

Nucleon-Nucleon Scattering Parameters in the Limit of $SU(3)$ Flavor Symmetry

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Parameters

- | L | T | beta | b(fm) | L(fm) | T(fm) | m_π | m_π L | m_π T | Ncfg | Nsrc |
|----|----|------|-------|-------|-------|----------------------|-----------|-----------|------|------|
| 24 | 48 | 6.1 | 0.145 | 3.4 | 6.7 | 806.5(0.3)(0)(8.9) | 14.3 | 28.5 | 3822 | 96 |
| 32 | 48 | 6.1 | 0.145 | 4.5 | 6.7 | 806.9(0.3)(0.5)(8.9) | 19.0 | 28.5 | 3050 | 72 |
| 48 | 64 | 6.1 | 0.145 | 6.7 | 9.0 | 806.7(0.3)(0)(8.9) | 28.5 | 38.0 | 1905 | 54 |

errors:(statistical)(fit systematic)(lattice spacing)

Three ensembles of isotropic gauge-field configurations, generated with a tadpole-improved Lüscher-Weisz gauge action and a clover fermion action [15], are used in this work and have been used previously to calculate the lowest-lying levels of the s-shell nuclei and hypernuclei [13]. This particular lattice-action setup follows closely the anisotropic clover action of the ensembles generated by the JLab group that we have used in our previous calculations [4, 11, 16–19]. The parameter tuning and scaling properties of this action will be discussed elsewhere [20]. One level of stout smearing [21] with $\rho = 0.125$ and tadpole-improved tree-level clover coefficient $c_{\text{SW}} = 1.2493$ are used in the gauge-field generation. Studies [20, 22, 23] of the partially-conserved axial-current (PCAC) relation in the Schrödinger functional indicate that this choice is consistent with vanishing $\mathcal{O}(b)$ violations, leading to discretization effects that are essentially $\mathcal{O}(b^2)$. The parameters of the ensembles are listed in Table I, and further details will be presented elsewhere [20]. As two-nucleon systems are

Nucleon operator

$$\mathcal{B}_N^{ijk}(\mathbf{p}, t; x_0) = \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} S_i^{(f_1),i'}(\mathbf{x}, t; x_0) S_j^{(f_2),j'}(\mathbf{x}, t; x_0) S_k^{(f_3),k'}(\mathbf{x}, t; x_0) b_{i'j'k'}^{(N)} \quad , \quad (1)$$

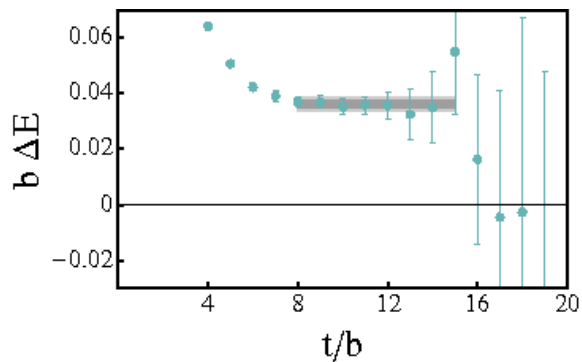
where $S^{(f)}$ is a quark propagator of flavor f , and the indices are combined spin-color indices running over $i = 1, \dots, N_c N_s$.¹ The choice of the f_i and the tensor $b^{(N)}$ depend on the

