Light-cone gauge superstring field theory in linear dilaton background

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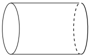
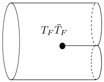
Light-cone gauge closed super SFT

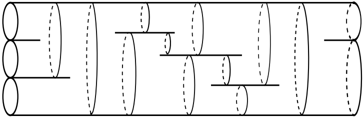
$$S = \int \left[\frac{1}{2} \Phi \cdot \left(i\partial_t - \frac{L_0 + \tilde{L}_0 - 1}{p^+} \right) \Phi + \frac{g_s}{3} \Phi \cdot (\Phi * \Phi) \right]$$



propagator



vertex



Feynman amplitudes diverge.

Feynman amplitudes for superstrings suffer from

1. infrared divergences
2. spurious singularities
   (a) collisions of the picture changing operators (or T_F)
   (b) divergences of the $\beta\gamma$ partition function

- formulations using supermoduli space (Witten)
- avoind the singularities patchwise (Sen-Witten)
- SFT with nonpolynomial interactions (Sen)

In the LC SFT, we do not have the problem (b) and may be able to deal with the problem with only the three string interaction.

We would like to get finite amplitudes

Strategy

We regularize the amplitudes, by considering the SFT in linear dilaton background

$$\Phi = -iQX^1$$

- The amplitudes become finite for $Q^2 > 10$.
- The amplitudes coincide with those obtained by the 1-st quantized approach in the limit $Q \to 0$.

Based on Murakami-N.I. JHEP 1606 (2016) 087

N.I. arXiv:106504666, Murakami-N.I. to appear

LC gauge super SFT in LD background

Linear dilaton background $\Phi = -iQX^1$ $(ds^2 = 2\hat{g}_{z\bar{z}}dzd\bar{z})$

$$S = \frac{1}{16\pi}\int dz \wedge d\bar{z} i\sqrt{\hat{g}}\left(\hat{g}^{ab}\partial_a X^1 \partial_b X^1 - 2iQ\hat{R}X^1\right)$$

We construct SFT (type II) with the worldsheet theory for $X^i, \psi^i, \bar{\psi}^i$ $(i = 1, \cdots 8)$

GO

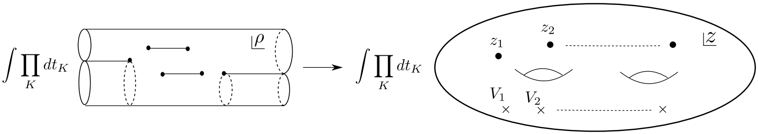
$$S = \int \left[\frac{1}{2}\Phi \cdot \left(i\partial_t - \frac{L_0 + \tilde{L}_0 - 1 + Q^2 - i\varepsilon}{p^+}\right)\Phi + \frac{g_s}{3}\Phi \cdot (\Phi * \Phi)\right]$$

LC gauge super SFT in LD background

Feynman amplitude GO

$$A_Q^{\mathrm{LC}} = \int \prod_K dt_K \left\langle \prod_{I=1}^{2g-2+N} \left| (\partial^2 \rho)^{-\frac{3}{4}} T_F^{\mathrm{LC}}(z_I) \right|^2 \prod_{r=1}^{N} V_r^{\mathrm{LC}} \right\rangle_{g_{z\bar{z}}} e^{-(1-Q^2)\Gamma}$$

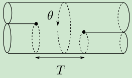
$$\equiv \int \prod_K dt_K F(\vec{t})$$



- $F(\vec{t})$ is expressed explicitly in terms of the theta functions defined on the Riemann surface.

Possible divergences arise from the combinations of

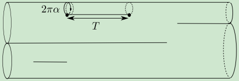
Contact term



$T \to 0$

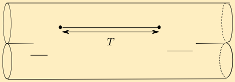
$\theta \to \theta_0$

Infinitely thin cylinder



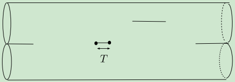
$\alpha \to 0$

Infinitely long cylinder

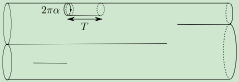


$T \to \infty$

Tiny neck



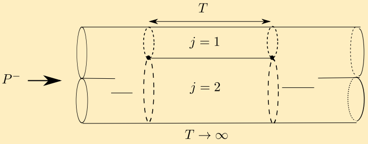
$T \to 0$



$T \sim \alpha \sim \epsilon \sim 0$

Finiteness

Infinitely long cylinder



$$\int_0^\infty dT \exp\left[-T\left(\sum_j \frac{L_0^{(j)} + \bar{L}_0^{(j)} - 1 + Q^2 - i\varepsilon}{\alpha_j} - P^-\right)\right]$$

