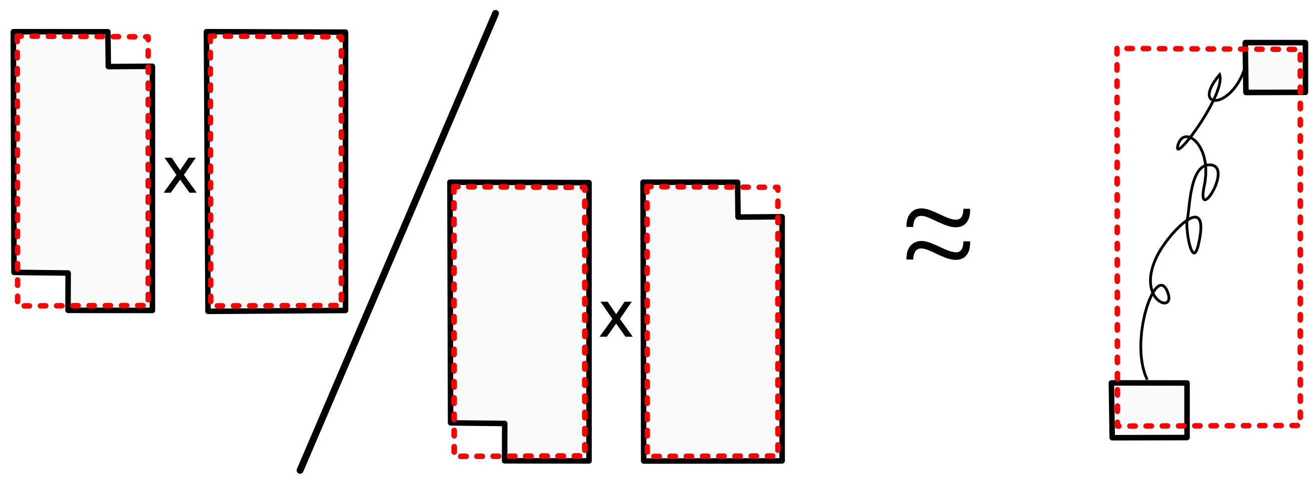
  $\Rightarrow$ WL can be expanded around $\tau \to \infty$ by eigenstates of (E,p,m) propagating in null square

  = flux–tube (FT) excitations



$$\langle W \rangle = \sum_n e^{-E_n\tau + ip_n\sigma + im_n\phi} C_n^{\text{top}} C_n^{\text{bottom}}$$

cf. appropriate combination of WL at leading order



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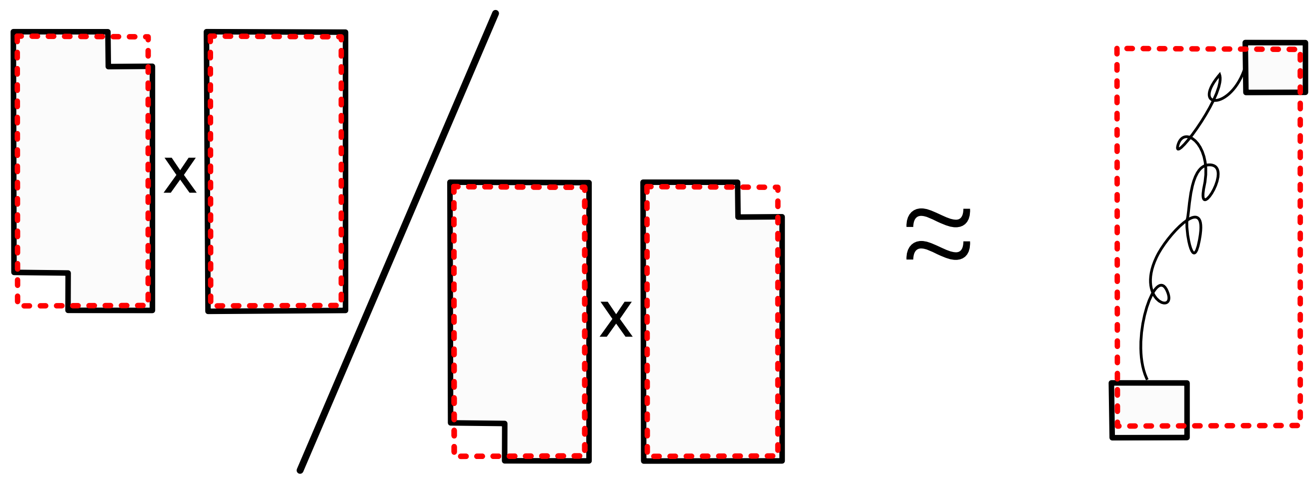

