Multiloop amplitudes of light-cone gauge superstring field theory: Odd spin structure contributions

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SFT@HIT

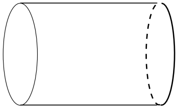
Light-cone gauge SFT for closed strings

- String field

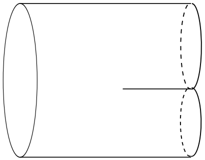
$$\Phi\left[x^{+}, p^{+}, X^{i}(\sigma)\right]$$

- Action

$$S = \int \left[\frac{1}{2}\Phi \cdot \left(i\partial_{x^{+}} - \frac{L_0 + \tilde{L}_0 - 1}{p^{+}}\right)\Phi + \frac{g_s}{3}\Phi \cdot (\Phi * \Phi)\right]$$

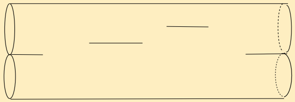


propagator



vertex

On-shell amplitudes for bosonic strings



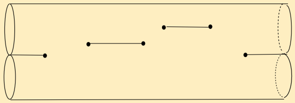
$$A^{\mathrm{LC}} = \int \prod_K dt_K F^{\mathrm{LC}}(t)$$

On-shell amplitudes coincide with the conformal gauge ones.

$$A^{\mathrm{conf.}} = \int \prod_a dm_a F^{\mathrm{conf.}}(m)$$

- t_K can be chosen to be the moduli parameters (Giddings-Wolpert)
- $F^{\mathrm{LC}}(t) = F^{\mathrm{conf.}}(t)$ (D'Hoker-Giddings)
- The integral itself is divergent.

On-shell amplitudes for superstrings



$$A^{\mathrm{LC}} = \sum_{\alpha_{\mathrm{L}}, \alpha_{\mathrm{R}}} \int \prod_K dt_K F^{\mathrm{LC}}(t, \alpha_{\mathrm{L}}, \alpha_{\mathrm{R}})$$

For superstrings, on-shell amplitudes

- with (NS,NS) external lines
- even spin structure

coincide with the conformal gauge ones. (Aoki-D'Hoker-Phong)

- The integral itself is divergent because of the contact term divergences.
- This can be remedied by dimensional regularization. (Murakami-N.I.)

On-shell amplitudes for superstrings

In this talk, I would like to show that the above results can be generalized to the odd spin structure case. (with (NS,NS) externall lines)

We would like to show

1. The LC amplitudes for odd spin structures can be recast into a conformal gauge expression.
2. Although the expression yields a divergent integral, we can make it well-defined by dimensional regularization.
   - Here we assume that there are no problems of mass renormalization or vacuum shift.

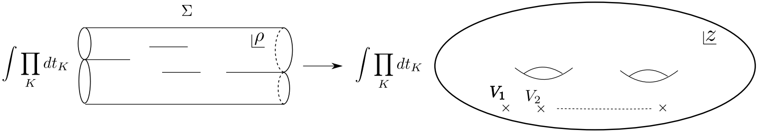
Based on Murakami-N.I. in preparation,

Outline

§1 LC gauge vs. conformal gauge

For bosonic strings ▸ GO

$$
\begin{aligned}
A^{\mathrm{LC}} &= \int \prod_K dt_K \, F^{\mathrm{LC}}(t) \\
F^{\mathrm{LC}}(t) &\propto \int_\Sigma \left[dX^i\right]_{\partial\rho\bar\partial\bar\rho} e^{-S^{X^i}} \\
&= e^{-\Gamma[\rho,\hat g_{z\bar z}]} \int \left[dX^i\right]_{\hat g_{z\bar z}} e^{-S^{X^i}} \prod_r V_r^{\mathrm{LC}}(Z_r,\bar Z_r)
\end{aligned}
$$



$ds^2 = \partial\rho\bar\partial\bar\rho dz d\bar z$ $\qquad$ $ds^2 = 2\hat g_{z\bar z} dz d\bar z$

