Onogi-san as a string theorist

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Outline

1. Onogi-san started his career as a string theorist.
2. Mirzakhani's idea
3. Fokker-Planck formalism for closed bosonic strings
4. Conclusion

1. Onogi-san started his career as a string theorist.

Graduate school days (1984-1989)

- Onogi-san was my classmate in University of Tokyo and we entered the graduate school of University of Tokyo in 1984.
- 1984 is the year of "the first superstring revolution". We (almost all the students at that time) studied and worked on string theory.
- Onogi-san had a good reason to start his career as a string theorist.
- His mentor was Yoneya-san.
- Yoneya-san is one of the fathers of string theory. He discovered that string theory can be used to describe quantum gravity.

His first paper

Progress of Theoretical Physics, Vol. 78, No. 1, July 1987

One Loop Integration Region in Closed Light-Cone String Field Theory

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(Received February 18, 1987)

We have studied the integration region of the one-loop two-point amplitude in closed light-cone string field theory and confirmed that it has one to one correspondence with the moduli space of the torus with two punctures.

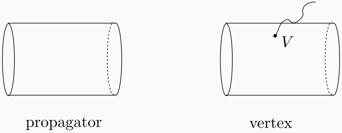
§ 1. Introduction

One of the problems in string field theory is to construct covariant string field theory which contains only closed strings. For open strings there are two theories proposed by Kyoto group[1] and by Witten.[2] If one tries to extend the formalisms to closed strings, severe difficulties arise. For example, in the string field theory which

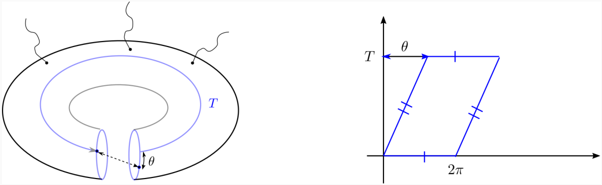
- Onogi-san's first paper is with Ito-san (TIT).

One-loop amplitude for strings

- Amplitudes are calculated using the “Feynman rule”



- One-loop Feynman graph



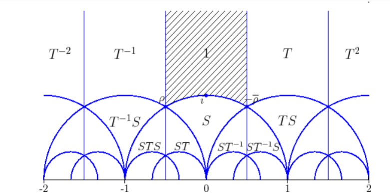
- The amplitude would be $A \sim \int_0^\infty dT \int_0^{2\pi} d\theta \langle V_1 \cdots V_n \rangle$

Modular invariance

$$A \sim \int_0^\infty dT \int_0^{2\pi} d\theta \langle V_1 \cdots V_n \rangle$$

$\tau = \frac{\theta + iT}{2\pi}$

$\lfloor \tau$



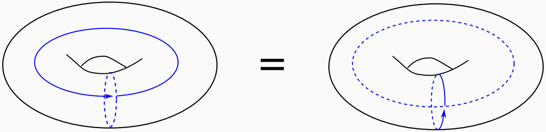
from "Reduction theory of binary forms" by Lubjana BESHAJ arXiv:1502.06289

- This is divergent because of the modular invariance

$$\tau \to \frac{a\tau + b}{c\tau + d}, \;\; a, b, c, d \in \mathbb{Z}, ad - bc = 1$$

Modular invariance

- modular transformation $\tau \to \frac{-1}{\tau}$

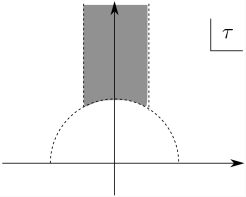


- We should mod out this symmetry

$$A = \int_{\mathcal{M}} \langle V_1 \cdots V_n \rangle = \int \langle V_1 \cdots V_n \rangle$$

$$\mathcal{T} = \{\tau | \mathrm{Im}\tau > 0\}$$

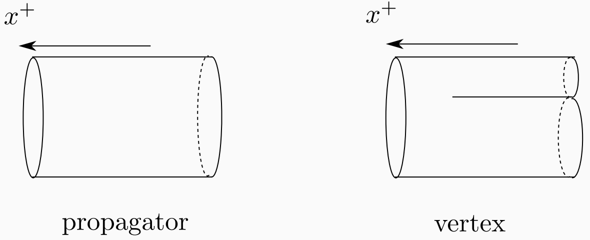
$$\mathcal{M} = \mathcal{T}/\text{modular group}$$