$$m_\pi = 805.9(0.6)(0.4)(8.9) \text{ MeV}$$

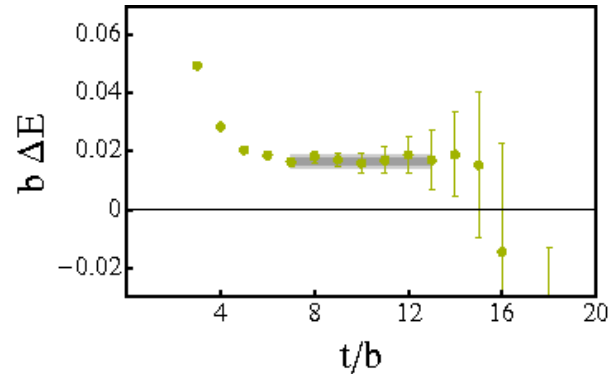
$$m_N = 1.635(0)(0)(18) \text{ GeV}$$

NN scattering in 1S0 channel

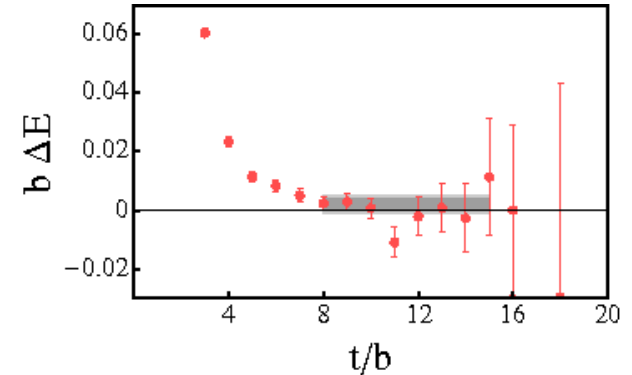
Effective mass of the first excited state



$L=3.4\text{fm}$



$L=4.5\text{ fm}$



$L=6.7\text{fm}$

Volume effects are observed in the smallest volume

The binding energy in the infinite volume is taken from the binding energy in the largest volume

Lowest-lying continuum states in $1S0$

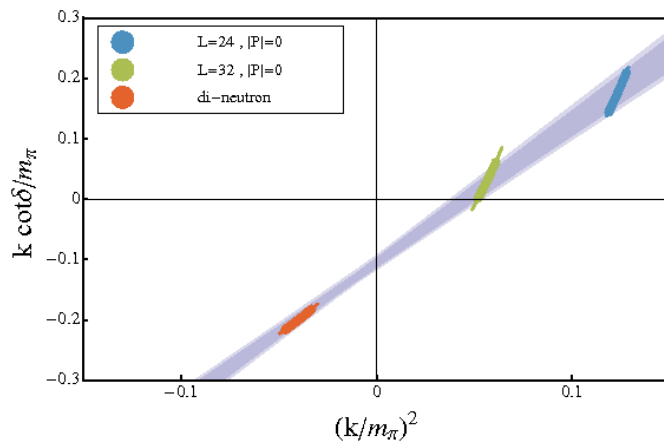
	ΔE b	$ k /m\pi$	$k\cot\delta / m\pi$
$24^3 \times 48$	0.0358(13)(16)	0.3506(64)(78)	0.175(+.034 -0.031)(+0.043 -0.036)
$32^3 \times 48$	0.0165(13)(22)	0.2373(92)(96)	0.030(+0.031 -0.028)(+0.057 -0.046)

binding energy

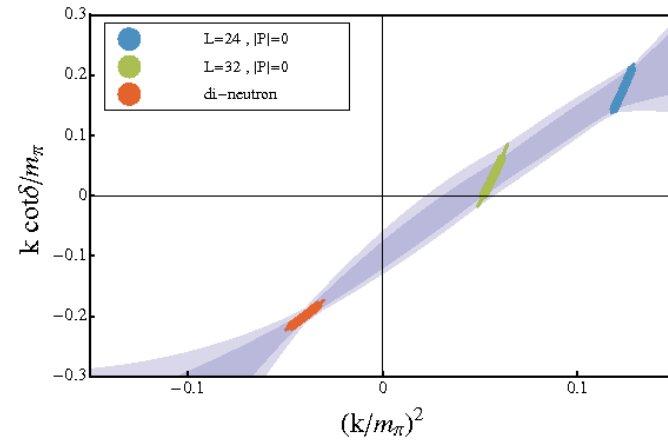
$B_{nn} = 15.9(2.7)(2.7)(0.2)$ MeV

from 3.4 fm, 4.5 fm, 6.7 fm

K cot delta in 1S0 channel



2-parameter fit



3-parameter fit

$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} r |k|^2 + P |k|^4 + O(|k|^6)$$