$F^{\mathrm{LC}}(t) = F^{\mathrm{conf.}}(t)$

$$
\begin{aligned}
F^{\mathrm{conf.}}(t) \quad &= \quad \int \left[dX^{\mu} db dc\right] e^{-S^{\mathrm{conf.}}} \\
&\quad \times \prod_K \left[\oint_{C_K} \frac{dz}{\partial\rho} b_{zz} + \varepsilon_K \oint_{\bar{C}_K} \frac{d\bar{z}}{\bar{\partial}\bar{\rho}} b_{\bar{z}\bar{z}}\right] \prod_r \left[c\bar{c}V_r^{\mathrm{DDF}}\left(Z_r, \bar{Z}_r\right)\right]
\end{aligned}
$$

- V_r^{DDF} is a $(1,1)$ matter primary
- $\varepsilon_K = \pm 1$
- This can be shown by just performing the integrations over $X^{\pm}$ and b, c in $F^{\mathrm{conf.}}(t)$.

Bosonic strings

$$\int [dX^{\pm}]\, e^{-S^{\pm}} \prod_r V_r^{\mathrm{DDF}}(Z_r, \bar{Z}_r)$$

$$= Z_{X^{\pm}} \left\langle \prod_r V_r^{\mathrm{DDF}}(Z_r, \bar{Z}_r) \right\rangle^{X^{\pm}} \propto Z_{X^{\pm}} \prod_{r=1}^{N} V_r^{\mathrm{LC}}(Z_r, \bar{Z}_r)$$

$$\int [dbdc]_{\hat{g}_{z\bar{z}}}\, e^{-S^{bc}} \prod_{r=1}^{N} c\bar{c}(Z_r, \bar{Z}_r) \prod_{K=1}^{6g-6+2N} \left[\oint_{C_K} \frac{dz}{\partial\rho} b_{zz} + \varepsilon_K \oint_{\bar{C}_K} \frac{d\bar{z}}{\bar{\partial}\bar{\rho}} b_{\bar{z}\bar{z}} \right]$$

$$\propto (Z_{X^{\pm}})^{-1}\, e^{-\Gamma[\rho,\ \hat{g}_{z\bar{z}}]}$$

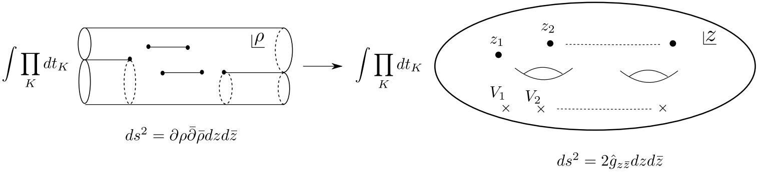
$$F^{\mathrm{conf.}}(t) \propto e^{-\Gamma[\rho, \hat{g}_{z\bar{z}}]} \int [dX^i]_{\hat{g}_{z\bar{z}}}\, e^{-S^{X^i}} \prod_r V_r^{\mathrm{LC}}(Z_r, \bar{Z}_r)$$

$$= F^{\mathrm{LC}}(t)$$

LC gauge amplitudes for type II strings

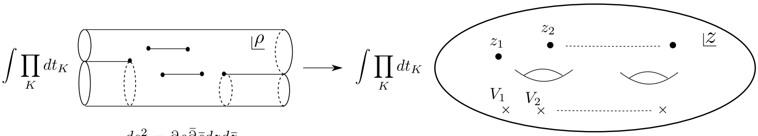
$$A^{\mathrm{LC}} = \sum_{\alpha_{\mathrm{L}},\alpha_{\mathrm{R}}} \int \prod_K dt_K F^{\mathrm{LC}}(t, \alpha_{\mathrm{L}}, \alpha_{\mathrm{R}})$$

- $\alpha_{\mathrm{L}}, \alpha_{\mathrm{R}}$: spin structures of left and right moving fermions ▸ GO
- Supercurrent for the LC variables $T_{\mathrm{F}}^{\mathrm{LC}}(z)$ are inserted at the interaction points.