Light-cone gauge string field theory (SFT)

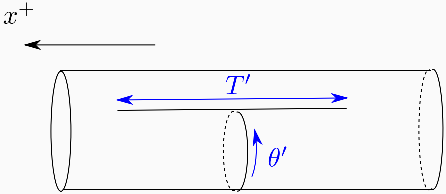
- The procedure in the previous slides is rather ad hoc.
- It is better to have a formulation in which we get the right answer without modding out the symmetry.
- The light-cone gauge SFT (Kaku-Kikkawa)

$$S = \int [\phi(\partial_{x^+} - H)\phi + \phi^3]$$



Ito-Onogi

- One-loop Feynman diagram for the light-cone gauge SFT



- They find the map $(T', \theta') \to \tau$ and show

$$\int_0^\infty dT' \int_0^{2\pi} d\theta' \langle V_1 V_2 \rangle = \int_{\mathcal{M}} \langle V_1 V_2 \rangle$$

- The light-cone gauge SFT yields the right answer.

Yoneya-san's oracle

- String theory we have at present is like the "old quantum theory".
- We should discover the "Schrödinger equation" for string theory.

- The light-cone gauge SFT will provide some insight into the "Schrödinger equation" .
- Yoneya-san himself wrote a paper on space-time local symmetry of the light-cone gauge SFT.

Light-cone gauge SFT

- Onogi-san studied the Lorentz algebra of the light-cone gauge SFT. (unpublished)
- from Kugo-san's paper "Lorentz Transformation in the Light-Cone Gauge String Field Theory"

17) T. Onogi, talk given at JPS annual meeting, 1987, held at Nagoya.

Note added: After completing this work the author received a preprint by Bengtsson and Linden[16] in which the Lorentz transformation M^{j-} of ϕ is constructed up to $O(g^1)$ by demonstrating the closure of the Poincaré algebra. Their result agrees with ours up to the order they have presented. The author also understands that Onogi[17] has essentially proved the Lorentz invariance of closed light-cone gauge SFT up to $O(g)$ in investigating the self-duality requirement in the case of torus compactification.

Dynamics

- Besides SFT things, Onogi-san worked on various aspects of string theory.
- We wrote two papers together on open string theory.
- After getting Ph.D, we both went to KEK as JSPS posdocs.
- At KEK, we had a lot of seminars and journal club talks on phenomenology, lattice theory, cosmology etc..
- Onogi-san was more and more inclined to study dynamics, probably influenced by people like Higashijima-san and Okawa-san.
- He started working on Nambu-Goto string on lattice. Hashimoto-san may tell us the rest of the story.

2. Mirzakhani's idea

Yoneya-san's oracle: 35+ years later

- String theory we have at present is like the "old quantum theory".
- We should discover the "Schrödinger equation" for string theory.

- Have we got the "Schrödinger equation" for string theory?
  $\longrightarrow$Not yet (in my opinion)
- The problem Onogi-san tackled is important and fundamental but actually very difficult.

String field theory

- For bosonic strings, we have SFT with a simple action

$$S = \phi Q \phi + \phi^3$$

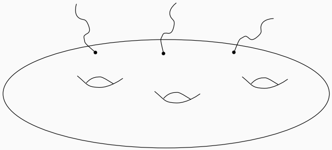
  - $(\alpha = p^+)$ HIKKO, covariantized light-cone
  - Witten's SFT

- Zwiebach proposed a SFT for closed bosonic strings with infinitely many interactions:

$$S = \phi Q \phi + \phi^3 + \phi^4 + \cdots + \hbar \phi^3 + \cdots$$

- If one tries to formulate SFT for superstrings, one runs into the "spurious singularity" problem. One should follow Zwiebach's formulation. Sen constructed such a SFT.
- We may need some new idea.