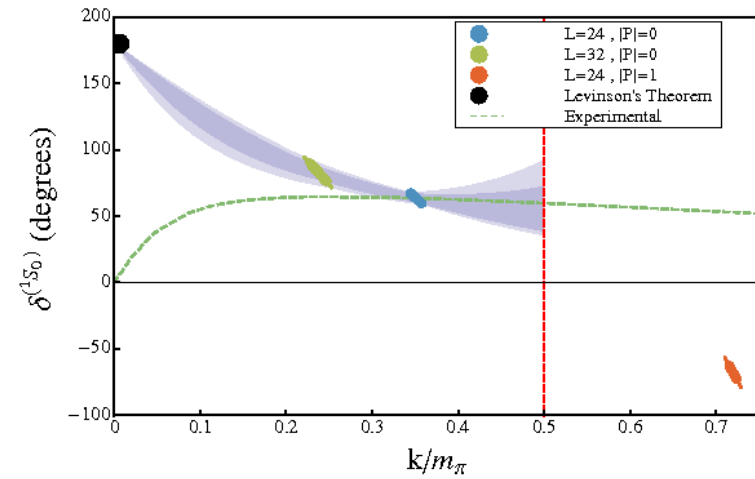
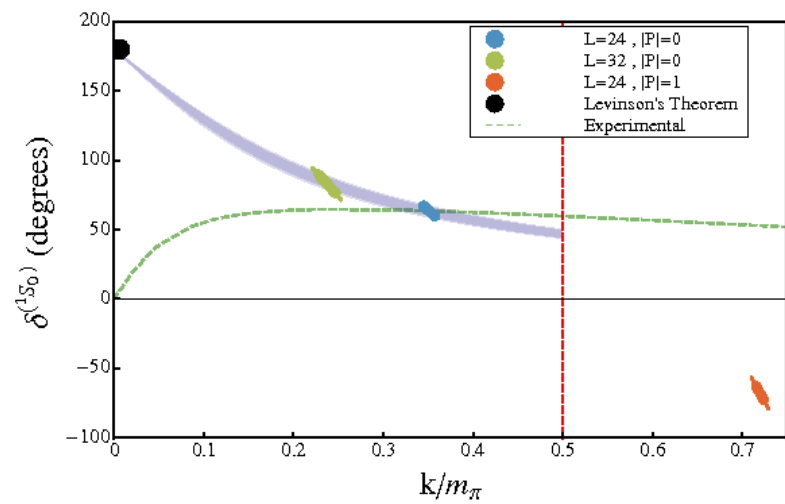
$$a({}^1S_0) = 2.33_{-0.17}^{+0.19} {}_{-0.20}^{+0.27} \text{ fm} \quad r({}^1S_0) = 1.130_{-0.077}^{+0.071} {}_{-0.063}^{+0.059} \text{ fm}$$

P: consistent with zero

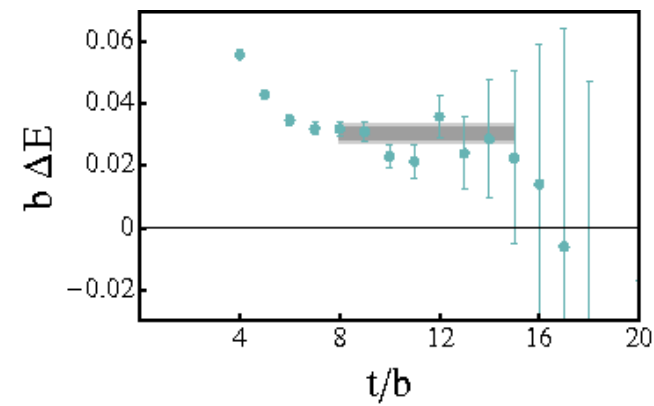
$$P m_\pi^3 = -1_{-5}^{+4} {}_{-8}^{+5}$$

cf. exp.: $a_{nn} = -18.9 \pm 0.4 \text{ fm}$ $r_{nn} = 2.75 \pm 0.11 \text{ fm}$
 $a_{np} = -23.740 \pm 0.020 \text{ fm}$ $r_{np} = 2.77 \pm 0.05 \text{ fm}$