LC gauge amplitudes for critical type II strings

$$F^{\mathrm{LC}}(t,\alpha_{\mathrm{L}},\alpha_{\mathrm{R}}) \propto e^{-\frac{1}{2}\Gamma[\rho;\hat{g}_{z\bar{z}}]}\int\left[dX^i d\psi^i d\bar{\psi}^i\right]_{\hat{g}_{z\bar{z}}} e^{-S^{\mathrm{LC}}\left[X^i,\psi^i,\bar{\psi}^i\right]}$$
$$\times\prod_{I=1}^{2g-2+N}\left(\left|\partial^2\rho(z_I)\right|^{-\frac{3}{2}} T_{\mathrm{F}}^{\mathrm{LC}}(z_I)\,\bar{T}_{\mathrm{F}}^{\mathrm{LC}}(\bar{z}_I)\right)$$
$$\times\prod_{r=1}^{N} V_r^{\mathrm{LC}}(Z_r,\bar{Z}_r)\;.$$



$ds^2=\partial\rho\bar{\partial}\bar{\rho}dzd\bar{z}$

$ds^2=2\hat{g}_{z\bar{z}}dzd\bar{z}$

$F^{\mathrm{LC}}(t,\alpha_{\mathrm{L}},\alpha_{\mathrm{R}}) = F^{\mathrm{conf.}}(t,\alpha_{\mathrm{L}},\alpha_{\mathrm{R}})$

$$\begin{aligned} F^{\mathrm{conf.}}(t,\alpha_{\mathrm{L}},\alpha_{\mathrm{R}}) \equiv & \int \prod_K dt_K \int \left[dX^\mu d\psi^\mu d\bar{\psi}^\mu dbdcd\beta d\gamma\right]_{\hat{g}_{z\bar{z}}} e^{-S^{\mathrm{tot}}} \\ & \times \prod_K \left[\oint_{C_K} \frac{dz}{\partial\rho} b_{zz} + \varepsilon_K \oint_{\bar{C}_K} \frac{d\bar{z}}{\bar{\partial}\bar{\rho}} b_{\bar{z}\bar{z}}\right] \prod_{I=1}^{2g-2+N} \left[X(z_I)\bar{X}(\bar{z}_I)\right] \\ & \times \prod_{r=1}^{N} \left[c\bar{c}e^{-\phi-\bar{\phi}} V_r^{\mathrm{DDF}}(Z_r,\bar{Z}_r)\right] . \end{aligned}$$

- V_r^{DDF} is a $\left(\frac{1}{2},\frac{1}{2}\right)$ matter primary in the (NS,NS) sector.
- $X(z) = \left[c\partial\xi - e^{\phi}T_{\mathrm{F}} + \frac{1}{4}\partial b\eta e^{2\phi} + \frac{1}{4}b\left(2\partial\eta e^{2\phi} + \eta\partial e^{2\phi}\right)\right](z)$
- The PCO's are inserted at the interaction points of the LC diagram.

$F^{\mathrm{conf.}} = F^{\mathrm{LC}}$

Proof involves two steps (Murakami-N.I.)

1. $X(z) = -e^{\phi} T_{\mathrm{F}}^{\mathrm{LC}}(z) + \triangle(z)$ One can show that $\triangle(z)$ does not contribute to the correlation function

$$
\begin{aligned}
F^{\mathrm{conf.}} &= \int \prod_K dt_K \int \left[dX^\mu d\psi^\mu d\bar{\psi}^\mu dbdcd\beta d\gamma\right]_{\hat{g}_{z\bar{z}}} e^{-S^{\mathrm{tot}}} \\
&\quad \times \prod_K \left[\oint_{C_K} \frac{dz}{\partial\rho} b_{zz} + \varepsilon_K \oint_{\bar{C}_K} \frac{d\bar{z}}{\bar{\partial}\bar{\rho}} b_{\bar{z}\bar{z}}\right] \prod_{I=1}^{2g-2+N} \left[e^{\phi} T_{\mathrm{F}}^{\mathrm{LC}}(z_I)\, e^{\bar{\phi}} \bar{T}_{\mathrm{F}}^{\mathrm{LC}}(\bar{z}_I)\right] \\
&\quad \times \prod_{r=1}^{N} \left[c\bar{c} e^{-\phi-\bar{\phi}} V_r^{\mathrm{DDF}}(Z_r, \bar{Z}_r)\right] .
\end{aligned}
$$