- expansion is rather natural from string point of view

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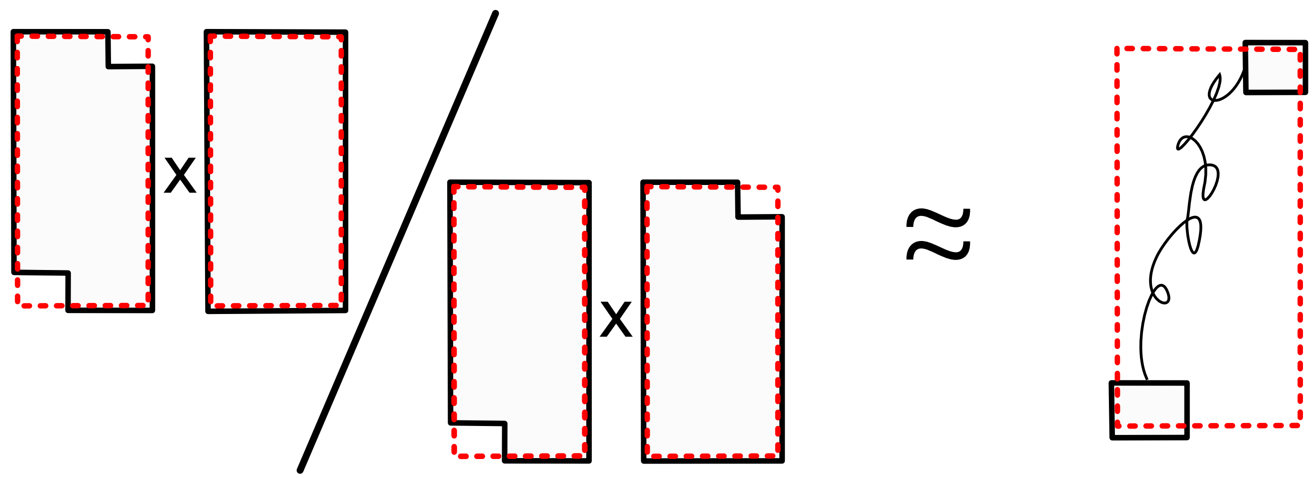


- expansion is rather natural from string point of view

- null Wilson line ≈ $\Phi D_+ D_+ \cdots D_+ D_+ \Phi$

  ⇒ FT excitation = excitation on SL(2)/GKP vacuum

  ⇒ E(p) is computed for ∀ coupling
  by machinery ("spin-chain") for spectral problem

[Beisert-Staudacher'05; Basso'10]

Based on WL OPE,

- numerical check for hexagon WL at 2 loops for large τ

  [Alday, Gaiotto, Maldacena, Sever and Vieira, arXiv:1006.2788]

- derivation of general n–gon WL in $R^{1,1}$ at 2 loops

  [Gaiotto, Maldacena, Sever and Vieira, arXiv:1010.5009]

- derivation of hexagon WL at 2 loops

  [Gaiotto, Maldacena, Sever and Vieira, arXiv:1102.0062]

- predictions for heptagon WL for large τ

  [Sever and Vieira, arXiv:1105.5748]