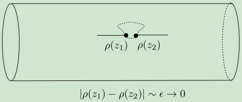
- Following Berera, Witten, we modify the contour as

$$\int_0^\infty dT \to \left(\int_0^{T_0} + \int_{T_0}^{T_0+i\infty}\right) dT$$

- The Feynman $i\varepsilon$ takes care of the divergences of this kind.

Finiteness

Contact term



$$F(\vec{t}) \sim \epsilon^{-\frac{10}{3}+\frac{1}{3}Q^2}$$

- For $Q = 0$, $F(\vec{t})$ becomes singular.
- For $Q^2 > 10$, $F(\vec{t})$ becomes regular.

- For $Q^2 > 10$, $\varepsilon > 0$, we find $F(\vec{t})$ is a continuous function without singularities and $A_Q^{\mathrm{LC}} = \int \prod_K dt_K F(\vec{t})$ is finite.
- We can define the amplitudes for $Q^2 > 10$ as analytic functions of Q and take the the limit $Q \to 0\, \varepsilon \to 0$.
- The results coincide with those of the first quantized approach.

Conclusions and discussions

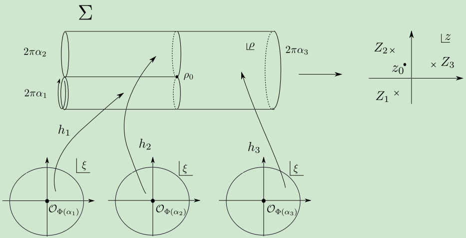
- In order to regularize the Feynman amplitudes, we consider light-cone gauge superstring field theory in linear dilaton background $\Phi = -iQX^1$.
- The amplitudes become finite for $Q^2 > 10$ and they can be defined as analytic functions of Q. The amplitudes without the background is given by the limit $Q \to 0$.
- The results coincide with those from the first quantized approach.

Outlook

- Equivalence of the amplitudes with odd spin structure.
- Our approach looks quite similar to the dimensional regularization in field theory, but there are crucial differences:

- The number of $\psi^i, \bar{\psi}^i$ is not changed. Therefore the number of the gamma matrices is not changed and we do not have any problems with fermions.
- We have a concrete theory for $Q \neq 0$. It may be possible to discuss nonperturbative problems using this approach.

Three-string vertex



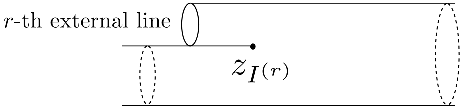
▸ BACK

$$
\begin{aligned}
\int \Phi_1 \cdot (\Phi_2 * \Phi_3) \quad = \quad & \int dt \prod_{r=1}^{3} \left(\frac{p_r^+ \, dp_r^+}{4\pi} \right) \delta \left(\sum_{r=1}^{3} p_r^+ \right) \\
& \times \left(p_1^+ p_2^+ p_3^+ \right)^{-\frac{1}{2}(1-Q^2)} e^{-(1-Q^2) \sum_r \frac{1}{p_r^+} \sum_{s=1}^{3} p_s^+ \ln \left| p_s^+ \right|} \\
& \times \left\langle \left| \partial^2 \rho(z_0) \right|^{-\frac{3}{2}} T_F^{\mathrm{LC}}(z_0) \, \bar{T}_F^{\mathrm{LC}}(\bar{z}_0) \right. \\
& \quad \left. \times \ \rho^{-1} h_1 \circ \mathcal{O}_{\Phi_1(t,\alpha_1)} \rho^{-1} h_2 \circ \mathcal{O}_{\Phi_2(t,\alpha_2)} \rho^{-1} h_3 \circ \mathcal{O}_{\Phi_3(t,\alpha_3)} \right\rangle_{\mathbb{C}}
\end{aligned}
$$