Volume of the moduli space



- In string theory, amplitudes are given by integrals over the moduli space of Riemann surfaces

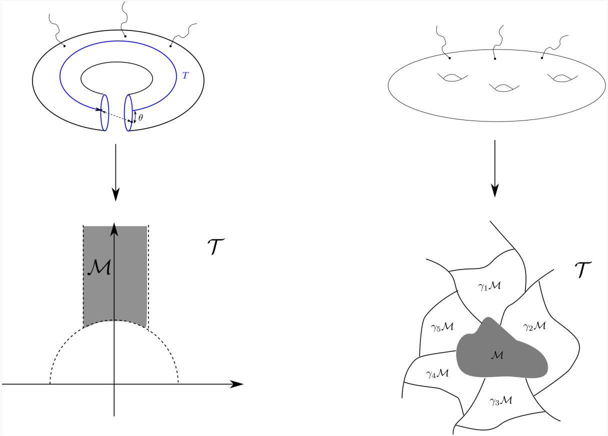
$$A_{g,n} = \int_{\mathcal{M}_{g,n}} dX \langle V_1 \cdots V_n \rangle$$

  - $\mathcal{M}_{g,n} = \mathcal{T}_{g,n}/\text{modular group}$

- Mathematicians were interested in calculating the volume of the moduli space

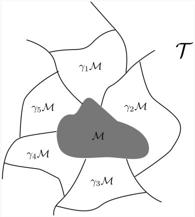
$$V_{g,n} = \int_{\mathcal{M}_{g,n}} dX$$

Volume of the moduli space



- Computing $V_{g,n}$ is very difficult for general g, n, because the fundamental domain of the modular group is very complicated.

Mirzakhani's idea



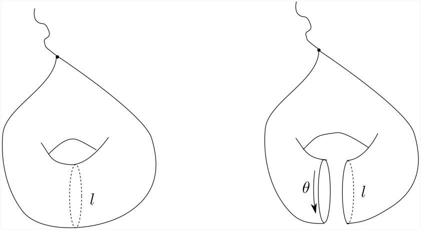
- Suppose we have a function $f(X)$ satisfying

$$\sum_{\gamma \in \text{modular group}} f(\gamma X) = 1$$

$$\begin{aligned} \int_{\mathcal{T}} dX f(X) &= \sum_{\gamma} \int_{\gamma\mathcal{M}} dX f(X) = \sum_{\gamma} \int_{\mathcal{M}} d(\gamma X) f(\gamma X) \\ &= \int_{\mathcal{M}} dX \sum_{\gamma} f(\gamma X) = \int_{\mathcal{M}} dX = V \end{aligned}$$

- The integral over $\mathcal{M}$ is unfolded to that over $\mathcal{T}$.

McShane identity ($g = n = 1$)

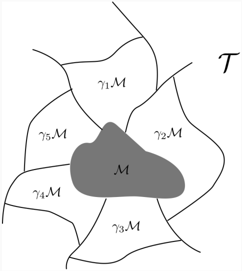


- Cut the surface along a closed geodesic (hyperbolic metric)
- $\mathcal{T} \longleftrightarrow l, \theta$ plane.
- McShane identity (1998): for $f(X) = f(l,\theta) = \frac{2}{1+e^l}$

$$\sum_{\gamma \in \text{modular group}} f(\gamma X) = 1$$

$$V = \int \frac{dl d\theta l}{2\pi} \frac{2}{1+e^l} = \frac{\pi^2}{6}$$

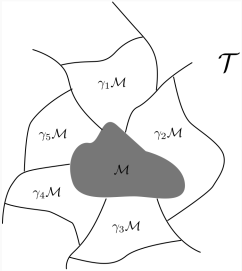
Mirzakhani's idea



- Mirzakhani generalized the McShane identity to general (g, n) and derived a recursion relation for $V_{g,n}$. (2007)
- Solving the recursion relation, one can compute $V_{g,n}$.

3. Fokker-Planck formalism for closed bosonic strings

String field theory



- So far, we were looking for SFT which yields amplitudes as integrals over $\mathcal{M}$.
- Is it possible to construct an SFT which yields amplitudes as integrals over $\mathcal{T}$ following Mirzaknani's idea?
- Yes, but we need to use the Fokker-Planck formalism.