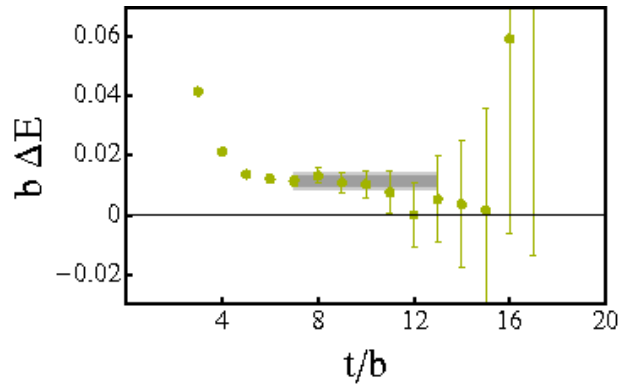
Phase shift in 1S0 channel



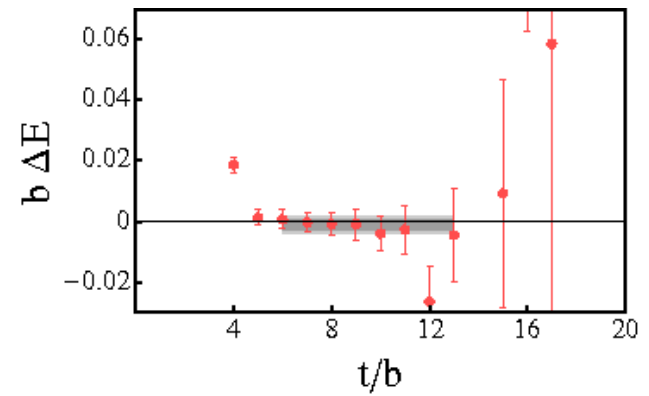
NN scattering in 3S1 channel



$L=3.4\text{fm}$



$L=4.5\text{ fm}$



$L=6.7\text{fm}$

Lowest-lying continuum states in 3S_1

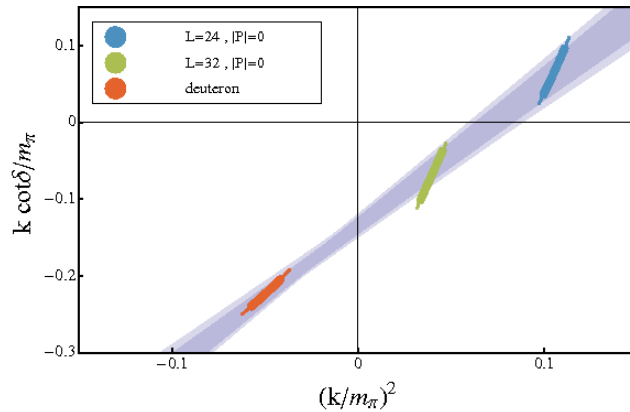
- | | ΔE b | $ k /m\pi$ | $k\cot\delta / m\pi$ |
|-----------|----------------|---------------|----------------------------------|
| 24^3x48 | 0.0306(16)(23) | 0.324(8)(12) | 0.065(+.031 -0.029)(+0.47 -0.40) |
| 32^3x48 | 0.0115(17)(23) | 0.198(15)(19) | -0.069(32)(43) |

binding energy

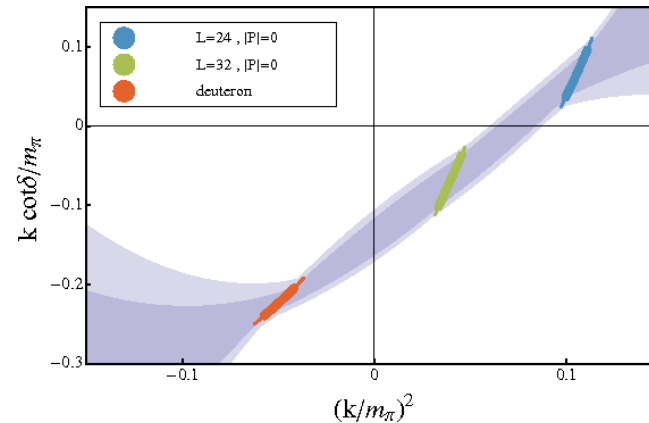
$B_{nn} = 19.5(3.6)(3.1)(0.2)$ MeV

from 3.4 fm, 4.5 fm, 6.7 fm

K cot delta in 3S_1 channel



2-parameter fit



3-parameter fit

$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} r |k|^2 + P |k|^4 + O(|k|^6)$$