Anomaly factor

$$e^{-\Gamma} \propto \prod_{r=1}^{N} \left[\alpha_r^{-1} (g^{\mathrm{A}}_{Z_r \bar{Z}_r})^{-\frac{1}{2}} e^{-\mathrm{Re}\bar{N}^{rr}_{00}} \right] \prod_{I=1}^{2g-2+N} \left[(g^{\mathrm{A}}_{z_I \bar{z}_I})^{-\frac{1}{2}} \left| \partial^2 \rho(z_I) \right|^{-\frac{1}{2}} \right]$$

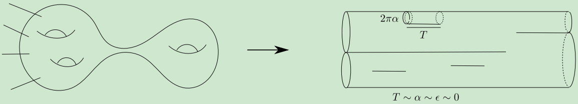
- $r = 1, \ldots, N$ label the punctures BACK
- $I = 1, \ldots, 2g-2+N$ label the interaction points, where $\partial\rho(z_I) = 0$.
- $g^{\mathrm{A}}_{z\bar{z}}$: Arakelov metric on the surface
- $\bar{N}^{rr}_{00} \equiv \frac{1}{p_r^+} \left(\rho(z_{I(r)}) - \lim_{z \to Z_r} \left(\rho(z) - p_r^+ \ln(z - Z_r) \right) \right)$



Remark BACK

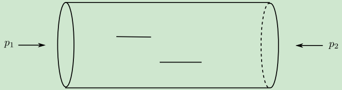
Tadpoles and mass renormalization are irrelevant to the limit $\varepsilon \to 0$.

- Tadpoles: belong to the "Tiny neck" category



- Mass renormalization: If p_1 is on-shell, p_2 is generically off-shell for $Q \neq 0$.

$$p_1^1 + p_2^1 + 2Q(1-g) = 0$$



$X^{\pm}$ CFT

$$
\begin{aligned}
S_{X^{\pm}} &= -\frac{1}{2\pi}\int d^2z d\theta d\bar{\theta}\left(\bar{D}X^{+}DX^{-}+\bar{D}X^{-}DX^{+}\right)-Q^2\Gamma_{\text{super}}[\Phi] \\
& X^{\pm}\equiv x^{\pm}+i\theta\psi^{\pm}+i\bar{\theta}\tilde{\psi}^{\pm}+i\theta\bar{\theta}F^{\pm} \\
& \Gamma_{\text{super}}[\Phi]=-\frac{1}{2\pi}\int d^2z d\theta d\bar{\theta}\left(\bar{D}\Phi D\Phi+\theta\bar{\theta}\hat{g}_{z\bar{z}}\hat{R}\Phi\right) \\
& \Phi\equiv\ln\left|\partial X^{+}-\frac{\partial DX^{+}DX^{+}}{(\partial X^{+})^2}\right|^2-\ln\hat{g}_{z\bar{z}}
\end{aligned}
$$

- This theory can be formulated in the case $\langle\partial_m X^{+}\rangle\neq 0$.
- In the case of the LC gauge amplitudes, we always have $\prod e^{-ip_r^{+}X^{-}}$ $(p_r^{+}\neq 0)$ and $\langle\partial_m X^{+}\rangle\neq 0$.

$X^{\pm}$ CFT ▸ BACK

$$\begin{aligned}
S_{X^{\pm}} &= -\frac{1}{2\pi}\int d^2z d\theta d\bar{\theta}\left(\bar{D}X^{+}DX^{-}+\bar{D}X^{-}DX^{+}\right)-Q^2\Gamma_{\text{super}}\left[\Phi\right] \\
T\left(z,\theta\right) &= G\left(z\right)+\theta T\left(z\right) \\
&= \frac{1}{2}:\partial X^{+}DX^{-}\left(\mathbf{z}\right):+\frac{1}{2}:DX^{+}\partial X^{-}\left(\mathbf{z}\right):+2Q^2S\left(\mathbf{z},\boldsymbol{X}^{+}\right)
\end{aligned}$$

- It is a superconformal field theory with $\hat{c}=2+8Q^2$.
- The worldsheet theory becomes BRST invariant

$$\begin{array}{ccccccccc}
 & & X^{\pm} & & X^{i} & & \text{ghosts} & & \\
\hat{c} & = & 2+8Q^2 & + & 8-8Q^2 & - & 10 & = & 0
\end{array}$$