- super WL/non–MHV, 6–pt. NMHV at 1 loop etc

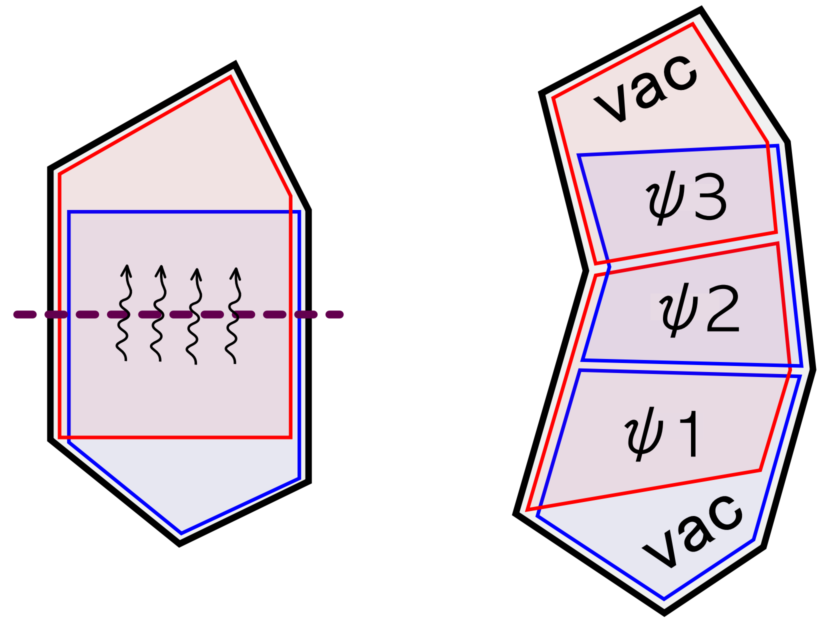
  [Sever, Vieira and Wang, arXiv:1108.1575]

- many–particle exchange, matching of "T–fn."
  btw weak and strong coupling in "FT lim."

  [Sever, Vieira and Wang, arXiv:1208.0841]

Pentagon transition and 3 axioms

- according to flux–tube (FT) picture, WL are decomposed as



- each pentagon represents transition of FT excitations

  = "pentagon transition" $\mathcal{P}(\psi_i|\psi_j)$

$$\Rightarrow \quad \langle W_{\text{n-gon}} \rangle = \sum_{\psi_i} e^{\sum_j (-E_j \tau_j + i p_j \sigma_j + i m_j \phi_j)}$$

$$\times \mathcal{P}(0|\psi_1)\mathcal{P}(\psi_1|\psi_2) \cdots \mathcal{P}(\psi_{n-4}|\psi_{n-5})$$

- excitation (state) is labeled by species and rapidities

$$\mathbf{a} = \{a_1, ..., a_N\}\,, \quad \mathbf{u} = \{u_1, ..., u_N\}$$

$$\Rightarrow \quad \mathcal{P}(\psi_i|\psi_j) \equiv P_{\mathbf{a}_i,\mathbf{a}_j}(\mathbf{u}_i|\mathbf{u}_j)$$

$$E = 2ig^2(1/x^+ - 1/x^-)\,, \qquad e^{ip} = x^+/x^-$$

$$x(u) = \frac{1}{2}(u + \sqrt{u^2 - 4g^2})\,, \qquad x^\pm(u) = x(u \pm i/2)$$

g : coupling

- in the following, consider gluonic transitions

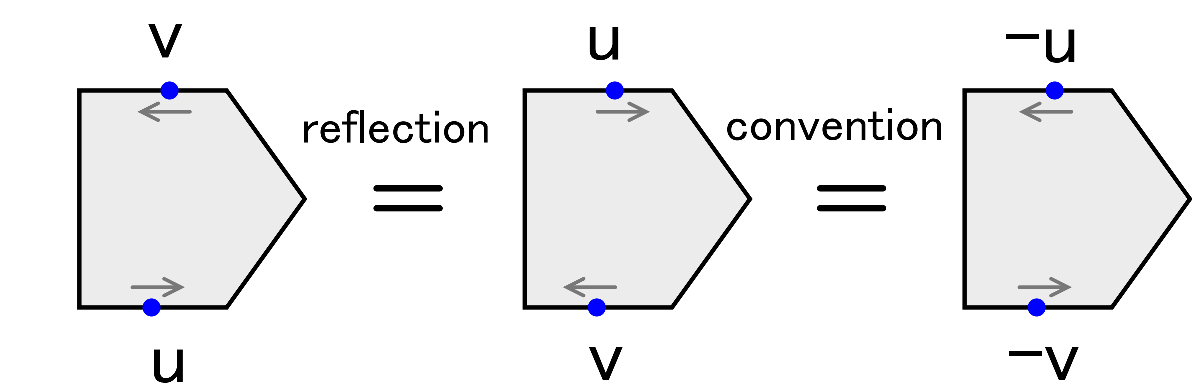
$$P(u|v) := P_{FF}(u|v)\,, \quad \overline{P}(u|v) := P_{F\bar{F}}(u|v)$$

$$F := F_{z-}\,, \quad \bar{F} := F_{\bar{z}-}$$

3 axioms :