- One can find a nilpotent fermionic charge $\hat{Q}$, s.t. all the insertions are $\hat{Q}$ invariant and $\triangle(z) = \left\{\hat{Q}, \mathcal{O}(z)\right\}$

$F^{\mathrm{conf.}} = F^{\mathrm{LC}}$

2. Integrating over $X^{\pm}, \psi^{\pm}$ and ghosts, we get $F^{\mathrm{conf.}} = F^{\mathrm{LC}}$

$$\begin{aligned} \int \left[dX^{\pm} d\psi^{\pm} d\bar{\psi}^{\pm}\right] e^{-S^{\pm}} \prod_{r=1}^{N} V_r^{\mathrm{DDF}}(Z_r, \bar{Z}_r) &\sim Z_{X^{\pm}} Z_{\psi^{\pm}} V_r^{\mathrm{LC}}(Z_r, \bar{Z}_r) \\ (b, c \text{ part}) &\sim (Z_{X^{\pm}})^{-1} e^{-\Gamma[\rho,\ \hat{g}_{z\bar{z}}]} \\ (\beta, \gamma \text{ part}) &\sim \left(Z_{\psi^{\pm}}\right)^{-1} e^{\frac{1}{2}\Gamma[\rho,\ \hat{g}_{z\bar{z}}]} \prod_{I=1}^{2g-2+N} \left|\partial^2 \rho(z_I)\right|^{-\frac{3}{2}} \end{aligned}$$

$$\begin{aligned} F^{\mathrm{conf.}} &\sim e^{-\frac{1}{2}\Gamma[\rho;\hat{g}_{z\bar{z}}]} \int \left[dX^i d\psi^i d\bar{\psi}^i\right]_{\hat{g}_{z\bar{z}}} e^{-S^{\mathrm{LC}}\left[X^i, \psi^i, \bar{\psi}^i\right]} \\ &\qquad \times \prod_{I=1}^{2g-2+N} \left(\left|\partial^2 \rho(z_I)\right|^{-\frac{3}{2}} T_{\mathrm{F}}^{\mathrm{LC}}(z_I)\, \bar{T}_{\mathrm{F}}^{\mathrm{LC}}(\bar{z}_I)\right) \prod_{r=1}^{N} V_r^{\mathrm{LC}}(Z_r, \bar{Z}_r) \\ &= F^{\mathrm{LC}} \end{aligned}$$

§2 Odd spin structure

- In order for the above procedure to be well-defined we need

$$Z_{\psi\pm} = \left(\frac{\det'\left(-g^{z\bar{z}}\partial_z\partial_{\bar{z}}\right)}{\det \mathrm{Im}\Omega \int d^2z\sqrt{g}} \right)^{-\frac{1}{2}} \vartheta[\alpha_{\mathrm{L}}]\,(0)\,\vartheta[\alpha_{\mathrm{R}}]\,(0)^* \neq 0$$

- The theta function satisfies

$$\vartheta[\alpha]\,(-\zeta) = (-1)^{4\vec{\alpha}'\cdot\vec{\alpha}''}\,\vartheta[\alpha]\,(\zeta)$$

- α is called even or odd, depending on whether $4\vec{\alpha}' \cdot \vec{\alpha}''$ is an even or odd integer.

When the spin structure α_{L} is odd, for example, $\vartheta[\alpha_{\mathrm{L}}]\,(0) = 0$ and we are in trouble.

Odd spin structures

$$Z_{\psi^{\pm}} = \left(\frac{\det'\left(-g^{z\bar{z}}\partial_z\partial_{\bar{z}}\right)}{\det \mathrm{Im}\Omega \int d^2z\sqrt{g}}\right)^{-\frac{1}{2}} \vartheta[\alpha_{\mathrm{L}}](0)\,\vartheta[\alpha_{\mathrm{R}}](0)^*$$

$$\int [d\beta d\gamma]\, e^{-S_{\beta\gamma}} \prod_{I=1}^{2g-2+N}\left[e^{\phi}(z_I)\, e^{\bar{\phi}}(\bar{z}_I)\right]\prod_{r=1}^{N}\left[e^{-\phi}(Z_r)\, e^{-\bar{\phi}}(\bar{Z}_r)\right]$$

$$= \left(Z_{\psi^{\pm}}\right)^{-1} e^{\frac{1}{2}\Gamma[\rho,\ \hat{g}_{z\bar{z}}]}\prod_{r=1}^{N} e^{-\mathrm{Re}\,\bar{N}_{00}^{rr}} \prod_{I=1}^{2g-2+N}\left|\partial^2\rho(z_I)\right|^{-\frac{3}{2}}\ .$$