Fokker-Planck formalism

- Euclidean field theory

$$\langle \phi(x_1)\cdots\phi(x_n)\rangle = \frac{\int [d\phi] e^{-S[\phi]}\phi(x_1)\cdots\phi(x_n)}{\int [d\phi] e^{-S[\phi]}}$$

- Fokker-Planck formalism

$$\langle \phi(x_1)\cdots\phi(x_n)\rangle = \lim_{\tau\to\infty}\langle 0| e^{-\tau \hat{H}_{\mathrm{FP}}}\hat{\phi}(x_1)\cdots\hat{\phi}(x_n)|0\rangle$$

$$[\hat{\pi}(x), \hat{\phi}(y)] = \delta(x-y)\,, [\hat{\pi},\hat{\pi}] = [\hat{\phi},\hat{\phi}] = 0$$

$$\langle 0|\hat{\phi}(x) = \hat{\pi}(x)|0\rangle = 0$$

$$\hat{H}_{\mathrm{FP}} = -\int dx \left(\hat{\pi}(x) + \frac{\delta S}{\delta\phi(x)}[\hat{\phi}]\right)\hat{\pi}(x)$$

- path integral

$$\langle \phi(x_1)\cdots\phi(x_n)\rangle = \frac{\int [d\pi d\phi] e^{-I_{\mathrm{FP}}}\phi(0,x_1)\cdots\phi(0,x_n)}{\int [d\pi d\phi] e^{-I_{\mathrm{FP}}}}$$

$$I_{\mathrm{FP}} = \int_0^\infty d\tau \left[-\int dx \pi \partial_\tau \phi + H_{\mathrm{FP}}\right]$$

$I_{\rm FP}$ for closed bosonic strings

$$
\begin{aligned}
I_{\rm FP} \quad = \quad & \int_0^\infty d\tau \Big[-\int_0^\infty dL \langle R|\pi_\alpha(L)\rangle \frac{\partial}{\partial\tau}|\phi^\alpha(L)\rangle \\
& + \int_0^\infty dL L \left(\langle R|\phi^\alpha(L)\rangle|\pi_\alpha(L)\rangle - \langle R|\pi_\alpha(L)\rangle|\pi_{-\alpha}(L)\rangle \right) \\
& - g_{\rm s} \int dL dL' dL'' {}_{123}\langle T_{LL'L''}|b_{-\alpha''} b_{\alpha'} b_\alpha |\phi^{\alpha''}(L'')\rangle_1 |\pi_{\alpha'}(L')\rangle_2 |\pi_\alpha(L)\rangle_3 \\
& - \frac{1}{2} g_{\rm s} \int dL dL' dL'' {}_{123}\langle D_{LL'L''}|b_{-\alpha'} b_{-\alpha''} b_\alpha |\phi^{\alpha'}(L')\rangle_1 |\phi^{\alpha''}(L'')\rangle_2 |\pi_\alpha(L)\rangle_3 \\
& + \int_0^\infty dL \Big(\langle R|\mathcal{Q}^\alpha(\tau,L)\rangle|\lambda_\alpha^{\mathcal{Q}}(\tau,L)\rangle + \langle R|\mathcal{T}^\alpha(\tau,L)\rangle|\lambda_\alpha^{\mathcal{T}}(\tau,L)\rangle \Big) \Big]
\end{aligned}
$$

$$
I_{\rm FP} \sim \int_0^\infty d\tau \left[-\pi\partial_\tau\phi + \pi\phi + \pi^2 + \phi^2\pi + \phi\pi^2 + \lambda J \right]
$$

- λ:auxiliary fields
  - $J \sim \phi + \pi + \phi\pi + \phi^2$
- $I_{\rm FP}$ is invariant under nilpotent BRST symmetry

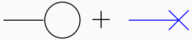
$S[\phi]$ for strings?

- In the conventional field theory, we have two formalisms.

$$\begin{aligned} \langle \phi(x_1)\cdots\phi(x_n)\rangle &= \frac{\int [d\phi] e^{-S[\phi]}\phi(x_1)\cdots\phi(x_n)}{\int [d\phi] e^{-S[\phi]}} \\ &= \frac{\int [d\pi d\phi] e^{-I_{\mathrm{FP}}}\phi(0,x_1)\cdots\phi(0,x_n)}{\int [d\pi d\phi] e^{-I_{\mathrm{FP}}}} \end{aligned}$$

- $S[\phi]$ is not well-defined in our setup.

$$S[\phi] \sim \phi^2 + \phi^3 + \hbar\phi + \cdots$$



$$\int_{\mathcal{T}} \langle V\rangle + \left(\int_{\mathcal{M}} \langle V\rangle - \int_{\mathcal{T}} \langle V\rangle\right) = \int_{\mathcal{M}} \langle V\rangle$$

4. Conclusion

Conclusion

- Onogi-san started his career working on string field theory.
- There is still much to be done in this field of study.