1. reflection symmetry

$$P(-u|-v) = P(v|u)$$



2. fundamental relation

$$P(u|v) = S(u,v)P(v|u)$$

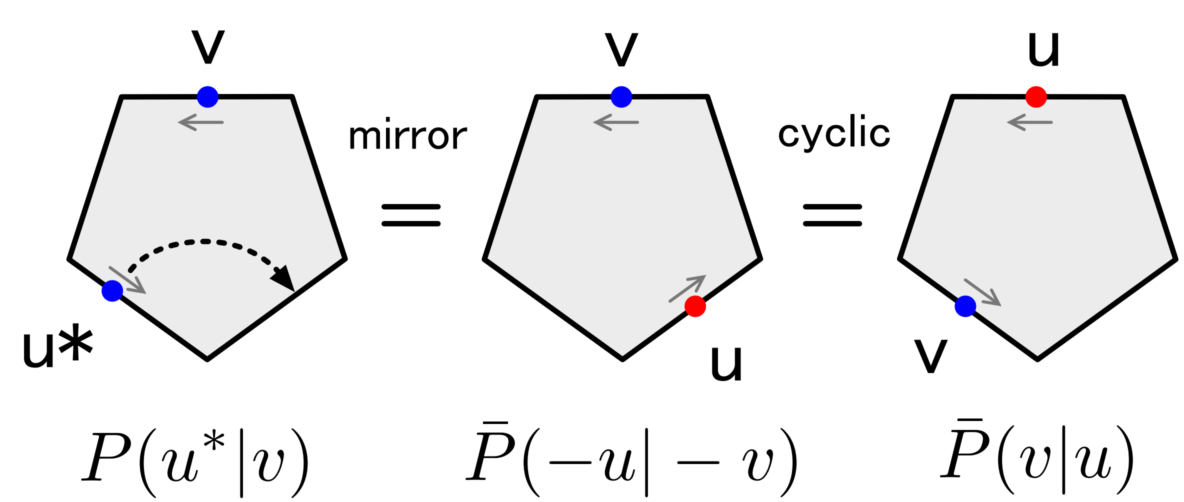
S(u,v) : 2 dim. S−matrix for spectral problem

3. mirror transformation

$$P(u^*|v) = \bar{P}(v|u)$$

$$E(u^*) = -ip(u)$$

$$p(u^*) = -iE(u)$$



Solution for gauge field

- following expression satisfies axioms

$$P(u|v)^2 = \left[\frac{f(u,v)}{g^2(u-v)(u-v-i)}\right]^\eta \frac{S(u,v)}{S(u,v^*)}\,, \quad \eta = 1$$

$$\bar{P}(u|v)^2 = [\eta = 1 \to -1]$$

$$\begin{aligned} f(u,v) = x^+ x^- y^+ y^- \left(1 - g^2/x^+ y^-\right)\left(1 - g^2/x^- y^+\right) \\ \left(1 - g^2/x^+ y^+\right)\left(1 - g^2/x^- y^-\right) \end{aligned}$$

Evidence

- comparison w/perturbative results for large τ
  - agreement w/ hexagon at 3 loops
  - agreement w/ the symbol of heptagon at 2 loops

- comparison w/ strong-coupling results for large τ
  - agreement w/ TBA results

Summary

- based on Wilson loop OPE, this paper announced that one can compute SAm/WL of N=4 SYM at $\forall$ coupling

- agreement w/ weak and strong coupling results

- details will be given in future publications

[Basso, Sever and Vieira, to appear]

[Basso and Rej, to appear]