- $\vartheta[\alpha_{\mathrm{L}}](0) = 0$ implies that $\psi^{\pm}, \beta, \gamma$ possess zero modes $h_{\alpha_{\mathrm{L}}}(z)\,, \partial\rho h_{\alpha_{\mathrm{L}}}(z)\,, (\partial\rho)^{-1} h_{\alpha_{\mathrm{L}}}(z)$ where

$$h_{\alpha_{\mathrm{L}}}(z) = \sqrt{\sum_{\nu} \partial_{\nu}\vartheta\,[\alpha_{\mathrm{L}}]\,(0)\,\omega_{\nu}(z)}$$

- The conformal gauge expression involves a combination $0 \times \infty$ and is ill-defined.

§3 Amplitudes for odd spin structures

- In order to deal with the problem, we need to insert $\psi^{\pm}, \delta(\gamma), \delta(\beta)$ to soak up the zero modes.
- This can be achieved in a BRST invariant way by changing the pictures of some of the external lines.
- In the following, we consider the case when $\alpha_{\rm L}$ is odd and $\alpha_{\rm R}$ is even

Amplitudes for odd spin structures

The conformal gauge expression is taken to be

$$F^{\text{conf.}}(t,\alpha_{\text{L}},\alpha_{\text{R}}) = \int [dX^{\mu}d\psi^{\mu}dbdcd\beta d\gamma]_{g^{A}_{z\bar{z}}} e^{-S^{\text{tot}}} \times \prod_{K=1}^{6g-6+2N}\left[\oint_{C_K}\frac{dz}{\partial\rho}b_{zz}+\varepsilon_K\oint_{\bar{C}_K}\frac{d\bar{z}}{\bar{\partial}\bar{\rho}}b_{\bar{z}\bar{z}}\right]\prod_{I}[X(z_I)\bar{X}(\bar{z}_I)] \times V_1^{(-2,-1)}(Z_1,\bar{Z}_1)\,V_2^{(0,-1)}(Z_2,\bar{Z}_2)\prod_{r=3}^{N}\left[V_r^{(-1,-1)}(Z_r,\bar{Z}_r)\right],$$

$$V_r^{(-1,-1)}(Z_r,\bar{Z}_r) = c\bar{c}e^{-\phi-\bar{\phi}}V_r^{\text{DDF}}(Z_r,\bar{Z}_r)$$

$$V_1^{(-2,-1)}(Z_1,\bar{Z}_1) = -\frac{2}{p_1^+}c\bar{c}e^{-2\phi}e^{-\bar{\phi}}\psi^+V_1^{\text{DDF}}(Z_1,\bar{Z}_1),$$

$$V_2^{(0,-1)}(Z_2,\bar{Z}_2) = \left[-\frac{1}{2}c\bar{c}e^{-\bar{\phi}}\left(p_2^+\psi^-+\left(p_2^--\frac{N_2}{p_2^+}-\frac{Q^2}{2p_2^+}\right)\psi^+-\vec{p}_2\cdot\vec{\psi}\right)+\frac{1}{4}\bar{c}\gamma e^{-\bar{\phi}}\right]V_2^{\text{DDF}}$$