$$a(^3S_1) = 1.82^{+0.14+0.17}_{-0.13-0.12} \text{ fm}$$

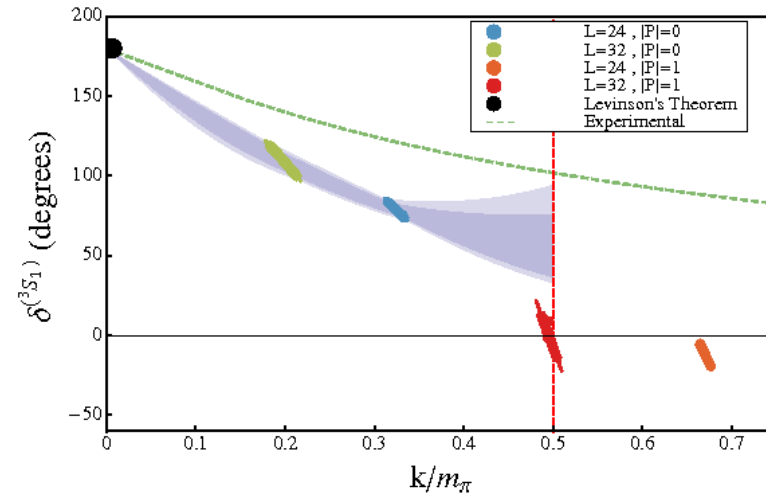
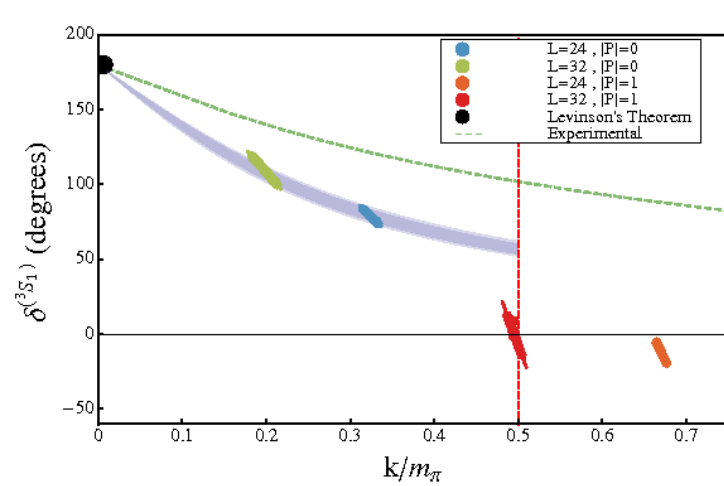
P: consistent with zero

$$r(^3S_1) = 0.906^{+0.068+0.068}_{-0.075-0.084} \text{ fm}$$

$$P m_\pi^3 = 3^{+5+5}_{-6-6}$$

cf. exp.: $a_{np} = 5.419 \pm 0.007 \text{ fm}$ $r_{np} = 1.753 \pm 0.008 \text{ fm}$

Phase shift in 3S1 channel



conclusions

- Low-energy NN scattering parameters at SU(3) symmetric point
 $m_{\pi} = 800 \text{ MeV}$
- Effective ranges are calculated for the first time
- Phase shifts change sign at higher momentum, indicating that the nuclear interactions have a repulsive core

$$a/r \sim 2.0$$

- Experimental values
 $a/r \sim -8.7 (1S0), +3.1(3S1)$
- Present work : performed at a single pion mass with one lattice spacing