Comparison with the first quantized approach

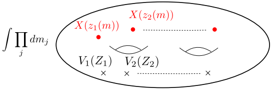
The LC amplitude can be recast into a conformal gauge expression (even spin structure)

$$
\begin{aligned}
A_Q^{\mathrm{LC}} &= \int \prod_K dt_K \left\langle \prod_{I=1}^{2g-2+N} \left| \left(\partial^2\rho\right)^{-\frac{3}{4}} T_F^{\mathrm{LC}}(z_I) \right|^2 \prod_{r=1}^{N} V_r^{\mathrm{LC}} \right\rangle_{g^{\mathrm{A}}_{z\bar{z}}} e^{-(1-Q^2)\Gamma} \\
&= \int \prod_j dm_j \left\langle \prod_j \oint \left(\mu_j b + \bar{\mu}_j \bar{b}\right) \prod_{I=1}^{2g-2+N} X(z_I)\bar{X}(\bar{z}_I) \prod_{r=1}^{N} V_r^{\mathrm{conf.}} \right\rangle^{X^\mu, \psi^\mu, \mathrm{ghosts}}
\end{aligned}
$$

- with a nontrivial CFT for $X^\pm, \psi^\pm$ ($X^\pm$ CFT). (Murakami-N.I.) GO
- $X(z) = -e^\phi G + c\partial\xi + \frac{1}{4}\partial b \eta e^{2\phi} + \frac{1}{4} b \left(2\partial\eta e^{2\phi} + \eta\partial e^{2\phi}\right)$: picture changing operator (PCO)
- PCO's are placed at the interaction points.

First quantized approach (Verlinde-Verlinde)

$$A_Q^{VV} = \int_{\mathcal{M}} \prod_j dm_j \left\langle \prod_j \oint (\mu_j b + \bar{\mu}_j \bar{b}) \prod_i^{2g-2+N} X(z_i(m)) \bar{X}(\bar{z}_i(m)) \prod_{r=1}^{N} V_r^{\text{conf.}} \right\rangle$$



- If the PCO's are placed at $z = z_i(m)$, but the amplitudes suffer from the so called spurious singularities.

- Sen-Witten gave a prescription to write down amplitudes placing PCO's avoiding the spurious singularities patchwise.

$$A_Q^{SW} = \sum_\alpha \int_{\mathcal{M}^\alpha} \prod_j dm_j \left\langle \prod_j \oint (\mu_j b + \bar{\mu}_j \bar{b}) \prod_i^{2g-2+N} X(z_i(m)) \bar{X}(\bar{z}_i(m)) \prod_{r=1}^{N} V_r^{\text{conf.}} \right\rangle + \cdots$$

$A_Q^{\mathrm{LC}} = A_Q^{SW}$

- When $Q^2 > 10$,

$$\begin{aligned} A_Q^{SW} &= \sum_\alpha \int_{\mathcal{M}^\alpha} \prod_j dm_j \left\langle \prod_j \oint \left(\mu_j b + \bar{\mu}_j \bar{b}\right) \prod_i^{2g-2+N} X(z_i(m))\bar{X}(\bar{z}_i(m)) \prod_{r=1}^{N} V_r^{\mathrm{conf.}} \right\rangle \\ &\quad + \cdots \\ &= \int_{\mathcal{M}} \prod_j dm_j \left\langle \prod_j \oint \left(\mu_j b + \bar{\mu}_j \bar{b}\right) \prod_{I=1}^{2g-2+N} X(z_I)\bar{X}(\bar{z}_I) \prod_{r=1}^{N} V_r^{\mathrm{conf.}} \right\rangle \\ &= A_Q^{\mathrm{LC}} \end{aligned}$$

because

- putting $z_i(m) = z_I$ does not make the amplitude diverge
- Sen-Witten prescription does no depend on the choice of $z_i(m)$ ▸ BACK

Therefore as an analytic function of Q, $A_Q^{\mathrm{LC}} = A_Q^{SW}$.

We can get $\lim_{Q\to 0} A_Q^{\mathrm{LC}} = A_0^{SW}$, if A_0^{SW} is well-defined.