$F^{\text{conf.}} = F^{\text{LC}}$

Proof involves two steps

1. One can show that $X(z_I)$ can be replaced by $-e^{\phi}T_{\text{F}}^{\text{LC}}(z_I)$ and $V_2^{(0,-1)}$ by the first term

$$
\begin{aligned}
F^{\text{conf.}}(t,\alpha_{\text{L}},\alpha_{\text{R}}) &= \int \left[dX^{\mu}d\psi^{\mu}dbdcd\beta d\gamma\right]_{\hat{g}_{z\bar{z}}} e^{-S^{\text{tot}}} \\
&\quad \times \prod_{K=1}^{6g-6+2N} \left[\oint_{C_K} \frac{dz}{\partial\rho} b_{zz} + \varepsilon_K \oint_{\bar{C}_K} \frac{d\bar{z}}{\bar{\partial}\bar{\rho}} b_{\bar{z}\bar{z}}\right] \prod_I \left[e^{\phi}T_{\text{F}}^{\text{LC}}(z_I)\, e^{\bar{\phi}}\bar{T}_{\text{F}}^{\text{LC}}(\bar{z}_I)\right] \\
&\quad \times \left[-\frac{2}{p_1^+} c\bar{c}e^{-2\phi}e^{-\bar{\phi}}\psi^+ V_1^{\text{DDF}}\right](Z_1,\bar{Z}_1)\, V_2^{(0,-1)}(Z_2,\bar{Z}_2) \\
&\quad \times \left[-\frac{1}{2} c\bar{c}e^{-\bar{\phi}} p_2^+ \psi^-\right] V_2^{\text{DDF}}(Z_2,\bar{Z}_2) \\
&\quad \times \prod_{r=3}^{N} \left[c\bar{c}e^{-\phi-\bar{\phi}} V_r^{\text{DDF}}(Z_r,\bar{Z}_r)\right]
\end{aligned}
$$

$F^{\text{conf.}} = F^{\text{LC}}$

2. Integrating over $X^{\pm}, \psi^{\pm}$ and ghosts, we get $F^{\text{conf.}} = F^{\text{LC}}$

$$\int \left[dX^{\pm} d\psi^{\pm} d\bar{\psi}^{\pm}\right] e^{-S^{\pm}} \prod_{r=1}^{N} V_r^{\text{DDF}}(Z_r, \bar{Z}_r) \psi^{+}(Z_1)\, \psi^{-}(Z_2)$$

$$\sim Z_{X^{\pm}} \prod_{r=1}^{N} V_r^{\text{LC}}(Z_r, \bar{Z}_r) \left(\frac{\det{}'\left(-g^{z\bar{z}}\partial_z\partial_{\bar{z}}\right)}{\det \text{Im}\Omega \int d^2z\sqrt{g}}\right)^{-\frac{1}{2}} \vartheta[\alpha_{\text{R}}](0)^* \, h_{\alpha_{\text{L}}}(Z_1)\, h_{\alpha_{\text{L}}}(Z_2)$$

$$(b, c \text{ part}) \sim \left(Z_{X^{\pm}}\right)^{-1} e^{-\Gamma[\rho,\, \hat{g}_{z\bar{z}}]}$$

$$(\beta, \gamma \text{ part})$$

$$\sim e^{\frac{1}{2}\Gamma[\rho,\, \hat{g}_{z\bar{z}}]} \prod_{I=1}^{2g-2+N} \left|\partial^2\rho(z_I)\right|^{-\frac{3}{2}}$$

$$\times \frac{p_1^{+}}{p_2^{+}} \left(\frac{\det{}'\left(-g^{z\bar{z}}\partial_z\partial_{\bar{z}}\right)}{\det \text{Im}\Omega \int d^2z\sqrt{g}}\right)^{\frac{1}{2}} \left(\vartheta[\alpha_{\text{R}}](0)^* \, h_{\alpha_{\text{L}}}(Z_1)\, h_{\alpha_{\text{L}}}(Z_2)\right)^{-1}$$

$$\begin{aligned} F^{\text{conf.}} \sim\ & e^{-\frac{1}{2}\Gamma[\rho;\hat{g}_{z\bar{z}}]} \int \left[dX^i d\psi^i d\bar{\psi}^i\right]_{\hat{g}_{z\bar{z}}} e^{-S^{\text{LC}}\left[X^i,\psi^i,\bar{\psi}^i\right]} \\ & \times \prod_{I=1}^{2g-2+N} \left(\left|\partial^2\rho(z_I)\right|^{-\frac{3}{2}} T_{\text{F}}^{\text{LC}}(z_I)\, \bar{T}_{\text{F}}^{\text{LC}}(\bar{z}_I)\right) \prod_{r=1}^{N} V_r^{\text{LC}}(Z_r, \bar{Z}_r)\,. \\ =\ & F^{\text{LC}} \end{aligned}$$

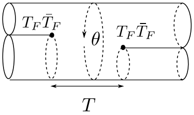
Contact term divergences

- The amplitude

$$A = \sum_{\alpha_{\rm L},\alpha_{\rm R}} \int \prod_K dt_K F^{\rm LC}(t,\alpha_{\rm L},\alpha_{\rm R}) = \sum_{\alpha_{\rm L},\alpha_{\rm R}} \int \prod_K dt_K F^{\rm conf.}(t,\alpha_{\rm L},\alpha_{\rm R})$$

is ill-defined because of the contact term divergences.

$\int dT d\theta$

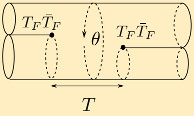


$T \to 0$

$\theta \to \theta_0$

Dimensional regularization

$\int dT d\theta$



$T \to 0$

$\theta \to \theta_0$

- The divergences can be regularized by dimensional regularization.
- By considering the theory in a linear dilaton background $\Phi = -iQX^1$, with a real constant Q, we can make the amplitudes well-defined for $Q^2 > 10$:

$$
\begin{aligned}
F^{\mathrm{LC}}(t, \alpha_{\mathrm{L}}, \alpha_{\mathrm{R}}) \quad \sim \quad & e^{-\frac{1-Q^2}{2}\Gamma[\rho;\hat{g}_{z\bar{z}}]} \\
& \times \int \left[dX^i d\psi^i d\bar{\psi}^i\right]_{\hat{g}_{z\bar{z}}} e^{-S^{\mathrm{LC}}\left[X^i,\psi^i,\bar{\psi}^i\right]} \\
& \qquad \times \prod_{I=1}^{2g-2+N} \left(\left| \partial^2 \rho(z_I) \right|^{-\frac{3}{2}} T_{\mathrm{F}}^{\mathrm{LC}}(z_I)\, \bar{T}_{\mathrm{F}}^{\mathrm{LC}}(\bar{z}_I) \right) \prod_{r=1}^{N} V_r^{\mathrm{LC}}(Z_r, \bar{Z}_r) \, .
\end{aligned}
$$

Dimensional regularization

- We can prove $F^{\mathrm{LC}}(t,\alpha_{\mathrm{L}},\alpha_{\mathrm{R}}) = F^{\mathrm{conf.}}(t,\alpha_{\mathrm{L}},\alpha_{\mathrm{R}})$

$$\begin{aligned} F^{\mathrm{conf.}}(t,\alpha_{\mathrm{L}},\alpha_{\mathrm{R}}) &= \int \left[dX^{\mu} d\psi^{\mu} dbdcd\beta d\gamma\right]_{g^{\mathrm{A}}_{\bar{z}\bar{z}}} e^{-S^{\mathrm{tot}}} \\ &\quad \times \prod_{K=1}^{6g-6+2N} \left[\oint_{C_K} \frac{dz}{\partial\rho} b_{zz} + \varepsilon_K \oint_{\bar{C}_K} \frac{d\bar{z}}{\bar{\partial}\bar{\rho}} b_{\bar{z}\bar{z}}\right] \\ &\quad \times \prod_I \left[X(z_I)\,\bar{X}(\bar{z}_I)\right] \prod_r e^{-\frac{iQ^2}{\alpha_r} X^{+}} \left(\hat{\tilde{\mathbf{z}}}_{I(r)}, \hat{\bar{\tilde{\mathbf{z}}}}_{I(r)}\right) \\ &\quad \times V_1^{(-2,-1)}(Z_1,\bar{Z}_1)\, V_2^{(0,-1)}(Z_2,\bar{Z}_2) \prod_{r=3}^{N} \left[V_r^{(-1,-1)}(Z_r,\bar{Z}_r)\right] . \end{aligned}$$

Dimensional regularization

- The longitudinal part is a super conformal field theory with $c = 3 + 12Q^2$ so that the total central charge vanishes.
- The LC amplitudes $A^{\mathrm{LC}}\left(Q^2\right)$ are well-defined for $Q^2 > 10$ and can be defined as analytic functions of Q^2.
- $A^{\mathrm{conf.}}\left(Q^2\right)$ can be made well-defined by avoiding the spurious singularities using Sen-Witten prescription for $Q^2 < 10$ and

$$A^{\mathrm{LC}}\left(Q^2\right) = A^{\mathrm{conf.}}\left(Q^2\right)$$

- $\lim_{Q\to 0} A^{\mathrm{LC}}\left(Q^2\right)$ is well-defined when there are no infrared divergences.

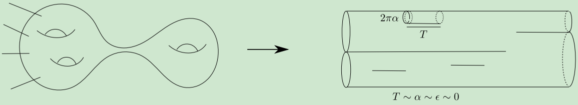
§4 Outlook

- We have shown that the odd spin structure contributions to the light-cone gauge amplitudes correspond to the conformal gauge expression using the vertex operators $V^{(-2,-1)}, V^{(0,-1)}$.
- The contact term divergences can be regularized by dimensional regularization.

- The wrong picture vertex operators $V^{(-2,-1)}, V^{(0,-1)}$?

Vacuum shift and mass renormalization

- Mass renormalization may be dealt with by making the external line off-shell.
- There exist no light-cone tadpole diagrams but there are divergences associated with them.



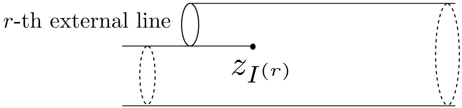
- We may have to deal with it in the same way as the UV divergences in usual field theory.

Jump

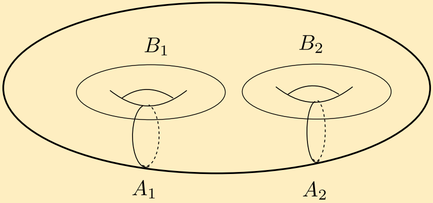
Anomaly factor ▸ BACK

$$e^{-\Gamma\left[\rho, g^{\mathrm{A}}_{z\bar{z}}\right]} \propto \prod_{r=1}^{N} \left[\alpha_r^{-1} (g^{\mathrm{A}}_{Z_r \bar{Z}_r})^{-\frac{1}{2}} e^{-\mathrm{Re}\bar{N}^{rr}_{00}}\right] \prod_{I=1}^{2g-2+N} \left[(g^{\mathrm{A}}_{z_I \bar{z}_I})^{-\frac{1}{2}} \left|\partial^2 \rho(z_I)\right|^{-\frac{1}{2}}\right]$$

- $g^{\mathrm{A}}_{z\bar{z}}$: Arakelov metric on the surface
- $r = 1, \ldots, N$ label the punctures
- $I = 1, \ldots, 2g-2+N$ label the interaction points, where $\partial\rho(z_I) = 0$.
- $\bar{N}^{rr}_{00} \equiv \frac{1}{p_r^+} \left(\rho(z_{I^{(r)}}) - \lim_{z \to Z_r} \left(\rho\left(z\right) - p_r^+ \ln\left(z - Z_r\right)\right)\right)$



Spin structure ▸ BACK



$$\alpha_{\mathrm{L}} = \begin{pmatrix} \alpha'_{\mathrm{L}1} \\ \vdots \\ \alpha'_{\mathrm{L}g} \\ \alpha''_{\mathrm{L}1} \\ \vdots \\ \alpha''_{\mathrm{L}g} \end{pmatrix}$$

- When z is moved around the cycles once, left moving fermion $\phi(z)$ transforms as

$$\begin{aligned} \phi(z) &\to e^{2\pi i \alpha'_{\mathrm{L}j}} \phi(z) \ \ (A_j \text{ cycle}) \\ \phi(z) &\to e^{2\pi i \alpha''_{\mathrm{L}j}} \phi(z) \ \ (B_j \text{ cycle}) \end{aligned}$$

  with $\alpha'_{\mathrm{L},j}, \alpha''_{\mathrm{L},j} = 0, \frac{1}{2}$. We label the spin structure by the vector α_{L}.

- α_{R} is defined in the same way.